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ENGINEERING ECONOMIC ANALYSIS

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BY

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ENGINEERING ECONOMIC ANALYSIS

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PREFACE

The young engineer is called upon early in his career to study various engineering projects. Some person in the company gets an idea which to him seems valuable. The young engineer is asked to study the idea and report upon it.

The report of the young engineer on the project should consist of two main parts: the technical solution and the economic solution. His training as an engineer has prepared him to discover the answer to the technical part of the project. The purpose of this text is to provide an understanding of the economics of engineering project analysis so that he can provide the economic part of the report.

The young engineer will seldom be asked to make a decision as a result of his study. He is asked simply to prepare a report on the project. This will be turned in to someone in authority above him, who will make the decision in terms of the material presented to him as interpreted by his own experience, idiosyncrasies, and more complete knowledge of some of the factors in the project.

The economic analysis of the project will, in general, consist of three separate yet definitely related analyses. The economy analysis will attempt to discover the worth-whileness of the project in terms of yield on the investment. The intangible analysis will attempt to discover the worth-whileness in terms of those factors which are difficult to evaluate because the factors of human judgment enter largely into their evaluation. The financial analysis will deal with the problem of providing funds with which to make the project possible.

The first two are definitely within the scope of training and experience of the young engineer. In fact, his primary training is in the technical analysis of the project. Practical experience in costs, wage rates, and prices of materials is essential to the economy analysis. His opinions concerning the evaluation of judgment factors are directly related to his length of contact with the projects studied and his fund of information concerning them and their relationships with society and industry. Very

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seldom does he have access to financial matters. Yet they should be understood and, hence, form part of the technique discussed.

It is assumed that the student or reader has a knowledge of the time value of money, commonly understood as the principles of compound interest. If he does not possess that understanding, then he should study it in Appendix A before proceeding beyond the introductory chapter. Appendix A presents the mathematical principles only. Applications of the mathematical principles will be presented in the text at appropriate places.

The author wishes to record the fact that his interest in this subject has been greatly stimulated by attendance at two of the Summer Schools for Engineering Teachers sponsored by the Society for the Promotion of Engineering Education. Particular acknowledgment is due C. A. Anderson, R. A. Hussey, and J. E. Crouch, members of the staff of the department of industrial engineering of The Pennsylvania State College, who tried out the material, here presented, for two years. J. E. Crouch also read and criticized the final manuscript. Thanks are also due to Miss Marguerite Shaffer for the preparation of the manuscript. Finally, acknowledgment must be made to Dean Emeritus R. L. Sackett whose encouragement and guidance are gratefully appreciated.

CLARENCE EDWARD BULLINGER.

STATE COLLEGE, PA.,
February, 1942.

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INTRODUCTION

THE PROBLEM

1. The Engineering Process.

The engineering process, carried on from the time a particular human need is recognized until the time when that need is satisfied through organized engineering effort, may be divided into a number of steps, each one of which calls for the exercise of a certain function. For example, suppose that a need is felt to supply a given human want by means of a product which can be produced under the direction of engineers. The whole process will include the following steps, which illustrate the main functions properly belonging to the engineer today:

1. *Research* on the problem of satisfying the need
2. *Invention* or adaptation of existing means and methods to perform the task efficiently
3. Preliminary *valuation* of the project
4. *Financial planning* of the project
5. *Organization* of effort for cooperation
6. *Promotion* of the project
7. *Design* of the project
8. Design and *construction* of plant and equipment
9. Supervision of operation and production, which also includes
    - *a.* *Maintenance* of plant
    - *b.* *Testing* of product
    - *c.* *Inspection* of plant
    - *d.* *Safety work*
10. *Sale* and *distribution* of the product.

Management may be considered from this point of view as being the art and science of directing and coordinating efficiently all these functions.

Now it matters little in this case whether the engineering product be a bridge or a telephone or an automobile; the functioning of research, invention, design, production, sale, and the rest are all required, and each function is performed according to the same principles wherever found. That is to say, a good bridge designer must not only know something about bridges, but he must also know the principles of design; a good sales engineer must know about his product, and he must also know how to sell. During such a project, a single engineer may perform one function, or several, or even, conceivably, all of them.[1]

2. An Example of the Engineering Process.—A need is felt for an apparatus to cool the air in a residence during the summer months. The working out of the engineering process would be somewhat as follows:

1. *Research.*—Studies would be made of all the literature on cooling. Search would be made through the records of the Patent Office to discover if any such devices had been patented. Other situations in which air is cooled would be studied to see if they offer a solution or the lead to a solution. The methods of refrigeration would be found to offer the principle necessary to a solution.

2. *Invention.*—Once the principle that appears to offer a solution has been discovered, study must be made in order to see how it may be applied. The adaptation of large-scale refrigeration apparatus to a small-scale installation in which noise is to be reduced to a minimum requires the ingenuity of the inventor engineer. A preliminary design of the apparatus is the result of this part of the process.

3. *Preliminary Valuation.*—The idea having proved to be a patentable one and it having been shown that the apparatus can be manufactured, further study is required in order to be sure that there will be a demand for it. How many people would desire to purchase such an air cooler? Would it find its greatest demand in the home or in the small business establishment? What are the potential demands for it? A study is made to determine the probable cost of manufacture and the probable selling price. A further study is made to determine how many persons or enterprises would be capable financially of purchasing the apparatus. Thus the probable demand is established.

4. *Financial Planning.*—It having been shown that the project is feasible, the matter of obtaining the necessary money and credit to finance the project must be studied. How much will it cost to set up the factory, man it for production, establish a sales force, and advertise and sell the apparatus?

5. *Organization.*—Some established enterprise may be asked to take over the project or a new enterprise may be established.

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[1] Hoover, T. J., The Structure of the Engineering Profession, *Jour. Eng. Education,* Vol. 25, No. 5, January, 1935.

Shall it be a single proprietorship, a partnership, or a corporation? Who shall be invited to go along with the inventor in the manufacture and sale of the apparatus?

6. *Promotion.*—In this phase of the process, the actual work of setting up the enterprise and obtaining the financial means is accomplished. Personnel is hired to man the enterprise.

7. *Design.*—The preliminary designs of the inventor are further developed and modified so that apparatus is planned that can be operated efficiently and may be manufactured at a reasonable cost.

8. *Design and Construction of Plant and Equipment.*—Methods of manufacture are determined and translated into equipment and industrial plant designs. These are then purchased, constructed, brought together, and installed in an industrial plant ready for manufacture.

9. *Supervision of Operation and Production.*—The organization that has been set up purchases raw materials, hires men, sets up production schedules, and produces the apparatus.

10. *Sale and Distribution.*—The organization set up for this purpose brings the new apparatus to the attention of the people, convinces them that they should buy, then sells and delivers the apparatus to them. It also services or sees to it that the apparatus, after having been installed, functions properly.

3. Engineering Economics.—At various places in the engineering process, the engineer is called upon to make decisions. These decisions should not be made on the basis of wishful thinking or of sudden thought or impulse. At each point in the process just described, the engineer is faced with the question of cost. The study and analysis of the economic or monetary aspects as well as other nonmonetary aspects of the foregoing functions are the field of *engineering economics.*

Usually several alternatives are discovered and suggested by the engineer in his study of a particular project. An economic study must be made in order to determine the advisability of adopting one of the proposed solutions. Such a study involves a comparison of costs and revenues pertaining to one plan with the costs and revenues of another plan or plans, or with some predetermined standard which may be used for comparison. It also includes a comparison of the nonmonetary aspects of the projects.

The problem of engineering economics is one of investment, namely, the determination of which of several proposed methods of carrying out a project is the most desirable. It is the problem of determining what combinations of goods and services will yield the greatest amount of satisfaction.

4. Types of Projects.—Engineering projects have been classified as follows by Professor Fish:[1]

1. Equipment required to provide a specified service
2. Method of operation of the equipment to provide a specified service
3. Rate of production of the equipment.

5. Equipment for a Specified Service.—Someone believes it a good idea to build a tunnel under the Delaware River from Philadelphia in Pennsylvania to Camden in New Jersey. He approaches the municipal authorities of the cities involved. He discusses the project in the daily newspapers. He arouses the public to the worth-whileness of the project. Those in authority become aware that the public favors such a project and make a study of it by means of a joint commission composed of persons from both cities as well as states. This commission, not being composed of engineers, probably engages consulting engineers and asks them to study the project and submit proposals. The engineers after much study have proposed several designs for the tunnel. The problem for engineering economy is to determine which of the several proposals is most desirable.

6. Method of Operation of Project.—The firm of consulting engineers when considering the above tunnel project is interested not only in the way in which the tunnel may be built but also in the way in which it may be operated. They, therefore, draw up plans for the operation of the tunnel after it has been built. In their consultation with the commission, they discover that it is possible to suggest several ways of operating the tunnel. It would be possible, for example, to let automobiles and trucks and busses pass through it one after the other just as they come along. Or, it would be possible to maintain two streams of traffic, automobiles in one stream and trucks in the other. The problem for engineering economy is to determine which of those two methods is the more desirable.

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[1] Fish, John C. L., "Engineering Economics," p. 5, Par. 112, McGraw-Hill Book Company, Inc., New York, 1923.

7. Rate of Production of Project.—When the method of operating the tunnel is considered, the question of speed of travel through it naturally arises. Should automobiles and trucks travel at the same speed? At what speed should each type of traffic travel? The speed at which the traffic moves may be considered as a problem in rate of production. Here, again, the problem of engineering economy is to determine which of the proposed rates of travel is most desirable.

8. Combination Projects.—In general, engineering projects like the one illustrated in the foregoing paragraphs involve all three phases or types of projects.

The family automobile is a good example of the fact that a particular project may be classified under all the types. When purchased to provide transportation for the family, the project is of the first type, a specified service. When a program of oiling, greasing, and general servicing is considered, the project is of the second type, method of operation. When it is to be decided at what speed the car is to be driven, the project is of the third type, rate of production.

9. The Technique of Engineering Economic Analysis.—The following chapters and sections of this book will develop the basic economic factors that appear in engineering projects, and their relationships with each other. They will suggest not one criterion but several, each with a particular point of view toward some factor or combination of factors.

It should be remembered that the technique developed in this text is presented for the purpose of providing some person with data and information prepared in such a way that it will assist him in making his decision. It should be borne in mind by the young engineer that, although he may be able to come to a decision from the data which he has collected, after all it is his superior who actually makes the decision. Hence the material prepared should include information on all the factors that may enter into the project. It should be borne in mind that the decision is influenced greatly by the sum total of current events, the environmental training, and the personal bias and inhibitions of the one who is authorized to make the final decision.

The technique of engineering economic analysis includes the following phases of the project:

1. The economy analysis

2. The intangible analysis
3. The financial analysis.

10. The Economy Analysis.—Such an analysis is the determination of all the factors that affect the economy of the project, measured in money units. It is aimed at finding out what it will cost to set up the project in good working order, what it will cost to keep it in good operating condition, what probable income will be received from it, and whether or not it is worth while doing anyway. It deals with the objective, measurable economic factors that are present in the project.

11. The Intangible Analysis.—This is an analysis of the facts concerning the project which have been found impossible to express in concrete figures or money values. As opposed to tangible facts, these are spoken of as intangible, subjective, or irreducible. They are also frequently referred to as judgment factors because the estimate of their relative importance depends upon the judgment of the person making the valuation.

12. The Financial Analysis.—This analysis is a study of the problem of financing a project. Its chief purpose is to determine the availability of the money necessary to finance the project and to determine which of several methods of financing the project is most desirable.

13. Relationship of the Several Analyses.—It is well not to confuse the economy analysis with the financial analysis. The economy analysis always comes first, followed by the intangible analysis. The economy analysis aims only to determine whether or not the project is worth while economically. It does not attempt to bring anything into the picture concerning the method of financing.

The intangible analysis closely follows the economy analysis and attempts to discover whether or not the project is worth while when consideration is given to those factors which are judgment ones depending upon human valuation.

The financial analysis follows the economy and intangible analyses and aims to discover the best method of financing. Obviously, it is impossible to make this analysis until it is known just what funds are necessary and this is an outcome of the economy analysis.

Thus there are three fundamental analyses to be made for any project. A favorable decision for any project implies that the several analyses are all favorable. It is quite possible to have a very favorable decision from an economy viewpoint but an unfavorable decision from an intangible one. Likewise, one may obtain favorable decisions from the economy and intangible analyses but get an unfavorable one from the financial report. Or, on the other hand, one might get an unfavorable decision from the economy analysis but favorable decisions from the intangible and financial analyses.

Thus it is practically impossible to make a decision based upon any one of the several analyses separately. They must all be made, studied, and related in the act of making a decision.

PART I

THE ECONOMY ANALYSIS

What is the cost of setting up the project?
What is the cost of operating the project?
What income may be expected for the project?
What is the return to the promoter from the project?
Is the project worth while?

CHAPTER 1

DEVELOPMENT AND PROMOTION COST

1.1. Development Cost.—Development cost is the sum of all the costs that are found necessary to prepare a project for consideration by a promoter. It is the cost incurred by an inventor or sponsor of a project up to the time that the project is accepted and plans made for its promotion by interested parties.

A suggested series of steps which are part of the development period and for which costs may have to be ascertained are[1]

1. Research on the problem of satisfying the need
2. Invention or adaptation of existing means and methods
3. Preliminary valuation of the project.

For illustrative purposes, consider the project of a tunnel under the Delaware River from Philadelphia to Camden. Consider a project for a vacuum cleaner for household use.

1.2. Research.—The need is voiced by persons having to go between Philadelphia and Camden for a better way to get there. Already they have the choice of a ferry service or a bridge service. What other way could there be for such travel? These people get together and make a study of the various methods of transportation as it is found under similar circumstances. They find that there is a comparable situation between the cities of New York and Hoboken. In order to have discovered this fact, it may have been necessary for someone to make a study of the cities, in the United States or other countries, that are located on the banks of rivers. Letters may have been written for information. Visits may even have been made to see the situation as it exists in these other cities. Thus the services of an investigator may be necessary to search for and to assemble the information as to how the job is being done in other places. An office may be set up and clerks hired. Letters will be written; the telephone used and telegrams or cablegrams sent out; plans and blueprints as well as photographs will without doubt be used.

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[1] Page 1.

The company that wishes to add vacuum cleaners to its line may also have expenditures for research. If it is already manufacturing a carpet sweeper, it may look around for some other means of cleaning carpets and rugs. In this case, the head of the company will probably appoint one of its engineering staff to make a study of the means of cleaning carpets. He will be allowed to set up an office and be given clerical assistance in making his study. He will have to make visits throughout the country. He will hear of this device and then of that device. He will investigate them, study them, and in the end make a report on their advisability. As in the case of the tunnel, he will have the usual office expenses.

In either case, the group of people or the industrial company may hire a professional firm of consulting engineers to make the study. This firm will charge the people who hire them a fee depending on the extent, difficulty, and time within which the research is to be made. The amount of information will also determine the remuneration.

1.3. Invention or Adaptation.—Is the tunnel scheme used in New York adapted to the Delaware River? More investigation, this time of a thoroughly technical nature, must be made. Problems involving the location of entrances, the composition of the river bed, the size of the tunnel, the materials to be used, the technical design, and numerous other matters are raised. Data are collected in sufficient quantity so that the feasibility of the project may be determined. Here again expenses of the general office variety are indicated. Engineers must be employed to make the investigations and plan the preliminary designs. Tests of the composition of the river bed may have to be made, involving the hiring of engineering firms that make a business of doing such work.

The carpet sweeper company, having studied the problem of carpet cleaning, has decided that it will manufacture a vacuum cleaner along the lines of its well-known principle. It, however, believes that a better vacuum cleaner can be built than any on the market. Accordingly it sets up an experimental department charged with the job of determining how this new cleaner shall be built. In the course of its design, the company discovers that a patent has been issued for a part of the device that they wished to use. It must, therefore, consider the payment of a fee or else find some other way to do the job.

An experimental shop may be set up. Machinery must be provided and mechanics hired. Draftsmen and experimenters must be employed. All the usual expenses that pertain to doing such work are indicated.

As in the case of research, so in this case, the company or the people may hire a firm of consulting engineers to do the job. In that case, the consulting engineers will charge a fee for their work.

1.4. Preliminary Valuation.—The group of people planning the tunnel have become very enthusiastic about their project and wish to obtain public backing for it. They realize, however, that before they go to the public with the idea, it is necessary for them to be able to prove that the project is wanted, is desirable, is economically sound, and is practicable. So they start another investigation. An investigatory group is set up to study the effect of the tunnel at various points in an attempt to show that traffic is congested. The volume of travel between the two cities is studied. The probability of diverting traffic from the present ferry and bridge service is considered.

The cost of operation of the tunnel is considered. How does it compare with the bridge or the ferry? Here again, an office force is required and the usual expenses incurred. What will the traveling public have to pay per trip? Likewise a consulting firm may do the work for a fee.

The vacuum cleaner company calls in its sales force. With much dramatization, it reveals to that group this new device that the experimental department has designed. What will be the demand for it? How will it sell compared with some other make? What interesting things in its design will attract the public? Another investigation is started to determine the answer to these questions. Again, the usual office expenses are involved. Here again, a consulting firm may do the work for a fee.

1.5. Promotion Cost.—Promotion cost is the sum of all the expenses found to be necessary to arrange for the financing and organizing of the business unit which will build and operate the project.

A suggested series of steps that are part of the promotion period, for which costs must be ascertained, is[1]

1. Financial planning of the project
2. Organization of effort for cooperation.

1.6. Financial Planning.—As a result of previous studies, the tunnel project has been shown to be feasible and at the same time highly desirable. The problem of financing is next. This involves a study into the legal aspects of the proposition. Two states and two cities are involved. Each has certain laws and regulations that may have a bearing on the project. Legal advice is secured through the employment of lawyers or by retaining as counsel the members of a law firm. The legality of the project having been determined and the steps necessary for its accomplishment having been laid out, the question of raising the funds comes next. The tunnel company is now ready to go before the public to raise the necessary funds with which to build the project. Usually an investment house handles the advertising work essential to bring before the public the opportunity to invest. It also takes care of printing the securities, their issuance, registry, sale, and all other expenses required to finance the project.

The vacuum cleaner company, convinced that it has a sure thing, may not need to call in a firm of lawyers. Its own organization should and probably has all the necessary talent to study and plan for the financing. True, if a new building is indicated, it becomes necessary to consult with the legal profession and with the investment people concerning their respective phases of the project.

Likewise, the vacuum cleaner company may have to acquire new funds to finance its new project. If it has sufficient capital already, of course, no such campaign is necessary. Note that in this case the advertisement of the new product is not considered; it is considered under the heading of selling expenses.

1.7. Organization.—In the case of the tunnel project, it is necessary that some organization be formed to carry on the work of building the tunnel and of operating it after it has been completed. The usual thing to do would be to form a corporation. Once again, legal advice is necessary. A charter for incorporation is asked from the courts. The charter having been granted, officers of the corporation must be selected and an operating organization set up. Someone must act as head, design the organization, secure the people to work in it, and see that it runs.

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[1] Page 1.

The vacuum cleaner company is not faced with so great a problem. It already has an organization. At best, all that it needs to do is to rearrange its present organization and possibly enlarge it.

In both of these cases, salaries must be paid for the persons who undertake the various things to be done and offices must be maintained.

1.8. Divisions of Development and Promotion Cost.—When the several divisions of development and promotion costs are studied, it will be seen that they fall into the following classifications: labor, materials, and expense.

1.9. Labor Cost.—By labor cost, in general, is meant the hire of any person to do whatever work he is skilled in doing. This includes physical as well as mental work. The work of the inventor, the investigator, the lawyer, the engineer, the designer, the draftsman, the clerk, the stenographer, the security salesman, and many others is included in this term. Labor cost is paid in various ways, such as salaries, wages, fee, hire, and commissions.

1.10. Materials Cost.—In this group belong the costs of everything of a material nature, such as are used by the several types of labor referred to in the preceding section.

1.11. Expenses.—In this group may be placed the cost of those things that are not either labor or materials. Such things include charges for transportation of both labor and materials, as well as charges for telephone, telegraph, and wireless messages. Another class of expense consists of things of a legal nature, as required by law, such as royalties, licenses, permits, charters, franchises, rights of way, rents, taxes, insurance, and others. Interest or borrowed monies which are had for short terms is an expense. Other miscellanecus costs are usually classed under this group.

1.12. Example of Development and Promotion Cost.—It is proposed to install a new piece of machinery in an industrial plan

Research.—The engineering department studies the possibilities of what to install.

Items of Cost.—Labor of members of the engineering department. Materials ordinarily required in an engineering office. Expense of operating the engineering office, which includes a part of general management expense.

Invention or Adaptation.—The engineering department adapts the data accumulated under Research to the project.

Items of Cost.—Items of cost are similar to those for Research.

Preliminary Valuation.—General manager or other high executives study the proposals developed under Invention or Adaptation and make a positive or negative recommendation.

Items of Cost.—Cost is a proportionate charge for general management.

Financial Planning.—The financial executive makes plans for the financing of the project.

Items of Cost.—Cost is a proportionate charge for general management.

Organization.—The organization already exists.

Items of Cost.—The cost of whatever changes are necessary is taken care of as a part of engineering and general management expense.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following projects, proposals, or situations has been brought to your attention.

You are asked to prepare an outline of the data that need to be collected, the estimates that must be made, and any pertinent information that seems necessary to enable you or your employers to evaluate the project, proposal, or situation and that will assist them in arriving at a decision and a plan of action.

Prepare a report on the development and promotion costs for your assigned project.

Projects, Proposals, or Situations

A. A proposed commercial hotel, 60 by 100 ft., with 250 rooms, having the usual facilities. It is to be located in a town of 25,000 inhabitants in a college community where the college population is 10,000 and the town population is 15,000.

B. A proposed central concrete-mixing plant. The materials are to be stored in a centrally and conveniently located building. They are placed in the concrete mixer mounted on a motor truck. The truck transports the mixer, while it mixes the materials, to the place where the concrete is wanted.

C. The selection of a type of prime mover for a 1,500-kw. central station from among Diesel and uniflow types of engines. Assume an installed capacity of 2,000 kw. in four 500 kw. units.

D. A proposed plant to manufacture milk-bottle caps. The plant capacity is to be 1,000,000 per day.

E. A proposed cold-storage locker plant, to have 320 lockers to be rented to families for the keeping of various foods by the quick-freezing process.

F. A proposed office building, 100 by 100 ft., with a basement, ground floor with stores, and 19 additional stories. The building is to have all the usual facilities of a modern office building.

G. A waterworks to supply a town of 13,000 population. The town is located in a valley between two mountain ranges. The plant is to be privately owned.

H. A college desires to build a power plant for the generation of steam for heating purposes and electricity for lighting and power purposes. Should it build a separate steam plant and purchase electricity or build a combined heating and electric power plant?

I. A proposed plant for the manufacture of carburetors. The capacity of the plant is to be 100,000 per year.

J. A proposed airport to serve a population of from 200,000 to 500,000. The airport is to have all the usual facilities, including a flying school.

K. The selection of an apartment project from two proposals, a 920- and a 940-room building. The apartment is to be designed for adequate living, including garages and recreational facilities.

L. A proposed toll bridge on a main highway crossing a river. The length of the bridge is to be about ½ mile. It may be privately owned.

M. The establishment of a fleet of delivery trucks for a manufacturing company. The estimated ton-mileage per year is to be 250,000.

N. The selection of a type of prime mover for a 1,500-kw. central station from among turbine and compound four-valve engines. Assume the installed capacity to be 2,000 kw. in four 500-kw. units.

O. A group of 25 boys, students at a college, propose to build a fraternity house. They wish to provide facilities for sleeping, eating, recreation, and social life.

P. A proposed toll road connecting two important cities which are 200 miles apart. The toll road will eliminate about 50 miles of additional distance by the use of tunnels under mountain ranges and will be so built as to eliminate many steep grades and dangerous curves. It is to be privately owned.

Q. The establishment of a blueprinting department for a large engineering concern. Two methods are under consideration: blueprinting and black-and-white printing. The engineering concern estimates its printing requirements to be equivalent to about 1,000 24- by 36-in. prints per 8-hr. day for 300 working days per year.

CHAPTER 2

CONSTRUCTION COST AND INVESTMENT

2.1. Construction Cost.—Construction cost is the sum of all the costs necessary to prepare a construction project for operation. Construction projects are of two types. One type is that which supplies a product or service and requires men to operate it; such as factories, power plants, hotels, garages, apartment houses, stores, and many others. The other type is that which produces a service and requires men to maintain it; such as dams, reservoirs, highways, waterworks, and sewage-disposal plants.

A suggested series of steps, which are part of the construction period and for which costs must be ascertained, is[1]

1. Design of construction phase of project
2. Construction of plant.

2.2. Design of Construction Phase of Project.—The tunnel company having been organized and finances having been arranged, the actual work of building may be begun. Up to this time, there has been a preliminary design, which was made when the project was first being considered. It, of course, was not detailed, but general. Detailed plans and specifications must be made so that the tunnel can be built. This naturally calls for the setting up of a design division or the hiring of consulting engineers. In case a firm of consulting engineers is hired, a fee is paid, which covers the usual expenses of an engineer's office. There are salaries for draftsmen and engineers as well as for clerks. Rent may have to be paid for the office. Compensation insurance will be required. Social security will have to be taken care of. Various other expenses will arise in this connection.

The vacuum cleaner company will transfer the developed design from its experimental department to its engineering design department. Here the design presented by the development engineers will be worked over by the design engineers and the production engineers and working plans made for its actual production. Again the usual expenses of engineering departments are incurred. In this case, the engineer will also design the manufacturing processes that are necessary to make the vacuum cleaner. The manufacturing design will be translated into the design of the industrial plant, including equipment and machinery.

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[1] **Page 1.**

2.3. Construction of Plant.—The construction of a plant will entail numerous costs of various kinds. If the building of the tunnel is let to a construction company, the contract price is then the cost of the tunnel. If it is built by a construction gang which it organizes itself, then the same costs that the construction company incurs will be incurred by the tunnel company. What are these costs? Materials will account for a large part of them. All kinds of labor will be employed to perform the various jobs required in the building of the tunnel. Machinery of various kinds, drills, air compressors, cranes, trucks, conveyors, will all be employed in this work. Machinery is paid wages just the same as labor is; its pay is called burden, overhead, or expense. The company may have to get permission from the cities to build the tunnel. It may have to acquire permission to cross over adjacent property by means of an easement. It may require permission to pile materials on adjacent property. The expense of obtaining these permissions is a charge to construction cost.

The vacuum cleaner company may have to build an addition to its existing plant. It may have its own engineering department do the work or give the job to a firm of engineering builders. In either case, the cost of this work is charged to construction. Like the tunnel project, there will be the usual costs of materials, labor, etc.

In both examples, the provision of equipment and machinery is considered a part of construction. The cost of acquiring the machinery and equipment consists of the price paid for the machinery, plus the cost of transportation and installation. The cost of materials includes the cost of transportation.

It is also usual to consider certain other charges incident to this part of the process as construction cost. The plant may require a certain period during which it is "broken in"; that is, various adjustments and trial operations may be made before the plant is ready to operate and produce the service for which it was designed. These charges include salaries for executives and others, expenses incurred during this period.

Usually it is necessary to pay for the cost of construction as the work progresses. In some cases it may be necessary to have all the money required to build the plant on hand and kept intact in one fund. Since the investor will of necessity forego or be unable to make any earning on this money, the interest on it during construction may be charged in as first cost. If the money is placed at interest, then such earned interest should be deducted from the charge for interest during construction. In either case, land or real estate will have to be secured or leased.

2.4. Divisions of Construction Cost.—When the several divisions of construction costs are studied, it will be seen that they fall into the following classifications: labor, materials, and expenses.

2.5. Labor Cost.—Labor cost was previously defined as the hire of any person to do whatever work he is skilled in doing. As thought of in connection with construction, it is the cost of the labor that is used directly in the construction of the project. It refers particularly to the labor that does the actual work. Such work may be done by laborers, carpenters, concrete workers, electricians, masons, plumbers, painters, truck drivers, crane and shovel operators, and many others. It does not, in general, include those persons or that form of labor which directs the work or which administers and manages the construction of the project.

2.6. Materials Cost.—Materials for construction cost are here the materials of construction. They may be lumber, sand, cement, stone, brick, plaster, electrical, plumbing, and heating supplies and equipment, and numerous others. It does not, in general, include office and drafting supplies. It also includes the cost of all equipment and machinery.

2.7. Expenses.—This division of construction costs includes costs for those things and services which it is difficult to allocate directly to some other category.

Labor is represented as "indirect" in such things as the administration and management group, foremen, draftsmen, office force, and engineers. Materials are represented as indirect in such things as office and drafting-room supplies, stationery, etc.

The cost of services and things that cannot readily be classified as labor or materials is also a part of this group. Such things are licenses, permits, charters, fees, franchises, rights of way, rents, taxes, insurance, lawyer's fees, consultant's fees, telephone and telegraph charges, and many others. Depreciation is also an expense.

2.8. Example of Construction Cost.—It is proposed to install a new piece of machinery in an industrial plant. The installation requires a new building.

Design of the Plant.—The engineering department studies what it is required to build.

Items of Cost.—Labor of members of the engineering department. Materials ordinarily required in an engineering office. Expense of operating the engineering office, which includes a part of management expense.

Construction of Plant.—This operating may be done by either of two organizations.

A. Construction Department of Company.—Labor: workmen directly employed on the project. Materials: construction materials of all kinds, concrete foundations, electric wiring, etc. Expenses: indirect labor; the proportionate share of supervisors and superintendent as well as office clerks.

Items of Cost.—Indirect materials: the usual office supplies and other things not directly chargeable to the specific project. Miscellaneous expenses: transportation for materials, equipment, etc.; depreciation charges for construction equipment; various legal papers such as permits, workmen's compensation, insurance, and others; interest on money tied up during construction; land.

B. Outside Contractor.—Practically the same items as in the preceding analysis plus the contractor's profit.

2.9. First Cost.—First cost in a project or enterprise may be defined as the sum total of the development, promotion, and construction costs. The inclusion of development costs depends upon the type of project under consideration, whether it is construction or production.

In the case of a construction project it is quite evident that development cost is a part of the first cost because it was a necessary prelude to the setting up of the project and because it is a cost that cannot be recovered in any other way than through the usual method of depreciation.

In the case of a production project, it is quite evident that development cost is not a part of the first cost. For, when the product ceases to be manufactured and sold, there is no justification for earning a return on its development cost. Development cost must be recovered, since it was incurred previous to production. Since it is not a part of first cost, the only way that it can be recovered is to include it as an operating cost and prorate it over an estimated amount of production. First cost for a production project is the sum of the development, promotion, and construction cost of the manufacturing plant.

2.10. Investment.—Investment in a project or enterprise may be defined as the sum total of first cost plus circulating capital. Investment is the total obligation upon which the enterprise must make a satisfactory return. It is that amount at which the investor looks and asks, "Will it pay me to make the investment?"

2.11 Circulating (Working) Capital.—Circulating or working capital may be defined as those funds that are required to make the enterprise or project a going concern. Development, promotion, and construction costs are sufficient to set up the project without any life in it.

Initial circulating capital permits the project to begin functioning and to function while it is supplying the product or service and before it is able to realize any income on its sales.

Regular circulating capital is that which is needed after the project has got started and has been running for sufficient time to normalize its operations. It is usually less than initial circulating capital.

Every business must be able to buy materials and labor and items of expense in order to operate. It is usually impossible to sell all of the product immediately as soon as it is produced; much of it must be stored. The cost of labor, materials, and expense to produce these goods must be paid for in many cases before the products can be sold. There is thus seen to be a considerable requirement of money so that the business can maintain such an inventory.

Much of the business is conducted on credit. This means that, when a bill of goods is sold to a customer, he has either to pay spot cash or be allowed 30, 60, or 90 days within which to settle. The company giving 30 days' credit must have sufficient funds because it has permitted an extension of 30 days in addition to the time of production within which to be paid for it. The company giving longer terms of credit must have large amounts of funds; the company doing a cash business needs only small amounts.

The amount of working capital depends upon the character of each business. The two principal factors are, as indicated above, credit terms to buyers of the company's product and the length of the inventory period. If the product is a seasonal one, the requirements are for a large and long-held inventory and, in consequence thereof, a large working capital. It has been estimated that for some chemical manufacturing plants the working capital may be equivalent to the first cost of the plant.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following projects, proposals, or situations has been brought to your attention. *See problems for Chap.* 1.

You are asked to prepare an outline of the data that need to be collected, the estimates that must be made, and any pertinent information that seems necessary to enable you or your employers to evaluate the project, proposal or situation and that will assist them in arriving at a decision and a plan of action.

Prepare a report on the construction cost and investment costs for your assigned project.

CHAPTER 3

DEPRECIATION COST AND DEPLETION COST

3.1. Depreciation Cost.—Depreciation cost may be defined as the annual charge that is made, during their period of useful life, for the maintenance of the investment in buildings, machinery, and equipment or the recovery of the investment at their retirement from service.

By maintenance is meant the keeping of the investment in buildings, equipment, and machinery in up-to-date condition and in line with improvements in the art. It means the continual renewal or replacement of the physical parts of the investment as times goes on. The project in which the investment is made is looked upon as a continuing one. It, therefore, must be continually remodeled, rebuilt, and renewed. Implied in the meaning of maintenance is the idea that the investment is always being kept at its most efficient earning power.

Repairs to the investment are for the purpose of keeping it in a productive condition. The depreciation charge is for keeping up with improvements by replacement.

By recovery is meant the return of the money value invested in buildings, equipment, and machinery when the project is of short life and where it is not expected to renew or replace the physical parts of the investment. In addition to short-lived projects, the idea of recovery also applies to investments in which there is a using up of the investment through depletion of the natural resources such as in mines or timber stands.

3.2. Meaning of Retirement.—Retirement, may be defined as the withdrawal from service of a unit of property at the end of its useful life. It is due mainly to the following causes:[1]

I. Physical causes
    A. Sudden physical damage, due to:
        1. Miscellaneous accidents
        2. Disasters.

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[1] Marston, A., and T. R. Agg, "Engineering Valuation," p. 83, McGraw-Hill Book Company, Inc., New York, 1936.

B. Physical decrepitude, due to:
    1. Physical deterioration
    2. Wear and tear
II. Functional causes, due to:
    A. 1. Inadequacy
        2. Obsolescence
        3. Supersession.

Retirements because of sudden physical damage are not predictable. Retirements because of any of the other causes are decided by the judgment of those in charge of operations.

Physical deterioration is caused by various chemical and mechanical effects on the materials of the property. Age is generally the prime factor. Wear and tear is caused by abrasion, friction, impact, vibration, or fatigue of materials. It is measured by the use rather than by age.

Inadequacy is simply lack of capacity. Obsolescence is due to the development and invention of improved devices designed to give the same general service. Supersession is the replacement of one piece of equipment by another that performs the same work but with greater efficiency.

3.3. Determination of Depreciation Cost.—Depreciation cost, is a function of the life and the first cost of the building or equipment.

The first cost is readily determinable. It is the invoice cost of the item, plus transportation charges to the site, plus installation charges, plus those costs necessary to put it into operating condition.

The life of the item is one of the most difficult things to determine. The policy of one industrial manufacturing company with regards to life is as follows:

Item	Life, Years
General-purpose machinery and equipment	10
Moderately specialized machinery and equipment	5
Specialized machinery and equipment	3
Highly specialized machinery and equipment	1

The first two classes in the tabulation may be thought of as being subject to depreciation because of physical causes. In such cases the life is easy to predict and its accuracy is very good. The second two classes may be thought of as being subject to depreciation because of functional causes. In such cases the life is difficult to predict with any high degree of accuracy. Consequently conservative practice predicts shorter lives for items subject to functional causes of depreciation.

The amount to be maintained or recovered depends upon whether or not the item under consideration will have any salvage value at the expiration of its estimated life. Salvage value is the estimated value at the end of the estimated economic life. Secondhand value is resale value at any time or what some other user of the item would be willing to pay for it. Scrap value is what a junkman would be willing to pay for the materials that are in it.

Salvage value is modified if any cost is involved in removing the item. In this case, if the cost of removal is sufficiently high, the salvage value may approach the scrap value and may even become zero. The amount to be maintained, therefore, will be equal to the first cost or investment minus the salvage value.

There are a number of methods in common use for the determination of depreciation cost. The most important of these are, the straight-line method, the sinking-fund method, and the production-units method. Other methods are percentage on diminishing value, the Gillette method, the Matheson method, and the Cole Method.[1]

3.4. Straight-line Method of Calculating Depreciation Cost.—This method is the simplest one to use.

$$\text{Annual depreciation cost} = \frac{\text{first cost} - \text{salvage value}}{\text{life in years}} \tag{3-1}$$

Example: First cost: \$20,000
Salvage value: 1,000
Life: 10 years

$$\text{Annual depreciation cost} = \frac{\$20,000 - \$1,000}{10} = \$1,900$$

3.5 Sinking-fund Method of Calculating Depreciation Cost.—This method assumes that the annual charge for depreciation is deposited in a fund drawing compound interest. A certain amount is deposited yearly and left in the fund at compound interest during the entire life of the project. The amount in the fund at the end of the life period is to be equal to the depreciated value of the structure or the difference between first cost and salvage value.

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[1] ALFORD, L. P., "Cost and Production Handbook," The Ronald Press Company, New York, 1934.

$$\text{Sinking-fund amount} = \text{first cost} - \text{salvage value} \tag{3-2}$$

$$\begin{pmatrix}\text{Annual}\\ \text{depreciation}\\ \text{cost}\end{pmatrix} = \begin{pmatrix}\text{sinking-fund}\\ \text{amount}\end{pmatrix}\left[\frac{i}{(1+i)^n - 1}\right]^* \tag{3-3}$$

Example: First cost: \$20,000
Salvage value: 1,000
Life (^{n}): 10 years
Interest rate (i): 3%

$$\text{Sinking-fund amount} = \$20{,}000 - 1{,}000 = \$19{,}000$$

$$\text{Annual depreciation cost} = (\text{sinking-fund amount})\left[\frac{i}{(1+i)^n - 1}\right]$$

Substituting,

$$\text{Annual depreciation cost} = (\$19{,}000)(0.0872) = \$1656.80$$

It is obvious that the rate of interest will affect materially the amount of the annual cost. If the interest rate is decreased from 8 to 6 per cent and then to 4, 2, and 0 per cent successively, the annual depreciation cost will change as indicated below:

Interest Rate, Percentage	Annual Depreciation Charge
8	\$1,311.00
6	1,442.10
4	1,582.70
2	1,734.70
0	1,900.00

Thus it is seen that, when the rate of interest becomes zero, the sinking-fund method becomes the straight-line method. An economy study should be made in the most conservative manner. Therefore, from the foregoing tabulation, it is seen that the degree of conservatism increases as the rate of interest used decreases until it reaches zero rate of interest. The most conservative practice, therefore, in an economy study is to evaluate the depreciation cost by the straight-line method.

3.6. Production-units Method of Calculating Depreciation Cost.—In this method the life is expressed in the number of units that may be produced.

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* See Appendix A, *S* series, p. 323.

$$\text{Depreciation cost} = \frac{\text{first cost} - \text{salvage value}}{\text{number units produced}} \tag{3-4}$$

$$\text{Depreciation cost} = \frac{\$2,000}{100,000} = \$0.02 \text{ per unit}$$

3.7. Accumulation of Depreciation Cost.—The maintenance or recovery of the investment in buildings, machinery, and equipment implies that the charge for depreciation cost must be collected year by year during the useful life of the item. One way to do this is for the enterprise or project to pay out in actual cash the amount of the annual depreciation cost into a fund which is kept for the specific purpose of accumulating such amounts. In this manner, when the item is retired there will have been collected the original cost of the item, and the money will be available toward the replacement of it.

The annual depreciation cost in this way is determined by using the sinking-fund method. The interest rate to be used would be the rate that could be obtained from the particular repository or from the reinvestment of these funds. Obviously, the rate of interest will, in general, be lower than the yield that the project makes on its investment. For this reason businessmen have been unwilling to set aside these funds at such a low interest rate and have contended that it is better to use a second way.

A second way in which the accumulation of depreciation cost is accomplished is one that keeps these funds in the business. In this method, a special account is set up in the books of the accounting system known as Reserve for Depreciation. In it are recorded, the annual depreciation cost charges against the property during each year of its life. Since the money representing the depreciation cost is not paid out of the business into a fund, it finds its way into the cash assets of the business. If at the end of any particular year, there is sufficient cash that could have been deposited into a fund but was not, then such cash augments the circulating capital and, if large enough, may diminish the need to borrow additional circulating capital. In consequence, this particular cash amount, representing depreciation cost, gets lost in the course of time in the assets of the business. The balance of the Reserve for Depreciation account may, therefore, be reflected in any of the assets of the business.

Such assets may be accounts receivable, notes receivable, materials inventory, or items of equipment.

It is easy to see that, by the first method, the money will be readily available when desired. In the second case, however, the money may not be readily available since the accumulated charges for depreciation represented by the amount in the Reserve for Depreciation may be in any one of the items of assets mentioned in the preceding paragraph.

3.8. Depreciation Cost and Federal Income Tax Regulations.—The relation of depreciation under the regulations of the Federal income tax regulations is shown by the following quotation from T. D. 4422:[1]

> Art. 205. Method of computing depreciation allowance. The capital sum to be recovered shall be charged off the useful life of the property, either in equal annual installments or in accordance with any other recognized trade practice, such as an apportionment of the capital sum over units of production. Whatever plan or method of apportionment is adopted must be reasonable and must have due regard to operating conditions during the taxable period. The reasonableness of any claim for depreciation shall be determined upon the conditions known to exist at the end of the period for which the return is made. Where the cost or other basis of the property has been recovered through depreciation or other allowances no further deduction for depreciation shall be allowed. The deduction for depreciation in respect of any depreciable property for any taxable year shall be limited to such ratable amount as may be reasonably considered necessary to recover during the remaining useful life of the property the unrecovered cost or other basis. The burden of proof will rest upon the taxpayer, to sustain the deduction claimed. Therefore, taxpayers must furnish full and complete information with respect to the cost or other basis of the assets in respect of which depreciation is claimed, their age, condition and remaining useful life, the portion of their cost or other basis which has been recovered through depreciation allowances for prior taxable years, and such other information as the Commissioner may require in substantiation of the deduction claimed.

3.9. Land and Wasting Assets.—Land cost includes, in addition to the original cost of real estate, the expenses made necessary for filling and grading and clearing the site and other carrying charges especially applicable to land during the period of organization. For pieces of land used as business sites it is assumed that the land is imperishable. In consequence, thereof, investment in land does not have to be maintained or recovered and hence no depreciation cost is incurred. Natural resources that are exhausted by the process of extraction are termed "wasting assets."

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[1] Issued by the Treasury Department, Office of the Commissioner of Internal Revenue, Washington, D. C.

3.10. Depletion Cost.—Depletion cost may be defined as the annual charge that is made for the maintenance of the investment in the wasting assets, such as mines, oil and gas wells, and timber projects, to cover the portion of such assets extracted or "cut" during the period.

Problems

1. A manufacturing company purchases an engine lathe for $3,500. The freight and haulage cost $100; installation charges amount to $150. Determine the following:

- *a.* Depreciation charge—straight-line method. The life is 15 years; the salvage value at end of life is $300.
- *b.* Depreciation charge—sinking-fund method. Use the same data as in (*a*). Annual interest rate is 3½ per cent.

2. An industrial company has a power plant consisting of a Diesel engine. Its first cost was $65,000, of which $1,000 was freight and haulage and $2,000 was installation charges. Determine the following:

- *a.* Depreciation charge—straight-line method. The life is 15 years; salvage value at end of life is $3,000.
- *b.* Depreciation charge—sinking-fund method. Same data as (*a*) and annual interest rate of 3 per cent.

3. A small water company has installed a gasoline-driven pump. Its purchase price was $1,000, freight and haulage charges were $50, and installation charges were $75. Determine the following:

- *a.* Depreciation charge—straight-line method. The life is 5 years; salvage value at end of life is $100.
- *b.* Depreciation charge—sinking-fund method. Same data as (*a*) and annual interest rate of 2½ per cent.

4. The Smith Construction Company has purchased a concrete truck mixer (including a truck) having 4 cu. yd. capacity, at a cost of $7,000, including freight charges of $200. Determine the following:

- *a.* Depreciation charge—straight-line method. The life is 4 years; no salvage value.
- *b.* Depreciation charge—sinking-fund method. Same data as (*a*) and interest at 4 per cent annually.

CHAPTER 4

VALUATIONS AND APPRAISALS

4.1. Valuation or Appraisals.—Valuation or appraisal is the art of estimating the fair exchange value of specific properties. The engineer will be confronted with the problem of valuation,

1. When purchasing or selling secondhand buildings, machinery, or equipment
2. When consolidation or merger of going industrial enterprises is under consideration
3. When buying or selling extractive industries, such as mines, quarries, timber, oil and gas properties
4. For fire-insurance purposes
5. For Federal income tax purposes
6. For Section 77 B of the Bankruptcy Act
7. For condemnation proceedings
8. For Securities Act registration
9. For the determination of fair value of a public utility
10. When determining the value of a bond
11. For penalties and bonuses.

4.2. Valuation of Used or Secondhand Equipment.—There are several viewpoints from which the valuation of used or secondhand equipment may be viewed.

1. *Salvage Value.*—This is the secondhand value. It is that sum of money which the machine will bring when sold to some other company or secondhand dealer. It is determined after giving consideration to several aspects surrounding the equipment, as follows:

a. Present condition of the equipment. When one buys a secondhand car, the present condition of the engine, body, upholstery, and tires is carefully determined. Good condition will increase the salvage value; poor condition will reduce it.

b. Selling price of new equipment of similar grade. If the price of new cars remains at a uniform level, the salvage value will be equivalent to the original cost minus the depreciable value as affected by its condition. If, however, the price of new cars has increased materially, the salvage value may be equivalent to the present price minus the depreciable value as affected by its condition. Thus the salvage value may approach and even be greater than the original cost of the car. Of course, if the price goes down, the opposite will result and the salvage value will be very low.

c. Reason for selling. The buyer of secondhand cars is suspicious of the seller. He is afraid that a poor buy is being unloaded on him. He suspects that the reason for selling may be that the car has been inefficient in its operation or that some other objectionable feature is the reason.

d. Cost of removal. The salvage value of the piece of equipment may be fairly high. The cost of removal may, however, be of sufficient amount to reduce materially the salvage value. The cost of removal will depend greatly on the location of the equipment. Its removal from locations that are hard to get at and where the shifting of other equipment is required may be very costly. If it is impossible to move other equipment easily, the ease with which the equipment may be disassembled or broken up will affect the cost of removal materially.

e. Supply and demand. Supply and demand will affect the salvage value. The nature of the market helps materially to set the price or value. In a seller's market the buyer must pay if he wishes the equipment. In a buyer's market the seller is at the mercy of the buyer. If the market is about even, neither one has the advantage over the other. The market price thus arrived at is a fair price.

2. *Scrap Value.*—Scrap value is a form of salvage value. It exists when the piece of equipment has no more value from an operating standpoint but is fit only to be broken up. Another name for this value would be "junk value."

3. *Depreciated Book Value.*—This is strictly an accounting value. It is equal to the first cost of the machine minus the accumulation of annual depreciation cost charges.

Example: An engine lathe.

Invoice cost	$3,500
Transportation	50
Installation	200
Accessories, not cutting tools	250
Total	$4,000

Estimated life 20 years; estimated salvage value at the end of 20 years to be $400.

First cost	$4,000
Salvage value	400
Amount to be recovered	$3,600

If the straight-line method is used, the annual depreciation cost charge is equal to

$$\frac{\$3,600}{20} \quad \text{or} \quad \$180$$

The depreciated book value is wanted at the end of 10 years.

First cost	$4,000
Accumulation of depreciation charges, 10 years at $180	1,800
Depreciated book value	$2,200

If the sinking-fund method is used, the accumulation of annual depreciation cost charges would be the amount in the sinking fund at the end of 10 years.

4. *In-place Value.*—This value is the one to be considered when the replacement of a piece of equipment is being considered. It is that value which is equivalent to the first cost to be incurred if the piece of equipment, now in place, had been bought from a secondhand dealer or some other business.

For purposes of replacement studies, it is necessary to place a value on the equipment to be replaced. This could be the salvage value, scrap value, or depreciated book value. This is then to be compared with some other piece of equipment in order to determine whether or not it should be replaced. But neither of these seems to be an adequate valuation. What is desired is a value that represents as nearly as possible the present economic worth of the machine.

Such a value can be determined if the equipment to be replaced is valued at the same amount that a competitor would spend as first cost to purchase and install a secondhand piece of equipment exactly similar to the one which it is proposed to replace. Thus, if a business determines that the salvage value for the engine lathe described previously is $1,600, then the "in-place" value of the engine lathe will be

Salvage value	$1,600
Transportation	50
Installation	200
In-place value	$1,850

This is the first cost which a competitor would pay for an exactly similar lathe as the one now owned. Hence, the in-place value may be considered as a good valuation for the machine that is infinitely better than the depreciated book value or the salvage value.

4.3. Valuation of a Going Industrial Enterprise.—Marston and Agg[1] suggest the following as factors that affect values:

1. The present original-cost value
2. The reproduction-cost-new-less-depreciation value
3. Earning value
4. Stock-and-bond value
5. Preliminary expense
6. Going concern value
7. Good-will value
8. Patents
9. Working or circulating capital
10. Other pertinent factors.

4.4. Present Original-cost Value.—The present original-cost value indicates the actual investment in the property at the date of valuation. It is in effect the book value of the property. This value begins with the original cost new, is modified by the accrued actual depreciation, the changes brought by the continual retirement and replacements of existing property, and the repeated additions of new units because of growth.

PRESENT ORIGINAL-COST VALUE

Original cost new of total installation		$100,000
Portions replaced 5 years ago		20,000
Original cost new of remaining portions		$ 80,000
Actual accrued depreciation		25,000
Depreciated value of remaining portions		$ 55,000
Original cost new of replaced portions 5 years ago	$25,000	
Actual accrued depreciation	4,000	21,000
Original cost new of added portion this year		10,000
Present original cost		$ 86,000

An individual plant had an original cost new 10 years ago of $100,000. Certain portions of the plant whose original cost new was $20,000 have been replaced at an original cost new 5 years ago of $25,000. The actual depreciation (not calculated) of these portions is now $4,000. The actual depreciation (not calculated) of the remaining part of the original installation is now $25,000. Additions have been made this year to the original-cost-new value of $10,000.

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[1] MARSTON, A., and T. R. AGG, "Engineering Valuation," Chap. XI, p. 305, McGraw-Hill Book Company, Inc., New York, 1936.

4.5. Reproduction-cost-new-less-depreciation Value.—The reproduction-cost-new-less-depreciation value represents the value on the basis of present prices. This value begins with the reproduction cost new of the properties which make up the present enterprise. This cost is then depreciated by the estimated actual depreciation for the life of the several properties.

The reproduction-cost-new values for the problem used in Sec. 4.4 are

PRESENT ORIGINAL-COST VALUE COMPARED WITH REPRODUCTION-COST-NEW VALUE

	Original-cost basis		Reproduction-cost-new basis	
Original cost new of total installation		$100,000		$120,000
Portions replaced 5 years ago		20,000		24,000
Original cost new of remaining portions		$ 80,000		
Reproduction cost new of remaining portion				$96,000
Actual accrued depreciation		25,000		
Actual accrued depreciation on reproduction cost new basis				30,000
Depreciated value of remaining portions		$ 55,000		$66,000
Original cost new of replaced portions 5 years ago	$25,000			
Actual accrued depreciation	4,000	21,000		
Reproduction cost new of replaced portions 5 years ago			$27,500	
Actual accrued depreciation			4,500	23,000
Original cost new of added portion which is also reproduction cost new		10,000		10,000
Present original cost		$86,000		
Reproduction cost new less depreciation				$99,000

4.6. Earning Value.—"The earning value of a property is the present worth of its probable future net earnings, as prognosticated on the basis of recent and present expense and earnings and the business outlook."

Capitalizing Annual Return.[1]—"When the net earnings are likely to continue at a uniform amount for a long future period, the earning value of the property is equal substantially to the capitalized annual net earnings; that is, annual earnings divided by the rate of net return expressed in decimal form."

$$\text{Capitalized value} = \frac{\text{annual earnings or return}}{\text{rate of return (decimal)}} \tag{4-1}$$

For the problem of Secs. 4.4 and 4.5, the annual net earnings are estimated at \$13,000. The desired return is 8 per cent.

$$\text{Capitalized value} = \frac{\$13,000}{0.08} = \$162,500$$

Present Worth of Annual Return.[1]—"When the net earnings are likely to continue at a uniform rate for a limited number of years, they constitute a uniform year-end annuity whose present worth, as computed by the proper formula, is earning value."

The proper equation is the present value of a uniform series of payments made at the end of a year. Reference to Appendix A indicates that it is the same as a series whose initial total is R. The equation is

$$R = D\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \tag{4-2}$$

where R is the present worth or value, D is the annual earning, n is the limited time, and i is the expected return.

Referring again to the example used in Secs. 4.4, 4.5, and 4.6, the value by this method will be (assuming limited time of 20 years)

$$R = (\$13,000)\left[\frac{(1+0.08)^{20} - 1}{0.08(1+0.08)^{20}}\right]$$
$$R = (\$13,000)(9.818) = \$127,634$$

Nonuniform Annual Return.—The present worth of a series of nonuniform annual returns is the sum of the separate present worths of each annual return.

Assume that, for the illustrative problem, the annual returns are as follows:

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[1] MARSTON and AGG, *op. cit.*, p. 11.

Year	Annual return	Year	Annual return
1	$14,000	11	$13,000
2	14,000	12	14,000
3	13,000	13	13,500
4	13,000	14	12,000
5	12,500	15	11,500
6	13,000	16	11,000
7	14,000	17	10,500
8	14,500	18	11,000
9	13,000	19	10,000
10	13,500	20	9,000

The present worth of each annual return at 8 per cent is then as follows:

Year	Annual Return	×	Factor	=	Present Worth
1	$14,000	×	0.9259	=	$12,962
2	14,000	×	0.8537	=	11,951
3	13,000	×	0.7938	=	10,319
4	13,000	×	0.7350	=	9,555
5	12,500	×	0.6806	=	8,507
6	13,000	×	0.6302	=	8,192
7	14,000	×	0.5835	=	8,169
8	14,500	×	0.5403	=	7,834
9	13,000	×	0.5002	=	6,502
10	13,500	×	0.4623	=	6,241
11	13,000	×	0.4289	=	5,575
12	14,000	×	0.3971	=	5,559
13	13,500	×	0.3677	=	4,963
14	12,000	×	0.3405	=	4,086
15	11,500	×	0.3152	=	3,624
16	11,000	×	0.2919	=	3,210
17	10,500	×	0.2703	=	2,838
18	11,000	×	0.2502	=	2,752
19	10,000	×	0.2317	=	2,317
20	9,000	×	0.2145	=	1,930
Total				=	$127,086

4.7. Stock and Bond Value.—Stock and bond value is the sum of the outstanding bonds multiplied by their current market value plus the issued stock multiplied by their current market prices.

For the illustrative problem used in preceding sections, the outstanding bonds amount to \$30,000 and the issued stock is 600 shares. The current market prices of the bonds whose par value is \$100 is \$103 and for the stock is \$160.

$$\begin{aligned}
\text{Bond value} &= \left(\frac{\$30{,}000}{\$100}\right)(\$103) &&= \$\ 30{,}900\\
\text{Stock value} &= (600)(\$160) &&= \underline{\quad 96{,}000}\\
\text{Total value} & &&= \$126{,}900
\end{aligned}$$

4.8. Preliminary Expense.—Preliminary expense is the fair present capital allowance for the actual, necessary, and legitimate expenses incurred for the development and promotion of the enterprise.

This value is obtained, if possible, from the accounting books of the enterprise. If not available, the amounts for this expense (see Chap. 1, for fuller discussion) may be determined by a survey of the probable conditions existing at the time of inception and by translating the factors into present prices.

For our illustrative problem, it is assumed that this expense amounts to \$5,000.

4.9. Going Concern Value.

> Going concern value is the difference between the value of a property as it stands possessed of its going elements and the value of the property alone as it would stand at completion of construction as a bare or inert assembly of physical parts.[1]

Going concern value has been well defined by the Supreme Court:

> This Court has declared it to be self-evident that there is an element of value in an assembled and established plant doing business and earning money, over one not thus advanced and that this element of value is "a property right." . . .
>
> Going concern value comprises among other factors, the estimated cost at today's prices of training and organization of personnel, season-

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[1] "Valuations and Appraisals," a pamphlet issued by Ford, Bacon, & Davis, Inc.

ing and integration of property, and attaching the business to the property.

The best measure of such going concern value seems to be, in the opinion of Marston and Agg and many others, the excess of present worths of the probable near-future net returns from an established plant over the present worths of those of a plant not so established.

The data for the determination of going concern value for the illustrative problem are as follows:

Year	Excess of returns	Present worth at 8 per cent
First	\$6,000 (0.9259) 1 year	\$ 5,555
Second	9,000 (0.8573) 2 years	7,715
Third	500 (0.7938) 3 years	396
Total		\$13,666

4.10. Good-will Value.—Good will is that "element of value which inheres in the fixed and favorable consideration of customers arising from an established and well-known and well-conducted business."[1]

"Customers going back to the old 'stand' to make additional purchases create the value good will.

"It is a value that is found in freely competitive enterprise where the customer has a variety of 'stands' to select from. It is found in well-known trade marks and in well-known products. It is advanced by advertising good products, sound policies, and good selling. It is, as its name defines, the value of the good will of the customer."[2]

"The correct measure of good-will value is the present worth of the probable future net profits in excess due to present existing good will, of fair net returns. It is customary to estimate the annual amount of such future excess profits at the three-year average of those actually being realized, in normal years."[3]

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[1] MARSTON and AGG, *op. cit.*, p. 357.

[2] From "Valuation and Appraisals," *op. cit.*

[3] MARSTON and AGG, *op. cit.*, p. 358.

It is calculated by using formula (4-2), the one used for the evaluation of earning value. The amount D is, in this case, the average excess profits.

By investigation it is found that for similar enterprises the average return to competitive businesses in the same field is about $11,500.

Return to enterprise	$13,000
Return to similar enterprises	11,500
	$ 1,500

Assuming that this excess return will continue for the next 10 years at an earning rate of 8 per cent the good-will value will be

$$\text{Good-will value} = (\$1{,}500)\left[\frac{(1 + 0.08)^{10} - 1}{0.08(1 + 0.08)^{10}}\right]$$
$$= (\$1{,}500)(6.71) = \$10{,}065$$

4.11. Patents.—The value of patent rights is the present worth of the estimated future returns for which the protection of the patent is responsible.

It is necessary first to estimate what portion of the net returns of the enterprise is obtained because of the protection of the patent right. It is then necessary to estimate how long these returns may be expected to continue. The present worths of these returns for the specified times is the value of patent rights.

For the illustrative problem it is estimated that $1,200 of the net returns can be definitely tied up to patent protection and that they will be good for 8 years. The formula that was used to evaluate earning value (4-2) is to be used; in this case D is the protected earnings or return.

$$\text{Patent value} = (\$1{,}200)\left[\frac{(1 + 0.08)^{8} - 1}{0.08(1 + 0.08)^{8}}\right]$$
$$= (\$1{,}200)(5.747) = \$6{,}896$$

4.12. Working or Circulating Capital.—This was discussed in Sec. 2.11. For the illustrative problem it is assumed to be $2,000.

4.13. Total Fair Value.—The total fair value of the enterprise consists of the cost of the physical factors and the intangible factors. For the illustrative problem the factors so far assumed or determined are

Land	$ 4,000
Original-cost value except land	86,000
Reproduction-cost value except land	99,000
Earning value, present worth	127,634
Stock-and-bond value	126,900
Preliminary expense	5,000
Going concern value	13,666
Good-will value	10,065
Patents	6,896
Working capital	2,000

Two fair values may be obtained by combining the foregoing figures and using original cost and reproduction cost for fair property values.

	Original-cost value	Reproduction-cost value
Land	$ 4,000	$ 4,000
Original-cost value	86,000	
Reproduction-cost value	....	99,000
Preliminary expense	5,000	5,000
Going concern value	13,666	13,666
Good-will value	10,065	10,065
Patents	6,896	6,896
Working capital	2,000	2,000
Total fair value	$127,627	$140,627

These values compare with the evaluations of earnings and stock and bond value:

Earning value	$127,634
Stock and bond value	126,900

If current prices are moving upward, the fair value may be considered to be near the one in which reproduction cost is used. If prices are likely to be lower in the near future, the fair value may lean toward the one in which original cost is used. In view of the general economic situation then existing, the fair value may be somewhere between the two values calculated and modified by other pertinent factors in the situation which are extremely hard to evaluate in specific monetary units.

4.14. Valuation of Extractive Properties.—The value of properties such as mines, quarries, timber lands, oil and gas lands, and others of the type where natural resources are extracted or cut is the worth today of the future profits.

Extractive properties are exhaustible. The probable output of the property must first be estimated by the proper technical engineer. The value of his estimate will depend largely upon the amount of development work he is allowed to do in the preparation of his estimate. The probable extent of the deposit is first determined. Next, by proper methods of assaying, the yield of the deposit is established and is given a value in the market. The product of the probable yearly production and its market value is the yearly income. Operating cost is determined. The present worth of the future income is a measure of the value of the property.

The determination of value may be made by using the method used for earning value. Hoskold's formula, which is found in the literature of mining, is applicable to extractive properties. It is a formula for finding the present worth of future incomes using two interest rates. The stipulated rate is the yield that the owner wishes to make on the investment. The practicable rate is the one that is used for the calculation of a sinking fund deposit to recover the capital.

The formula is derived from the fact that out of yearly return, a portion must be paid as return to those who own the investment and the balance is set aside in a sinking fund to recover the investment.

$$\text{Return on investment} = rP \tag{4-3}$$

where r is the stipulated rate and P the investment.

$$\text{Sinking fund deposit} = P\left[\frac{i}{(1+i)^n - 1}\right] \tag{4-4}$$

where i is the practicable rate and P is the investment to be recovered. Therefore,

$$\text{Yearly return } R = rP + P\left[\frac{i}{(1+i)^n - 1}\right] \tag{4-5}$$

Since the quantity P is desired, the equation must be rearranged as follows:

$$P = R\left[\frac{1}{r + \dfrac{i}{(1+i)^n - 1}}\right] \tag{4-6}$$

It should be here observed that the return R which is used in this formula or equation is equal to gross income for the year minus operating cost for the year. Operating cost in this case includes the usual elements but does not include a charge for depletion, since the recovery of the investment for which a depletion charge would be made is taken care of by the sinking fund of the equation.

Example: Data for a mine are as follows:
Estimated probable output: 20,000 tons yearly
Time to exhaust property: 15 years
Selling price of mined, dressed, and smelted ore: $10 per ton
Management costs not including charge for depletion: $5,000 per year
Costs of operating the mine and smelter: $7.53 per ton
Desired return on investment: 8% (stipulated rate)
Sinking fund interest rate: $2\frac{1}{2}$ per cent (practicable rate)

Yearly income (20,000 tons)($10 per ton)		$200,000
Yearly operating cost:		
Management costs	$ 5,000	
Operating costs	150,600	
Total		$155,600
Net yearly return		$ 44,400

Substituting in the equation,

$$P = (\$44{,}400)\left[\frac{1}{0.08 + \dfrac{0.025}{(1.025)^{15} - 1}}\right]$$

$$P = (\$44{,}400)\left(\frac{1}{0.08 + 0.0558}\right)$$

$$P = (\$44{,}400)\left(\frac{1}{0.1358}\right) = \$326{,}951.39$$

The accuracy of this amount may be checked as follows:

Net income per year	$44,400
Return of 8 per cent on $326,951	26,156
Balance	$18,244

This balance will accumulate, at $2\frac{1}{2}$ per cent, ($18,244) $\left[\frac{(1+i)^n - 1}{i}\right]$, or ($18,244)(17.9211), or $326,952.

4.15. Valuation for Other Purposes.—The methods of valuation which have been presented are made for many other purposes.

For Consolidations and Mergers.—Sometimes it is proposed that two or more going enterprises be merged into a larger and more comprehensive one. The reason for this merger may be the broadening of the market by acquiring enterprises producing commodities that would supplement an already established line. The use of patents may often lead to a merger. The pooling of patents may result in a better product and hence a broader market.

For Fire Insurance.—A program of insurance must rest on well-calculated values of the things that are to be protected and insured. Because of fluctuations in price levels, valuation for fire-insurance purposes requires constant attention. Reproduction cost is the principal method used.

For Federal Income Tax Purposes.—The changes in property ownership are reflected in income tax procedure through the deductions in operating cost for depreciation. A whole new field of property accounting methods has been opened up because of this. There is also the problem of determining in the absence of recorded data the fair value in the past property.

For Section 77B of the Bankruptcy Act.—In the reorganization proceedings coming under this act, valuation is a necessary part to the determination of the claims of the several parties to the reorganization.

For Condemnation Proceedings.—"The taking, by condemnation, of private property for public use is wholly a legal proceeding. The owner has definite constitutional rights and legislative protection, but he should have competent counsel. The laws of eminent domain are complex and vary from state to state."[1] Valuation is, therefore, an inherent part of such proceedings. Market value has been generally held by the courts to represent the measure of just compensation.

For Securities Act Registration.—When an issue of stock or bonds is registered under the Securities Act, certain information must be given as to the values that lie behind the statements of the enterprise. Book figures are not real value or worth. Valuation procedures will, in general, give the best estimate of the true values that inhere in the enterprise.

For Fair Value of a Public Utility.—A municipality wishes to take over and purchase a water company now operated by a private corporation. If a fair price cannot be agreed upon by the two sides, recourse may be made to legal procedure. This is accomplished by taking the case to a government agency such as a public-service commission or to the courts. The procedure is established by the several states.

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[1] From "Valuation and Appraisals," *op. cit.*

Likewise, if the rates charged by a public utility seem to be more than they should, recourse is had to similar procedure for the determination of the fair value of the utility for the purposes of rate making.

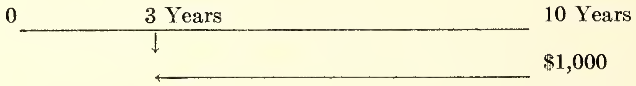
4.16. Valuation of a Bond.—Many times it becomes necessary to place a valuation on bonds which are up for sale or which comprise the assets of a company. The solution is to use the principle of valuation on the basis of present worth of earnings and future payments.

The bond is made up of two parts, the principal and the interest. The principal is a specific value at the termination of the term of the bond. The interest payment is a specific value at the time it is paid to the bondholder. The problem of valuation is to determine their value at the time the bond is bought.

Example: Bond data:
Face amount: $1,000
Interest rate: 6 per cent
Interest paid semiannually: $30
Term: 10 years
Up for sale at end of third year, 7 years to go

First Situation.—The buyer of the bond wishes to earn the same rate as the bond, namely, 6 per cent semiannually.

Since the principal will be paid at the end of the term of the bond, its value is the face amount of the bond at that time.



What is its value at the end of the third year when it is offered for sale? Obviously, it is the present worth of the principal at that time.

$$\text{Present worth} = (\$1{,}000)\,\frac{1}{(1.03)^{14}},\ \text{or } \$661.10$$

The interest payment or return is paid at the end of each 6 months or half year during the term of the bond. The valuation of the series of payments is the present worth of the payments at the interest rate which the investor wishes to earn. This is equivalent to an R series:[1]

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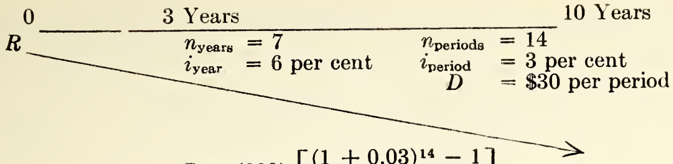
[1] See Appendix A, p. 324.


0
3 Years
10 Years
R
$n_{\text{years}} = 7$
$i_{\text{year}} = 6$ per cent
$n_{\text{periods}} = 14$
$i_{\text{period}} = 3$ per cent
$D = \$30$ per period

$$R = (\$30)\left[\frac{(1+0.03)^{14}-1}{0.03(1+0.03)^{14}}\right]$$
$$R = (\$30)(11.30) = \$339.00$$

The total valuation of the bond will be the sum of the separate valuations of the principal and interest payment.

Valuation of the principal......	$ 661.10
Valuation of interest payments.	339.00
	$1,000.10, or, in effect, $1,000

Second Situation.—The buyer of the bond wishes to earn at the rate of 8 per cent a year compounded semiannually. The method of solution is the same except that the interest rate is changed.

$$\text{Valuation of principal} = (\$1{,}000)\left[\frac{1}{(1+0.04)^{14}}\right] = \$577.50$$

Valuation of interest payment:

$$R = (\$30)\left[\frac{(1+0.04)^{14}-1}{0.04(1+0.04)^{14}}\right]$$
$$R = (\$30)(10.560) = \$316.80$$

Total valuation:

Valuation of principal...........................	$577.50
Valuation of interest payments..................	316.80
	$894.30

Third Situation.—The buyer of the bond is willing to earn at the rate of 4 per cent a year compounded semiannually.

Valuation of principal $(\$1{,}000)\dfrac{1}{(1.02)^{14}}$..........	$ 757.90
Valuation of interest $(\$30)(12.11)$..............	363.30
Total valuation............................	$1,121.20

To recapitulate:

Situation	Interest rate, paid semiannually	Valuation
First.....................	6%	$1,000.10
Second..................	8%	894.30
Third....................	4%	1,121.20

Thus, the valuation of the bond depends upon the rate of yield which the buyer expects. When the rate of expected yield is the same as that of the bond, the valuation is the face amount of the bond. When the rate of expected yield is higher, the valuation of the bond is less than its face amount. Likewise, when the rate of expected yield is lower, the valuation of the bond is greater than the face amount.

4.17. Valuation of Penalties or Bonuses.—When a piece of equipment is purchased, a guarantee of performance may go with it. Performance may be of several kinds, such as time to complete an installation, steam or water rate per horsepower, electric current consumed per horsepower, or some other factor. After the equipment has been installed, it may be found that the performance is not up to standard. More steam or electric current per horsepower is being used than was originally guaranteed. The obvious result of such poor performance is that operating costs are increased. The customer has made his plans on the guaranteed operating costs. This increase means that the return which he expected will not be realized. The guarantee which the seller gave him specified a penalty for operating costs above the guaranteed figure.

The amount of the penalty should be such as to compensate for the increased operating cost, over the life of the equipment. It is usually paid in a lump sum at or near the time when the equipment is purchased after sufficient time has elapsed for determining operating conditions. The amount of the penalty is the present worth of the increased operating cost, using an interest rate conforming to what the owner is earning on his capital or on the interest rate for borrowed capital.

It is determined by using the principles discussed in Sec. 4.6. Usually the period is not perpetual but short and the increased cost is uniform. The formula to be used is

$$R = D\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \tag{4-9}$$

where R is the penalty, D is the increased cost, n is the period, and i is the interest rate.

Example: A piece of equipment is sold with a guaranteed operating cost of $3,000 per year for a period of 10 years. The actual operating cost is later found to be \$3,300, an increase of \$300. The buyer expects to earn 7 per cent on his investment.

$$\text{Penalty} = (\$300)\left[\frac{(1+0.07)^{10}-1}{0.07(1+0.07)^{10}}\right]$$
$$= (\$300)(7.024) = \$2{,}107.20$$

In case the operating costs are less than guaranteed, it may be specified in the contract that the builder or seller be compensated by a bonus for the value of this decrease. In this case, the factor D in the foregoing calculation represents the decrease in operating cost.

Problems

1. For Prob. 1 in Chap. 3 determine
  - *a.* Depreciated book value at the end of 8 years.
    - (1) Straight-line method.
    - (2) Sinking-fund method.
  - *b.* In-place value at the end of 8 years assuming that the salvage value at that time would be \$1,000.

2. For Prob. 2 in Chap. 3 determine
  - *a.* Depreciated book value at the end of 5 years.
    - (1) Straight-line method.
    - (2) Sinking-fund method.
  - *b.* In-place value at the end of 5 years, assuming that the salvage value at that time was \$10,000.

3. For Prob. 3 in Chap. 3 determine
  - *a.* Depreciated book value at the end of 3 years.
    - (1) Straight-line method.
    - (2) Sinking-fund method.
  - *b.* In-place value at the end of 3 years, assuming that the salvage value at that time was \$150.

4. For Prob. 4 in Chap. 3 determine
  - *a.* Depreciated book value at the end of 2 years.
    - (1) Straight-line method.
    - (2) Sinking-fund method.
  - *b.* In-place value at the end of 2 years, assuming that salvage value at that time is \$2,500 and no charges will be made for freight or installation, but a charge of \$300 is made for overhauling.

5. If a property has a first cost of \$250,000 and a salvage value of \$25,000 at the end of 25 years, find its appraised (book) value to the nearest \$500 by the sinking-fund method of depreciation, at the end of 5, 10, 15 and 20 years, assuming an interest rate of 4 per cent. (From examination for Professional Engineer's license, Part III, New York State, June 28, 1939.)

6. If a property has a first cost of \$100,000 and a salvage value of \$10,000 at the end of 25 years, find to the nearest \$100 by the sinking-fund method of depreciation the appraised (book) value of the property at the end of 5, 10, 15, and 20 years, assuming an interest rate of 4 per cent. (From examination for Professional Engineer's license, Part III, New York State, June 27, 1934.)

7. Determine the valuation of the following situations by capitalizing the annual return. Assume indefinite time.

Situation	Annual return	Expected yield, per cent
a	36,000	9
b	125,000	15
c	44,600	12
d	12,600	8
e	75,000	16

8. Determine the valuation of the situations in Prob. 7 by the present-worth method of valuation of earning power, assuming the following periods during which the return can be expected to continue:

a. 15 years
b. 6 years
c. 24 years
d. 3 years
e. 20 years

9. John Jones has been offered a royalty of 8 cents per piece for the rights to a patent. It is estimated that production of the article will be about 100,000 per year for a period of 5 years. He, however, would like a cash payment in lieu of the royalty payments over the 5 years because he is afraid that the life period of the article may not be 5 years.

What should the amount of the cash payment be if the valuation is made at the rate of 10 per cent?

10. If in Prob. 9 the company purchasing the patent right agrees to pay an immediate sum of $25,000, how much production must it make in order to be equivalent to a royalty of 6 cents per piece for a period of 4 years? Interest rate is assumed at 12 per cent.

11. A mine will produce a net income of $70,000 annually for 30 years. Calculate its value if the annual dividend rate is to be 7 per cent, payable annually, and the sinking fund is to accumulate at the rate of 3 per cent annually.

12. A mine will produce an income of $55,000 annually for 20 years. Calculate its value if the annual dividend rate is to be 8 per cent, payable annually, and the sinking fund is to accumulate at the rate of $3\frac{1}{2}$ per cent annually.

13. A bond, whose face value is $1,000 and which pays interest at 5 per cent annually at the end of each year and whose term is 20 years, is offered for sale at the end of its eighth year. What should a buyer pay for it if he wishes to earn on his purchase price (*a*) 4 per cent? (*b*) 5 per cent? (*c*) 6 per cent?

14. A $500 bond which pays 6 per cent interest per year semiannually and whose life is 15 years is offered for sale at the end of its sixth year. What should the buyer pay for it if he expects to earn (*a*) 5 per cent, (*b*) 6 per cent, or (*c*) 7 per cent, on his purchase price, provided the earnings are semiannual?

15. A $2,500 bond at 6 per cent interest annually and whose life term is 20 years is offered for sale. If $2,000 is offered, what is the earning rate, or yield? If $2,500 is offered, what is the yield? If $3,000 is offered, what is its yield?

CHAPTER 5

FIRST-COST PATTERNS

5.1. First Cost vs. Varying Capacity or Size.—One of the fundamental facts of engineering economics is that as the size or capacity of engineering projects increases, the unit cost per some measure of size or capacity decreases.

Figure 5.1 is the relationship between first cost and size for open squirrel-cage motors. It is to be noted that the relationship between first cost (expressed in dollars) and size (expressed in horsepower) is a straight-line one, with a positive intercept at the zero ordinate.

Table 5.1.—First Cost of Open Squirrel-cage Motors

Size	Cost
1	$37
1½	44
2	52
3	58
5	71

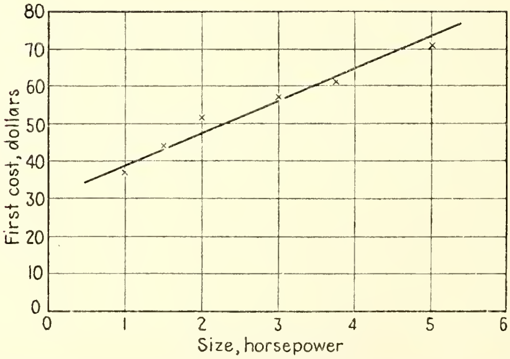


Fig. 5.1.—First cost vs. size for open squirrel-cage motors. See Table 5.1.

5.2. Unit Cost vs. Varying Capacity or Size.—This relationship can be shown by superimposing, upon the diagram showing the relationship between first cost and size, the unit costs for the various sizes or capacities. These are found for each size or capacity by dividing the first cost by the measure of capacity or size.

$$\frac{\text{First cost}}{\text{Measure of capacity or size}} = \text{unit cost}$$

In Fig. 5.2 it is to be noted that the relationship is a curved one descending to the right in such a way that the larger the thing is, the less the unit cost becomes. In other words, large sizes are less costly per unit than small units.

TABLE 5.2.—UNIT COST PER HORSEPOWER OF OPEN SQUIRREL-CAGE MOTORS

Size	First cost	Unit cost
1	$37	$37
1½	44	30
2	52	26
3	58	19.3
5	71	14.2

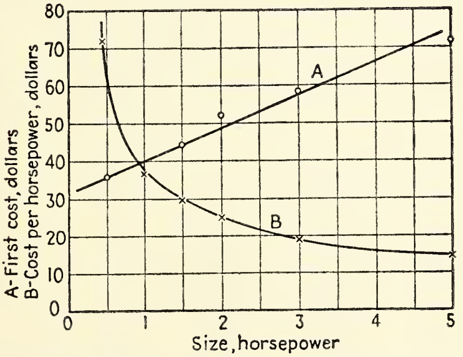


FIG. 5.2.—Unit cost vs. size for open squirrel-cage motors. See Table 5.2.

In Fig. 5.3 there are two curves representing cost per some unit of size. Curve *B* shows the cost per inch of diameter. Note that it does not conform to the unit-cost curve found in the case of the motors. Curve *C*, however, which is the cost per square inch of area of valve opening, does conform to the motor curve. The reason for the difference is that nominal size in diameters is not a true measure of the capacity or size of the valve opening. Area in square inches is a true measure of capacity.

TABLE 5.3.—IRON CITY GATE VALVES SCREWED

Size, inches	Area, square inches	Curve *A*, cost	Curve *B*, cost per inch diameter	Curve *C*, cost per square inch
2	3.14	$ 10.00	$ 5.00	$3.18
2½	4.90	11.50	4.60	2.35
3	7.06	14.00	4.60	1.98
3½	9.60	17.00	4.80	1.77
4	12.57	19.00	4.75	1.55
5	19.63	27.50	5.50	1.40
6	28.27	32.50	5.50	1.15
8	50.27	54.00	6.75	1.075
10	78.54	90.00	9.00	1.14
12	113.10	125.00	10.40	1.10

TABLE 5.4.—STANDARD SCREWED PIPE (BLACK)

Size, inches	Internal area	Weight per foot	Curve *A*, cost per 100 ft.	Curve *B*, cost per inch diameter	Curve *C*, cost per area	Curve *D*, cost per weight
¼	0.36	0.42	$ 3.08	$12.30	$8.55	$7.34
⅜	0.49	0.57	3.08	8.21	6.29	5.40
½	0.62	0.85	3.88	7.76	6.25	4.57
¾	0.82	1.13	4.83	6.84	5.89	4.28
1	1.05	1.68	6.71	6.71	6.39	3.99
1¼	1.38	2.27	9.08	7.26	6.59	4.00
1½	1.61	2.72	10.86	7.25	6.75	4.00
2	2.07	3.65	14.61	7.30	7.06	4.00
2½	2.47	5.70	23.10	9.25	9.35	3.99
3	3.07	7.57	30.21	10.07	9.81	3.97

Curves *B* and *C* (Fig. 5.4) show a minimum point at about the ¾ in. size. The typical decreasing-cost curve in this case is weight per foot or curve *D*, which after a certain point indicates a level uniform cost.

In the case of common nails, as shown in Fig. 5.5, the first-cost curve does not obey the trend shown in the other examples; it is

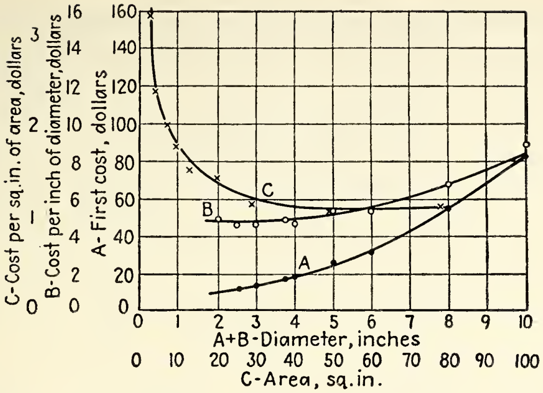


FIG. 5.3.—First cost and unit cost versus size for iron-body screwed gate valves. See Table 5.3.

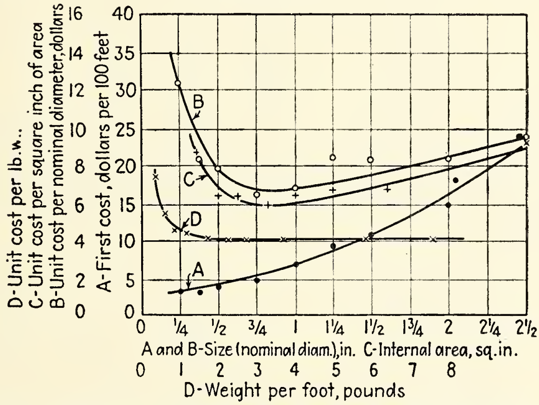


FIG. 5.4.—First cost and unit cost versus size for standard black screwed pipe. See Table 5.4.

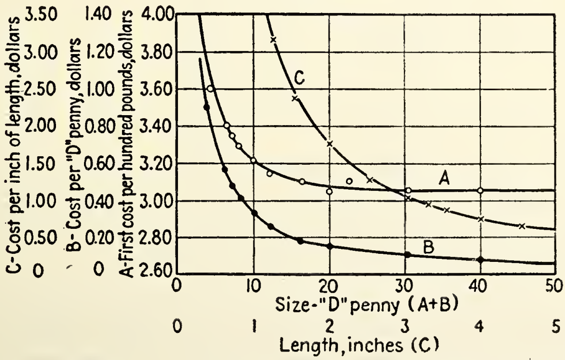


FIG. 5.5.—First cost and unit cost vs. size for common nails. See Table 5.5

high for the small size and decreases for the larger sizes. The unit-cost curves, however, show descending lines for sizes in terms of the common designation of *D* and of inches in length.

Table 5.5.—Common Nails
100-pound Keg

Size *D*	Length, inches	Curve *A*, cost per 100 lb.	Curve *B*, cost per *D*	Curve *C*, cost per length
3	1.25	$3.95	$1.33	$3.16
4	1.50	3.60	0.90	2.40
6	2.00	3.40	0.57	1.70
7	2.25	3.55	0.48	1.49
8	3.00	3.30	0.41	1.32
10	3.25	3.20	0.32	1.07
12	3.50	3.15	0.26	0.97
16	4.00	3.10	0.19	0.88
20	4.50	3.05	0.15	0.76
30	5.00	3.05	0.10	0.67
40	6.00	3.05	0.07	0.61

The curve in Fig. 5.6 is based on the same data as in Fig. 5.5, but reduced and modified after determining the number of nails in a 100-lb. keg.

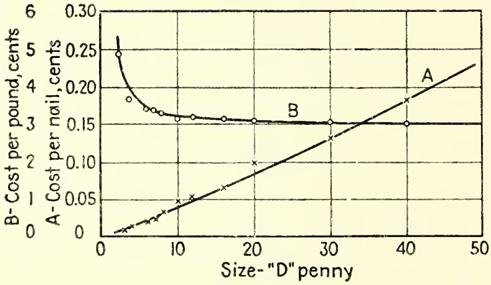


Fig. 5. —First cost and unit cost vs. size for common nails. See Table 5.5.

5.3. First Cost vs. Fixed Capacity or Size.—When a structure, of fixed capacity or size, is made up of several members each of which may vary in size, the resulting first cost is not necessarily the same for all combinations of the members.

Example: A cross-country transmission line for electric energy (Fig. 5.7). In the design of such a transmission line, the clearance between the conductor and the ground is fixed, either by custom or by state law. The minimum clearance in a span occurs when the deflection is a maximum. The maximum deflection is a function of the conductor and the critical catenary curve.

As the length of the span increases, the maximum deflection increases. This increased deflection, when added to the permissible clearance, requires that the height of the transmission poles shall also be increased. As the span increases, the number of poles per mile decreases. The data on first cost are given in Table 5.6.

TABLE 5.6.—CROSS-COUNTRY TRANSMISSION LINE

Length of span, feet	Cost per pole	Poles per mile	Cost per mile
100	$2,098	52.80	$110,774
150	2,102	35.20	73,990
200	2,146	26.40	56,654
250	2,152	21.12	45,450
300	2,238	17.60	39,388
350	2,291	15.08	34,548
400	2,358	13.20	31,125
450	2,435	11.73	28,562
500	2,529	10.56	26,706
550	2,631	9.60	25,257
600	2,737	8.80	24,085
650	2,879	8.12	23,377
700	3,041	7.54	22,929
750	3,206	6.60	22,570
800	3,400	6.21	22,440
850	3,605	5.86	22,387
900	3,824	5.55	22,408
950	4,092	5.28	22,710
1,000	4,388	5.02	23,168
1,050	4,699	4.80	23,588
1,100	5,041	4.59	24,196
1,200	5,828	4.30	25,643

The data in Table 5.6 show that the cost per pole increases, as the length of the span increases. The relationship is not a straight line but a parabola, indicating that some factor or factors in the design and production of towers increase the cost faster than a straight-line relationship.

It will be noted that curve *B*, cost per mile, decreases to the right in the typical manner but then turns upward, thus showing a minimum point at a span length of 850 ft. It should be noted that the difference in total cost per mile is at nearly the same value for spans from 550 to 1,200 ft., the gross variation from the minimum of $22,387 for a span of 850 ft. to $25,257 for

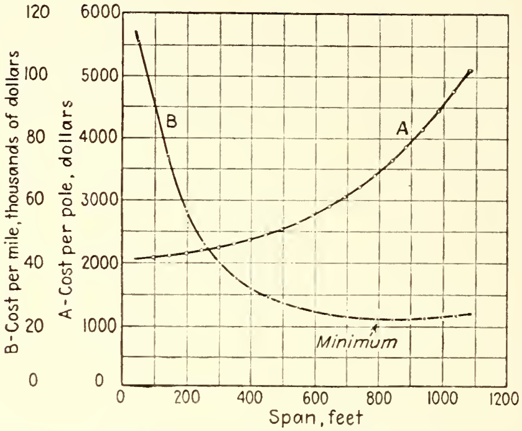


Fig. 5.7.—First cost of cross-country electric transmission line. See Table 5.6.

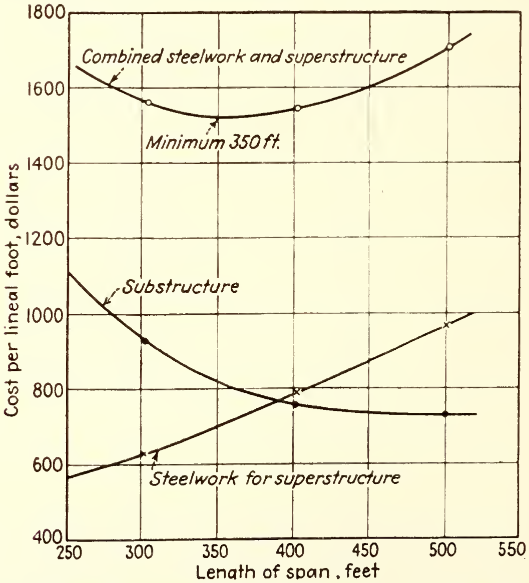


Fig. 5.8.—A low-level combined bridge of masonry or concrete piers and steel trusses. *Reprinted by permission from Economics of Bridgework by the late J. A. L. Waddell, published by John Wiley & Sons, Inc., New York.)*

a span of 550 ft. and $22,387 for a span of 850 ft. and $25,643 for a span of 1,200 ft. is only $2,870 to $3,256, or roughly 13 to 14 per cent of the minimum cost.

Example: A low level steel and concrete bridge.

It will be noted from the curves in Fig. 5.8 that the bridge is made up of two structures: a foundation and a bridge truss. The curves for each structure show that, as the span is increased, the cost of steelwork increases and that the cost of substructure decreases.

The combination of the first cost of the two structures produces a typical curve which shows a minimum at a span of 350 ft.

It is to be noted that any ordinate on the curve represents the total first cost for the particular combination of the variable elements in the bridge. The length of the bridge span is fixed in length. Other factors, such as length of individual arch, may vary within the confines of the fixed over-all length. It is these changes made in the elements of a structure that result in a decreasing total cost for one element as against an increasing total cost for another element.

Problems

L. (Refer to Prob. *L* in Chap. I.) The bridge is to be constructed of masonry or concrete arches. The engineer reporting on the cost of construction suggests that the first cost complete for the bridge may be estimated as follows:

Cost of land abutments in dollars = \$25 × span length + \$400
Cost of piers in dollars = \$40 × span length + \$500
Cost of superstructure in dollars = total length of bridge in feet to the 1.7th power

$$L^{1.7} = \text{cost}$$

The bridge length is to be $\frac{1}{2}$ mile, or 2,640 ft. What number of spans will give the lowest cost? Draw a first-cost curve. What is the first cost of the bridge?

1. Select some two or three pieces of equipment or machinery. Obtain from one of the following sources the cost for various sizes of the equipment:

- *a.* Catalogue of Sears, Roebuck and Company. (Borrow, do not send for one.)
- *b.* Catalogue of Montgomery Ward & Co., Inc. (Borrow, do not send for one.)
- *c.* Catalogue of any machinery supply company.

Draw the first-cost curves of the specific items of equipment or machinery.

2. The cost of a certain type of bridge superstructure in dollars is equal to the square of the total length of the bridge. At a certain location where a valley 1,000 ft. across must be bridged, end abutments for this type of bridge cost \$200 plus \$30x each, where x = length of one span in feet. Intermediate piers cost \$$(400 + 50x)$. Into how many spans of equal length should the 1,000 ft. be divided in order to give the cheapest bridge and what will be the total cost of the structure? Draw the first-cost curve of the bridge. (From examination for Professional Engineer's license, Part III, No. 4, New York State, Jan. 25, 1939.)

3. A railroad company proposes to build a double-track, half-through, plate-girder bridge 980 ft. long, with spans of equal length. The cost of the steel in place is estimated at 10 cents per pound. Disregarding the cost of track as constant, find the economical length of span. For the superstructure use the equation, $W = 25S + 2{,}050$, where W is the weight of the steel in pounds per linear foot of bridge and S is the span length in feet. For the center piers use, $C = 30{,}000 + 50S$, and $C = 20{,}000 + 50S$ for the end piers in which C is cost in dollars of one pier and S is the span length in feet. Draw the first-cost curve of the bridge. (Modified from examination for Professional Engineer's license, Part III, New York State Feb. 2, 1938.)

CHAPTER 6

ESTIMATING FIRST COST

6.1. Estimating.

The term estimate is used in engineering valuation in the sense in which it is employed in general engineering practice: that is, to indicate a carefully considered computation of some quantity the exact magnitude of which cannot be determined. The estimate represents the true magnitude as closely as it can be determined by the exercise of sound judgment based on approximate computations and is not to be confused with offhand approximations that are little better than outright guesses.[1]

6.2. A Cost Summary.—The methods of estimating may be illustrated by referring to the cost summary prepared after the completion of a project. For illustrative purposes, such a summary prepared by the Purdue Housing Research Project will be used.[2]

The illustration material indicates the manner in which the several items of first cost were assembled. The data were gathered from various papers that recorded transactions concerning the supply of material, labor, and equipment. The information was collected, classified, and tabulated as the work progressed. On the completion of the work, the information was brought together in proper divisions, summations were made, and the results presented in a complete cost summary, which was broken down into several analyses, such as material cost, labor cost, and trade cost.

It will be of interest to study the cost summary and its several breakdowns from the standpoint of what they contain. If it is possible to understand how the cost summary was obtained, then it should be easy to understand estimating since it is the same

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[1] Marston, A., and T. R. Agg, "Engineering Valuation," p. 10, McGraw-Hill Book Company, Inc., New York, 1936.

[2] By permission.

TABLE 6.1.—CONSTRUCTION-COST SUMMARY
House No. 5, Purdue Housing Research Project

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Labor	(5) Material	(6) Labor and material	(7) Profit and overhead	(8) Subtotal	(9) Total cost	(10) Per cent of cost
Excavation			$ 48.40		$ 48.40			$ 48.40	1.0
Foundations	Footings		8.80	$ 37.80	46.60		$ 46.60		
	Walls		71.85	132.85	204.70		204.70		
	Girder and post		2.70	8.20	10.90		10.90	262.20	5.2
Concrete floors	Fill	Backfill	11.20	8.95	20.15		20.15		
	Concrete work	Mixing and pouring	18.25	57.30	75.55		75.55	95.70	1.9
Accessories	Steps, etc.		12.70	11.35	24.05			24.05	0.5
Outside walls	Framing		48.00	87.95	135.95		135.95		
	Sheathing		41.70	84.01	125.80		125.80		
	Siding	Flush and bevel	117.90	185.65	303.55		303.55		
	Insulation		8.40	34.35	42.75		42.75		
	Interior finish	Plywood and board	58.95	116.05	175.00		175.00	783.05	15.7
Partitions	Framing		39.10	35.05	74.15		74.15		
	Wall finish	Plywood and board	76.95	135.20	212.15		212.15	286.30	5.8
1st floor	Framing	Joist, sub-floor, etc.	23.30	45.25	68.55		68.55		
	Wood flooring		25.05	39.85	64.90		64.90		
	Linoleum	Linoleum and base	8.85	42.05	50.90	*	50.90	184.35	3.7
2nd floor	Framing	Joist, sub-floor, etc.	34.30	74.80	109.10		109.10		
	Ceiling finish	Plywood and board	7.20	16.25	23.45		23.45		
	Wood flooring		29.95	50.20	80.15		80.15		
	Linoleum	Linoleum and base	2.25	13.90	16.15	*	16.15	228.85	4.6
Roof	Framing	Joist and rafters	27.50	90.60	118.10		118.10		
	Roofing	Shingles, flashing	24.75	58.60	83.35		83.35		
	Ceiling finish	Plywood	20.70	54.75	75.45		75.45		
	Insulation		1.80	9.45	11.25		11.25	288.15	5.8

* Profit and overhead included in material cost.

TABLE 6.1.—CONSTRUCTION-COST SUMMARY.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Labor	(5) Material	(6) Labor and material	(7) Profit and overhead	(8) Subtotal	(9) Total cost	(10) Per cent of cost
Millwork	Doors	Complete	$ 54.90	$ 229.75	$ 284.65		$ 284.65		
	Sash	Complete	55.80	150.00	205.80		205.80		
	Trim	Molds, shelving, etc.	30.85	4.70	35.55		35.55		
	Cabinets	Kitchen and bath	8.10	76.00	84.10		84.10		
	Screens	Complete	12.15	64.00	76.15		76.15		
	Shutters	Complete	8.55	56.85	65.40		65.40	$ 751.65	15.0
Stairs	1st story	Frame and finish	53.55	84.50	138.05		138.05		
	Basement		7.20	30.45	37.65		37.65	175.70	3.5
Heating	Chimney†	Brick work	28.25	39.50	67.75		67.75		
	Fuel bin	Framing	4.95	11.65	16.60		16.60		
	Rough	Steam pipe, etc.	28.15	25.45	53.60	$ 18.90	72.50		
	Finish	Furnace and radiators	21.40	220.80	242.20	85.30	327.50	484.35	9.7
Fireplace	Chimney†, mantel		33.20	75.45	108.65			108.65	2.2
Plumbing	Rough		40.65	54.95	95.60	28.85	124.45		
	Finish	Fixtures and fittings	17.55	110.00	127.55	38.60	166.15	290.60	5.8
Electrical	Rough		25.50	35.10	60.60	18.25	78.85		
	Finish	Plates, fixtures	6.00	51.60	57.60	17.20	74.80	153.65	3.1
Painting	Exterior	3 coats	45.00	38.55	83.55	48.10	131.65		
	Shutters, screens	3 coats	21.00	9.10	30.10	17.10	47.20		
	Interior	Stain, shellac, wax	70.50	29.20	99.70	57.45	157.15	336.00	6.8
Totals			$1343.80	$2828.10	$4171.90	$329.75		$4,501.65	
			General contractor's overhead and/or profit					$ 484.35	9.7
Grand total and contract price								$4,986.00	100.0

† One-half chimney cost charged to heating and one-half to fireplace.

process. The principal difference is that in the cost summary the quantities and rates are actual and in the estimate they are probable.

6.3. Construction-cost Summary.—Upon examination the following things are readily seen in Table 6.1:

1. Col. (1). As one reads the items in this column, he finds

Excavation
Foundations
Concrete floors
Accessories
Outside walls
Partitions
1st floor
2d floor
Roof
Millwork
Stairs
Heating
Fireplace
Plumbing
Electrical
Painting
General contractor's overhead and/or profit

These names of the major divisions of the work indicate that the contractor, in order to get a good picture of the cost of construction, divided the work into a number of natural divisions.

Hence, the first rule of estimating is to divide the work that is to be estimated into several natural divisions.

2. Col. (2). As one examines the subdivisions of Outside Walls, he finds

Framing
Sheathing
Siding
Insulation
Interior finish

These are the names of the subdivisions of that particular major division. They indicate that the contractor, in order to get a better picture than that given by major divisions, divided them into natural subdivisions.

Hence, the second rule of estimating is to divide the major divisions into natural subdivisions.

3. Col. (3). The information here is an explanation of the items appearing in col. (2). It is useful when it is necessary to compare the estimates of several houses.

4. Cols. (4) to (7). These columns give the costs of the items listed as subdivisions of the work in terms of labor, material, and profit and overhead. Notice that some items show profit and overhead; others do not. The profit and overhead for those not specifically marked is included in the general contractor's overhead listed at the bottom of the summary.

5. Col. (8). The amounts in this column are the totals of labor, material, profit, and overhead costs for each subdivision.

6. Col. (9). In this column will be found the total cost of each of the major divisions. They are made up of the sum of the subtotal costs of the subdivisions.

7. Col. (10). In this column is indicated the per cent which the total cost of each major division bears to the grand total cost.

6.4. Breakdown of Labor Cost.—An examination of the breakdown of the costs of labor, given in Table 6.2, shows the following:

1. Col. (1). The items in this list are the same as in col. (1) of Table 6.1.
2. Col. (2). The items in this list are the same as in col. (2) of Table 6.1 with a few additions.
3. Col. (3). The information here explains the items appearing in col. (2). In addition it gives a further breakdown into specific types of work or materials.
4. Col. (4). These items should be self-explanatory. They indicate the kind of labor required for the specific jobs indicated in col. (3).
5. Col. (5). The figure is a summation of all the time that was necessary for the class of labor indicated in col. (4) to do the work indicated in col. (3). The data for this column were collected through some form of time recording, such as a time book or time cards.
6. Col. (6). The hourly rates of pay for the several classifications of labor are here entered. These values are the actual rates paid in the community in which the house was erected.
7. Col. (7). In this column will be found the total wages paid to the several labor classes required for the specific jobs indicated in col. (3). Where the specific job required only one kind of labor, the total cost is shown in one of the next two columns. This value equals the hours from col. (5) multiplied by the wage rate from col. (6).
8. Col. (8). In this column will be found the total cost for the specific jobs as indicated in col. (3).
9. Col. (9). The figures in this column give the costs of the work as indicated by the subdivisions of col. (2).
10. Col. (10). The figures shown here give the total cost of each major division of the work.

The money values from this breakdown were transferred to col. (4) of Table 6.1.

6.5. Breakdown of Material Cost.—An examination of Table 6.3 shows the following:

1. Columns (1) to (3) are similar to those in Tables 6.1 and 6.2.
2. Col. (4). Brief specifications of the materials to be used for several specific jobs listed in col. (3) are shown in col. (4).
3. Col. (5). The amounts of the materials used as shown in col. (4) are shown here. The units used are those commonly employed and associated with the respective materials.
4. Col. (6). The amount shown in this column is found by multiplying the quantity from col. (5) by the unit cost of the material. An additional column between cols. (5) and (6) giving the unit costs or prices would have been advantageous.
5. Col. (7). These figures represent the total costs of the specific jobs as shown in col. (3).

TABLE 6.2.—BREAKDOWN OF LABOR COST
House No. 5, Purdue Housing Research Project

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Labor classification	(5) Total hours	(6) Wage per hour	(7) Cost	(8) Job cost	(9) Sub-total	(10) Total
Excavation	Basement	Pick and shovel	Common	50	$0.40		$ 20.00		
		Team and man		16½	1.00		16.50	$ 36.50	
	Footings	Pick and shovel	Common	29½	0.40			11.90	$ 48.40
Foundations	Footings	Concrete work	Common	22	0.40			8.80	
	Walls	Masonry	Mason Mason's helper Common	35 15 16½	1.20 0.80 0.40	$12.00 12.00 6.60	60.60		
		Dampproofing Backfilling	Common Mixed	8½ 17	0.40		3.40 7.85	71.85	
	Girder and post	Framing	Carpenter	3	0.90			2.70	83.35
Concrete floors	Fill	Shovel work	Common	28	0.40			11.20	
	Concrete slabs	Mixing and pouring	Common Skilled	18½ 7	0.40 0.75	7.40 5.25	12.65		
		Finish	Skilled	7½	0.75		5.60	18.25	29.45
Accessories	Steps, etc.		Skilled	16½					12.70
Outside walls	Framing	Sills, plates, studs, etc.	Carpenter Apprentice carpenter	46 12	0.90 0.40	11.40 4.80	46.20		
		Nonproductive time	Carpenter	2	0.90		1.80	48.00	
	Sheathing		Carpenter Apprentice carpenter	36½ 20¼	0.90	32.85 8.85		41.70	
	Siding	Flush boards	Carpenter	64	0.90		57.60		
		Beveled siding	Carpenter	67	0.90		60.30	117.90	
	Insulation	Installation	Common	21	0.40			8.40	
	Interior finish	Plywood and board	Carpenter	65½	0.90			58.95	274.95

TABLE 6.2.—BREAKDOWN OF LABOR COST.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Labor classification	(5) Total hours	(6) Wage per hour	(7) Cost	(8) Job cost	(9) Sub-total	(10) Total
Partitions	Framing	Studs, plates, etc.	Carpenter	41	$ 0.90	$ 36.90			
			Apprentice carpenter	5½	0.40	2.20		$ 39.10	
	Wall finish	Plywood and board	Carpenter	85½	0.90			76.95	$ 116.05
1st floor	Framing	Joist, bridging, etc.	Carpenter	18½	0.90	16.65			
			Apprentice carpenter	2½	0.40	1.00	$ 17.65		
		Subfloor	Carpenter	4½	0.90	4.05			
			Apprentice carpenter	4	0.40	1.60	5.65	23.30	
	Flooring	Hardwood	Carpenter	22	0.90	19.80			
		Sanding	Carpenter	2½	0.90	2.25			
			Apprentice carpenter	7½	0.40	3.00	25.05		
		Linoleum base	Carpenter	6		6.60			
		Linoleum	Skilled	3	0.75	2.25	8.85	33.90	57.20
2d floor	Framing	Exposed beams	Carpenter	17½	0.90		15.75		
		Joist, etc.	Carpenter	11½	0.90		10.35		
		Subfloor	Carpenter	6	0.90	5.40			
			Apprentice carpenter	7	0.40	2.80	8.20	34.30	
	Ceiling finish	Plywood	Carpenter	8	0.90			7.20	
	Flooring	Hardwood	Carpenter	26	0.90	23.40			
		Sanding	Carpenter	3½	0.90	3.15			
			Apprentice carpenter	8½	0.40	3.40	29.95		
		Linoleum base	Carpenter	2		1.10			
		Linoleum	Skilled	1½	0.75	1.15	2.25	32.20	73.70
Roof	Framing	Joists, rafters, etc.	Carpenter	19	0.90	17.10			
			Apprentice carpenter	5¾	0.40	2.30	19.40		
		Sheathing	Carpenter	9	0.90		8.10	27.50	

TABLE 6.2.—BREAKDOWN OF LABOR COST.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Labor classification	(5) Total hours	(6) Wage per hour	(7) Cost	(8) Job cost	(9) Sub-total	(10) Total
Roof (*Continued*)	Roofing	Shingles	Carpenter	27½	$ 0.90			$ 24.75	
	Ceiling finish	Plywood	Carpenter	23	0.90			20.70	
	Insulation	Installation	Carpenter	2	0.90			1.80	$ 74.75
Millwork	Doors	Set frames	Carpenter	18	0.90		$16.20		
		Hanging and hardware	Carpenter	43	0.90		38.70	54.90	
	Windows	Set frames	Carpenter	17	0.90		15.30		
		Fitting and hardware	Carpenter	45	0.90		40.50	55.80	
	Trim	Ceiling and corner mold	Carpenter	21¾	0.90		19.60		
		Base mold	Carpenter	12½	0.90		11.25	30.85	
	Cabinets	Set cabinets	Carpenter	9	0.90			8.10	
	Screens	Hanging and hardware	Carpenter	13½	0.90			12.15	
	Shutters	Hanging	Carpenter	9½	0.90			8.55	170.35
Stairs	1st story	Rough	Carpenter	16	0.90		14.40		
		Finish	Carpenter	43½	0.90		39.15	53.55	
	Basement	Complete		8	0.90			7.20	60.75
Heating	Chimney*	Masonry	Mason Mason's helper	14½ 11½	1.20	$ 17.40 8.60	26.00		
		Scaffolding	Carpenter	2½	0.90		2.25	28.25	
	Fuel bin	Framing	Carpenter	5½	0.90			4.95	
	Pipe and fittings	Steam pipe	Steam fitter Helper	17½ 31	0.90 0.40	15.75 12.40		28.15	
	Equipment	Set boiler	Steam fitter Helper	6½ 11	0.90 0.40	5.85 4.50	10.35		
		Set radiators	Steam fitter Helper	6½ 13	0.90 0.40	5.85 5.20	11.05	21.40	82.75

* One-half labor cost charged to heating and one-half to fireplace.

TABLE 6.2.—BREAKDOWN OF LABOR COST.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Labor classification	(5) Total hours	(6) Wage per hour	(7) Cost	(8) Job cost	(9) Sub-total	(10) Total
Fireplace	Chimney*							28.25	
	Finish	Mantel	Carpenter	5½	0.90			4.95	33.20
Plumbing	Rough	Excavation	Common	5	0.40		2.00		
		Soil pipe	Plumber	18	0.90	16.20			
			Plumber's helper	16	0.40	6.40	22.60		
		Waterpipe	Plumber	12	0.90	10.80			
			Plumber's helper	5	0.40	2.00	12.80		
		Gas pipe	Plumber	2½	0.90	2.25			
			Plumber's helper	2½	0.40	1.00	3.25	40.65	
	Finish	Fixtures and fittings	Plumber	13½	0.90	12.15			
			Plumber's helper	13½	0.40	5.40		17.55	58.20
Electrical	Rough	Bx cable	Electrician	19	1.00		19.00		
		Switch boxes	Electrician	5	1.00		5.00		
		Fuse box	Electrician	1½	1.00		1.50	25.50	
	Finish	Switches and plates	Electrician	2	1.00		2.00		
		Fixtures	Electrician	4	1.00		4.00	6.00	31.50
Painting	Exterior	Prime coat	Painter	15	1.00		15.00		
		2d coat	Painter	15½	1.00		15.50		
		3d coat	Painter	14½	1.00		14.50	45.00	
	Shutters	Screens, etc.	Painter	21	1.00			21.00	
	Interior	Stain walls and ceiling	Painter	7	1.00		7.00		
		Fill and stain floors	Painter	8½	1.00		8.50		
		Shellac walls, ceiling, floor	Painter	34	1.00		34.00		
		Wax walls, ceiling, floor	Painter	21	1 00		21.00	70.50	136.50
Total labor cost									$1,343.80

* One-half labor cost charged to heating and one-half to fireplace.

TABLE 6.3.—BREAKDOWN OF MATERIAL COST
House No. 5 Purdue Housing Research Project

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Material used	(5) Quantity	(6) Cost	(7) Job cost	(8) Sub-total	(9) Total
Foundations	Footings	Concrete materials	Cement	25 sacks	$ 18.75			
			No. 6 gravel	31,130 lb.	15.55			
			Mixer		3.50		$ 37.80	
	Walls	Masonry	Prepared mortar	18 sacks	11.70			
			Cement	6 sacks	4.50			
			No. 3 sand	5,000 lb.	2.50			
			8″ × 16″ conc. block	980	106.65	$125.35		
		Dampproofing	Asphalt emulsion	10 gal.		7.50	132.85	
	Girder and post	Frame	6″ × 6″ fir	8 lin. ft.	1.45			
			2″ × 10″ and 4″ No. 1 Y.P.	150 bd. ft.	6.75		8.20	$ 178.85
Concrete floors	Fill	Fill materials	Gravel	17,890 lb.			8.95	
	Concrete slabs	Concrete materials	Cement	53 sacks	39.75			
			No. 6 gravel	31,650 lb.	15.80			
			Mixer		1.75		57.30	66.25
Accessories	Steps, etc.		Miscellaneous					11.35
Outside walls	Framing	Termite shield	20 gauge G.I :	120 ft.		10.00		
		Sills	2″ × 8″ No. 1 Y.P.	216 bd. ft.	8.65			
			Prepared mortar	2 sacks	1.30	9.95		
		Studding, etc.	2″ × 4″ No. 1 Y.P.	1,475 bd. ft.	59.00			
			16 and 8d nails	200 lb.	9.00	68.00	87.95	
	Sheathing		1″ × 8″ No. 2 Y.P.	2,280 bd. ft.	82.10			
			Nails	50 lb.	2.00		84.10	
	Building paper			3 rolls			7.50	

TABLE 6.3.—BREAKDOWN OF MATERIAL COST.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Material used	(5) Quantity	(6) Cost	(7) Job cost	(8) Sub-total	(9) Total
Outside walls (*Continued*)	Siding	Tongue-and-groove flush boards	1" × 6" redwood Belt coarse W. P. quoins Crown molds Nails	737 bd. ft. 72 lin. ft. 46 bd. ft. 20 lb.	$ 55.25 2.50 14.20 9.50 1.00	$ 82.45		
		Beveled siding	¾" × 8" redwood Nails	1,457 bd. ft. 20 lb.	94.70 1.00	95.70	$178.15	
	Insulation	Loose type	Rock wool	1,510 lb.			34.35	
	Interior finish	Plywood and board					116.05	$ 508.10
Partitions	Framing	Studs, plates, etc.	2" × 4" No. 1 Y.P. 16 and 8.1 nails	807 bd. ft. 55 lb.	32.30 2.75		35.05	
	Wall finish	Plywood and board					135.20	170.25
1st floor	Framing	Joists, etc.	2" × 8" No. 1 Y.P. Nails	585 bd. ft. 12 lb:	23.40 0.60	24.00		
		Subfloor	1" × 6" No. 2 Y.P. Bldg. paper	555 bd. ft. 1 roll	20.00 1.25	21.25	45.25	
	Flooring	Hardwood	1" × 2¼" oak Nails	478 bd. ft. 5 lb.	39.60 0.25	39.85		
		Linoleum base	1" × 3" T. & G. pine	150 bd. ft.	7.50			
		Linoleum	Std. gauge	12 sq. yd.	34.55	42.05	81.90	127.15
2d floor	Framing	Joist, etc.	2" × 8" No. 1 Y.P.	243 bd. ft.		9.70		
		Fir beams	2" × 4" × 8" fir	358 bd. ft.		33.75		
		Subfloor	1" × 6" No. 2 Y.P. 1" × 6" car siding Paper and nails	285 bd. ft. 353 bd. ft.	10.60 19.40 1.35	31.35	74.80	

TABLE 6.3.—BREAKDOWN OF MATERIAL COST.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Material used	(5) Quantity	(6) Cost	(7) Job cost	(8) Sub-total	(9) Total
2d floor (*Continued*)	Ceiling finish	Plywood					$ 16.25	
	Flooring	Hardwood	$\frac{13}{16}'' \times 2\frac{1}{4}''$ oak	247 bd. ft.	$ 20.55			
			$\frac{13}{16}'' \times 2\frac{1}{4}''$ maple	166 bd. ft.	16.20			
			1″ × 3″ E. G. pine	144 bd. ft.	12.95			
			Nails	10 lb.	0.50	$ 50.20		
		Linoleum base	1″ × 3″ T. & G. pine	50 bd. ft.	2.50			
		Linoleum	Standard gauge	4 sq. yd.	11.40	13.90	64.10	$ 155.15
Roof	Framing	Joists, rafters, etc.	2″ × 6″ No. 1 Y.P.	1,612 bd. ft.		64.50		
		Sheathing	1″ × 6″ No. 2 Y.P.	725 bd. ft.		26.10	90.60	
	Roofing	Shingles	16″ cedar shingles	38 bdles.	48.45			
			Galv. nails	26 lb.	2.35	50.80		
		Flashing	Metal			7.80	58.60	
	Ceiling finish	Plywood					54.75	
	Insulation	Granulated type	Rock wool	14 bags			9.45	213.40
Millwork	Doors	Frames, doors, garage door	W.P. fir panels	16		190.75		
		Hardware	Bronze			39.00	229.75	
	Windows	Frames, sash, weights	White pine	16		135.00		
		Hardware	Bronze			7.00		
		Louver	Copper back			8.00	150.00	
		Trim	Molds, shelving, etc.				4.70	
		Cabinets	Kitchen and bath	5			76.00	
	Screens	Door and window		16		45.00		
			Storm door	2		15.00		
			Hardware			4.00	64.00	
	Shutters		Pine				56.85	581.30

TABLE 6.3.—BREAKDOWN OF MATERIAL COST.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Material used	(5) Quantity	(6) Cost	(7) Job cost	(8) Sub-total	(9) Total
Stairs	1st story	Rough	2″ × 8″ No. 1 Y.P.			$ 4.50		
		Finish	Oak and cypress			80.00	$ 84.50	
	Basement	Complete	Yellow pine				30.45	$ 114.95
Heating	Chimney*	Masonry	Common brick	1,030	$ 13.90			
			Prepared mortar	6½ sacks	4.25			
			Cement	1 sack	0.75			
			No. 3 sand	2,125 lb.	2.15			
			Flue tile	14	15.55			
			Cleanout	1	1.60	38.20		
		Scaffolding	2″ × 4″ No. 1 Y.P.	32 bd. ft.		1.30	39.50	
	Fuel bin	Framing	2″ × 4″ No. 1 Y.P.	64 bd. ft.	2.55			
			1″ × 6″ No. 2 Y.P.	100 bd. ft.	3.60			
			Coal chute	1	5.50		11.65	
	Pipe and fittings	Steam pipe	1″ to 2″ steel pipe				25.45	
	Equipment		Furnace, etc.		120.00			
			Radiators		100.80		220.80	297.40
Fireplace	Chimney*						39.50	
	Finish	Brickwork	Firebrick and clay	150	10.25			
			Damper, etc.	3 pcs.	10.70	20.95		
		Mantel	Cypress	4 pcs.		15.00	35.95	75.45
Plumbing	Rough	Soil pipe, etc.	Std. C.I.			29.65		
		Water pipe	¾″ G.C.I.		15.00			
			½″ G.C.I.		7.20	22.20		
		Gas line	Black pipe			3.10	54.95	

* One-half materials and cast charged to heating and one-half to fireplace.

TABLE 6.3.—BREAKDOWN OF MATERIAL COST.—(*Continued*)

(1) General heading of work	(2) Subheading of work	(3) Specific job	(4) Material used	(5) Quantity	(6) Cost	(7) Job cost	(8) Sub-total	(9) Total
Plumbing (*Continued*)	Finish	Fixtures and fittings	Tub, lavatory, sink—steel					
			Water closet—vit. china		$ 86.70			
			Laundry tubs		8.30			
			Gas water heater		15.00		$110.00	$ 164.95
Electrical	Rough		BX cable	610 lin. ft.	20.70			
			½" conduit	10 lin. ft.	2.25			
			Outlet boxes	54	5.05			
			Transformer	1	1.60			
			Fuse and switch	1	5.50		35.10	
	Finish		Switch and plates	13	2.45			
			Duplex outlets	21	4.55			
			Door bells	2	0.95			
			Fixtures	15	43.65		51.60	86.70
Painting	Exterior	Prime coat	Tung oil	3 gal.	12.00			
			Lead and oil	1 gal.	2.45	$ 14.45		
		2d and 3d coats	Lead and oil	1½ gal.	3.70			
			Mixed paint	7½ gal.	20.40	24.10	38.55	
	Shutters	Screens, etc.	Mixed paint	3 gal.			9.10	
	Interior	Floors, walls, ceilings	Wax	2½ gal.	8.15			
			Filler	20 lb.	3.40	11.55		
			Shellac	6 gal.	11.40			
			Stain	2½ gal.	6.25	17.65	29.20	76.85
Total material cost								$2,828.10

6. Col. (8). These figures represent the total costs of the subdivisions of the work as shown in col. (2).

7. Col. (9). These figures are the total costs of the major divisions of the work as listed in col. (1).

The money values from this breakdown were transferred to Table 6.1.

6.6. Breakdown by Trades.—The breakdown in Table 6.4 is self-explanatory. The various items that have been developed on the breakdowns of labor and material cost have been reassembled by construction trades. This breakdown is very useful in estimating, as will be shown later.

TABLE 6.4.—COST BREAKDOWN BY TRADES

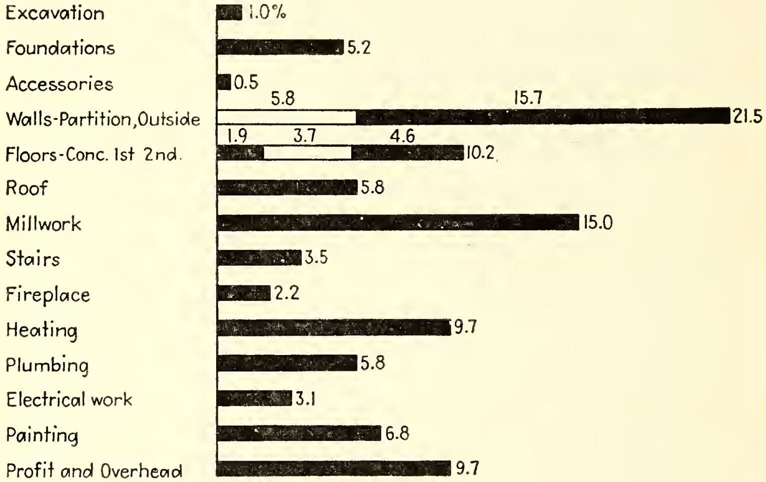
Construction item	Cost	Per cent
Excavation and backfill	$ 68.55	1.4
Masonry	507.35	10.2
Lumber and supplies	812.35	16.3
Millwork	630.80	12.6
Plywood and interior finish	322.25	6.5
Carpenter labor	826.75	16.6
Insulation	54.00	1.1
Hardware	50.00	1.0
Linoleum	49.35	1.0
Heating	400.00	8.0
Plumbing	290.60	5.8
Electrical work	153.65	3.1
Painting	336.00	6.7
Profit and/or overhead (general contractor only)	484.35	9.7
Total	$4,986.00	100.0

6.7. Relative Cost Percentage of Construction.—The breakdown in Table 6.5 is self-explanatory. The various cost items which have been developed on the breakdowns of labor and material costs have been reassembled by major divisions of the structure. The dollar values of this data are contained in col. (9) of Table 6.1.

6.8. Square Foot and Cubic Foot Costs.—Costs of houses per square foot and per cubic foot offer only a quick rule of thumb or rough approximation to the cost of a structure. Their usefulness is limited to an approximate comparative cost of similar construction. When conditions vary in any particular, due allowances must be made if the figures are to be used in arriving at the approximate cost of other new construction.

Based upon the total area of the basement, first floor, and second floor, the cost per square foot of House No. 5 was $2.72. This figure is based upon the contract cost of the house, $4,986, and a gross area of 1,836 sq. ft. The gross area was taken as the space enclosed within the lines of the outside face of the walls of the basement, first story, second story, and garage. Since basement construction is less than that for living spaces above, it can, if desired, be assumed to cost about one-half as much. The basement area is 543 sq. ft. Upon the basis of the total area of the first and second floors and garage and one-half the area of the basement, the cost per square foot was $3.18.

Table 6.5.—Relative Cost Percentages of Construction



The cubic content is the space enclosed with the outer surfaces of the outside walls, a plane 6 in. below the basement floor, and the outer surfaces of the roof. In arriving at the cost per cubic foot, the basement may be included at either its full or one-half of its full cubage. Using the full cubage of the basement, the cost was $0.28 per cubic foot, the cubic contents being 17,800 cu. ft. If the cubic contents of the basement are assumed to be one-half of the actual, the amount is 15,790 cu. ft. and the cost $0.31 per cubic foot.

6.9. Detailed Cost Estimate.—The several analyses of the Purdue House No. 5 have shown how the actual costs for its construction have been assembled and later summarized. In the same manner, an estimated cost for construction may be made. The principal difference, as before stated, is that quantities and rates are probable rather than actual. The estimator uses his knowledge of the work to approximate the probable values for quantities of labor and materials and for the probable unit costs of the same. The steps that must be followed in the preparation of a detailed cost estimate are as follows:

1. Divide the work that is to be estimated into several divisions (usually structural rather than trade).
2. Further divide the major divisions into natural subdivisions.
3. Again divide the subdivisions into specific jobs (usually distinguished by types of materials).
4. Determine, for each specific job, the kind of material and quantity required and the class and amount of labor necessary.
5. Ascertain, for the several kinds of materials and classes of labor, the unit rates for which they should be charged. Unit costs of materials may be obtained from those who sell building supplies. Hourly rates of labor wages may be obtained from constant contact with labor and with other builders.
6. Determine the job cost of materials and labor for the specific jobs.
7. Determine the total cost for subdivisions.
8. Determine the total cost for major divisions.
9. Summarize the major costs to get the gross total cost estimate.

Thus it is seen that the preparation of a detailed estimate of cost consists of four major operations:

1. The analysis of the completed project into major divisions or assemblies and these into subdivisions or subassemblies, and these in turn into specific jobs
2. The analysis of the specific jobs into kinds of materials and classes of labor necessary to do them
3. The assignment of rates of labor wages and unit costs of materials
4. The summarization of the quantities and rates for the respective specific jobs to subassemblies to major assemblies to the total gross cost estimate.

6.10. Method of Estimation.—The method of estimating just described is a detail method. There are a number of short methods which may be used if justified. The method of estimation may depend upon[1]

1. The purpose of the estimate.
2. The available data
3. The time available for making the estimate

6.11. Purpose of the Estimate.—The purpose of the estimate may be such that it requires any one of the following classifications:[1]

1. Roughly approximate
2. Fairly approximate
3. Closely approximate.

The order of accuracy and detail is from a roughly approximate to a closely approximate estimate. This variation may be illustrated by referring to the several steps that make up the engineering process as presented in the Introduction. In the early stages of the process any estimates that are made are very rough because they are made on insufficient data. They are made then with the idea of trying to get a picture of the project at that stage even though it may be inaccurate and will require clarification later.

As the development of the project advances, the estimates required become fairly approximate and then closely approximate when the project is actually put into construction or operation.

6.12. Available Data for the Estimate.—The ability to make an estimate depends largely upon the amount of data available. Obviously, in the early stages of the engineering process the available data are scanty and subject to wide variations. But as the project proceeds, the data become more accurate and more are available. It is not unusual for estimates to be constantly revised as new data become available.

6.13. Time Available for Making Estimates.—The choice of a method may very often depend on the time that is available for making the estimate. Thus, the detailed method is the most accurate and at the same time requires the most time to prepare. Short methods may be used when time is short, but it should be clearly understood that their accuracy is impaired by the haste required in their preparation.

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[1] Fish, J. C. L., "Engineering Economics." p. 66, McGraw-Hill Book Company, Inc., New York, 1923.

6.14. Short Methods of Estimating.—When the detailed method of estimating previously discussed is examined, a number of short methods will be suggested, as follows:

1. Subassemblies
2. Ratio
3. Similar
4. Units
5. Index number method.

6.15. Subassemblies Method.—When the costs of parts, subassemblies, or assemblies are known, the estimating may start at that point. Obviously this reduces the time in which the estimate may be made. Its accuracy depends upon the accuracy of the data. If the data include things that have a stable price level or show very slight fluctuations, then the estimate may be said to be closely approximate. If, however, the materials have wide fluctuations, then the estimate may be said to be fairly approximate. Estimates for projects requiring the assembly of fabricated parts may be prepared by this method with speed and accuracy, since the prices for such parts are easily obtained from manufacturers.

6.16. Ratio Method.—In col. (10) of Table 6.1 will be found a per cent figure, which represents the part that each major division bears to the entire cost. If the estimator has in his files sufficient data for the type of work that he does, he may be able to use this method. These data should contain cost summaries such as have been illustrated in the case of the house. The ratio that each major part bears to the total cost should be a part of these data. It is necessary to figure the cost of only one major division in detail. By applying the percentage figure that it bears to total cost, an estimate can be made which will be fairly accurate depending upon the wealth of data on file.

Example: It is required to estimate the cost of building a house similar to the one illustrated. Suppose the estimator uses the major division, Outside Walls. From the data, in this case Table 6.1, it is seen that the ratio is 15.7 per cent.

The estimator calculates in a detailed manner that the cost of the outside walls of the new house figured at present prices of materials and labor will be $1,000. The total cost of the new house under the same conditions would be found as follows:

$$\text{Estimated cost} = (\text{new detailed estimate})\left(\frac{100 \text{ per cent}}{15.7 \text{ per cent}}\right)$$

$$\frac{(\$1{,}000)(100)}{15.7} = \$6{,}639$$

In a like manner the date in Table 6.4 may be used. The estimator calculates in a detailed manner that the cost of carpenter labor of the new house figured at present wage rates will be $1,050. The total cost of the new house would be found as follows:

From Table 6.4, carpenter labor is 16.6 per cent.

$$\begin{aligned} \text{Ratio} &= 100 \text{ per cent}/16.6 \text{ per cent} \\ \text{New estimate} &= (\$1{,}050)(100)/16.6 = \$6{,}325 \end{aligned}$$

6.17. Similar Method.—Sometimes the project for which an estimate is wanted is very much like another project previously constructed. An estimate of the cost of the new one may be derived as the cost of the previous one plus or minus an amount which represents the difference between the projects. This difference may be estimated by a detailed method or by the ratio method.

This method of estimating also may be used to take care of cases where parts are added to or substracted from the original project. When the type of material is changed, this method may be used to estimate the difference.

6.18. Unit Method.—In Sec. 6.8 another method of estimating was indicated. The total cost, by this method, is determined by multiplying the total number of units, in that case square feet or cubic feet, by an estimated cost per unit. Other units may be used depending upon the nature of the thing to be estimated. Machinery, equipment, and power-plant piping may be estimated in units of pounds or tons.

6.19. Index Number Method.—Index numbers of construction cost are designed to measure the variations in costs of construction. They are based on a selected list of commodities and services, costs for which are assembled for specific quantities of each.

One of the most successful services is that of the Boeckh system,[1] sponsored by E. H. Boeckh and Associates, Inc., Appraisal Engineers. They have prepared a manual containing index numbers for various types of construction and for the various parts of the country. These indices are basic and represent comparative average costs for the years 1926 to 1929. After having made an analysis in accordance with the Boeckh routine, a cost per cubic foot is obtained. Multiplied by the cubic contents, this gives the cost as of the base year. This figure must now be corrected by applying an index of change since the base year. This change index is made up by the Boeckh company on the basis of certain local prices of a specific list of commodities and services. It can be obtained only from them.

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[1] Boeckh, E. H., "Manual of Appraisals," 3d ed. The Rough Notes Company, Inc., Indianapolis, Ind., 1937.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following projects, proposals, or situations has been brought to your attention. *See problems for Chap.* 1.

You are asked to prepare an outline of the data that need to be collected, the estimates that must be made, and any pertinent information that seems necessary to enable you or your employers to evaluate the project, proposal, or situation and that will assist them in arriving at a decision and a plan of action.

Prepare a report on a plan for estimating the first cost for your assigned project.

Determine the first cost for the following projects using the data given in the same lettered problems in Chap. 1.

A. A proposed commercial hotel, 60 by 100 ft. The cost of land is $300,000; building construction, 800,000 cu. ft. at 85 cents per cubic foot; architect's fee, 5 per cent of cost of construction; taxes during construction, $8,000; carrying charges during construction, $115,500; furniture and furnishings, $100,000; contingencies, $32,500.

B. A central concrete-mixing plant. The plant will consist of the following parts for a $1\frac{1}{2}$-cu. yd. plant:

Storage and scales building	$3,500
For cement storage, scales, sand and stone bins, ramp to top of bins	
Feeder truck	2,000
To haul sand and stone to bins	
Mixer truck	4,500
Heater	500
To keep materials warm in winter	

C. Selection of central-station prime mover; Diesel vs. uniflow engine. The installed capacity is 2,000 kw.

Since the first cost of equipment generally follows the form of the curve $y = mx + b$, in which x is in kilowatt, it is possible to estimate the first cost if the elements m and b are available. Determine the first cost of the two plants from the following data which are in terms of installed capacity from 500 to 15,000 kw.

Diesel Plant

Item	b	m	x
Real estate	$ 5,600	$ 1.20	kw.
Building	50,000	28.50	kw.
Piping	4,750	7.12	kw.
Generating units (4)	100,000	111.00	kw.
Switchboard	13,500	2.00	kw.
Wiring	9,975	1.60	kw.
Engine-room crane	1,900	0.56	kw.
Equipment foundations	6,700	3.50	kw.
Cooling water supply	1,500	7.30	kw.
Fuel oil storage	3,000	3.34	kw.
Railroad siding	640	9.50	kw.

Uniflow Engine

Item	b	m	x
Real estate	$ 5,600	$ 1.20	kw.
Buildings	75,000	56.00	kw.
Boilers and settings	21,000	8.00	kw.
Stokers	2,550	4.15	kw.
Condensers	4,500	3.70	kw.
Piping	43,500	7.25	kw.
Generating units (4)	85,700	25.00	kw.
Switchboard	13,500	2.00	kw.
Wiring	9,975	1.60	kw.
Chimney and flue	15,950	1.00	kw.
Forced-draft fans and ducts	4,100	9.30	kw.
Coal and ash handling equipment	28,300	3.00	kw.
Engine-room crane	1,900	0.32	kw.
Feed-water heater	640	0.96	kw.
Feed pumps	3,500	0.25	kw.
Equipment foundations	12,000	5.00	kw.
Oil filters and tanks	2,250	0.13	kw.
Cooling water supply	400	3.05	kw.
Railroad siding	640	9.50	kw.

D. A milk-bottle cap plant. The cap consists of two pieces, an outer one that fits on the bottle top and seals it and an inner one that fits inside the outer one and has imprinted on it the name of the dairy. The following equipment is estimated to be sufficient for the production of 1,000,000 tops per day of 8 hr.

Outer cap: 1 blanking press at $3,400, dies $1,600
Inner cap: 1 blanking press at $2,300, dies $500
Printing inner disk: 1 printing press at $1,200, dies $200
Forming outer disks: 14 forming presses at $1,530, dies $1,400
Tubing and carbon packing: 1 tubing machine at $500
Building (100 by 100 ft.) at $2.24 per square foot
Working capital: cost of one month's production

E. A cold-storage locker plant. A study of the plant shows that each locker should be large enough to store about 300 lb. of meat or measure about 18 in. by 18 in. by 3 ft. deep. For this size of plant it is found that a single-story building 42 by 50 ft. will be required to house the lockers, refrigeration machinery, preparing room, and office. The building will have about 20,000 cu. ft.

Determine the first cost of the proposition with the following data:
Building: $0.261 per cubic foot
Lockers and insulation: $7.15 each
Refrigerating machinery and other equipment: $16.55 per locker
Land (50 by 100 ft.): $1,000
Working capital: $1,200

CHAPTER 7

OPERATING COST

7.1. Operating Cost.—Operating cost is the sum of all the costs that are incurred periodically and continuously in order that the project in which an investment has been made may be operated, produce a commodity or service, and distribute and sell that commodity or service.

7.2. Producton Cost.—This part of the operating cost includes all periodic costs that are necessary for the production of the product or service. The examples used in Chaps. 1 and 2 will be used here to illustrate production costs.

The tunnel (see page 10) under the Delaware River between Philadelphia and Camden has been completed. The building contractor is about to turn over to the company the completed project. The business of putting the tunnel into operation and continuing its operation is the problem of operating cost. There will have to be many people employed for various things. Some will collect tolls, others will police the tunnel and keep traffic moving, others will operate the exhaust fans, others will take care of the lighting system, an office will be maintained for the usual office work of a company, and others will clean the tunnel. Supplies of many kinds will be necessary so that these various people may function properly. An office building will have to be maintained. It and the tunnel will have to be lighted, ventilated, and heated as well as cleaned. The property, if privately owned, will have to pay taxes. Fire insurance and workmen's compensation will be carried by the company. Fees for the use of other property or of payments to municipalities will probably be necessary. There will be officers of the company to serve in various capacities.

The company that has added vacuum cleaners to its line (see page 11) will also face a multitude of expenses of various kinds. Labor of various kinds will be needed to operate the machines with which the product is made. The services of other persons will be required to assist them in their work. Materials will have to be specified and purchased. On their receipt, these must be checked and stored. Later they are delivered to the production departments where the machine operators fabricate them into the finished product. Supplies of various kinds, such as brooms, oil, paper, pencils, and ink, will be needed by the supervisory force. Since investment in equipment and machines must be recovered during their useful life, a charge must be made to cover it. This is called the "allowance for depreciation." Other miscellaneous expenses are also to be included: taxes, fire insurance, workmen's compensation, royalties for the use of patents owned by others, the services of accountants, lawyers, engineers, and others. Interest must be paid on the short-term borrowings of the company.

Another item of operating cost to be considered is development cost, which was discussed in Chap. 1. As was explained in Sec. 2.9 a distinction must be made between development cost for a construction project for a production project. In accordance with that explanation development cost is considered a part of operating cost for a production project but not for a construction project.

7.3. Sale and Distribution of Product.—Miles before a motorist reaches the tunnel, he sees signboards and notices calling his attention to it. He reads of it in his automobile guidebook and sees it marked on the road maps. Advertising like this is to bring to the attention of the public the fact that there is such a tunnel under the river. People must be hired to design and place such advertising material. Signboards, literature, and other materials must be bought. Offices must be maintained to carry on this work. Salaries are paid for those who supervise the work; taxes and fire insurance are charged on the office. Workmen's compensation is carried for its employees.

In order to sell the vacuum cleaner, it must be brought to the attention of the public. Accordingly, the company secures the services of an advertising concern to plan advertising campaigns and place advertisements in the daily papers and various magazines. A group of persons is trained to go from door to door to sell the product. Arrangements are made with wholesale houses and department stores to distribute the product. Advertising literature is furnished to dealers throughout the country.

A sales office is maintained for this work. As in the case of the tunnel company, it must pay taxes, various forms of insurance, and other miscellaneous expenses.

7.4. Management.—In both cases, tunnel as well as vacuum cleaner, it is necessary to have a group of people called an "organization" to administer and manage the respective company. Each will be obliged to pay salaries and wages. Office expenses for materials and other items are included in the operating cost of management.

7.5. Divisions of Operating Cost.—When production costs and distribution costs are studied, it will be seen that they fall into classifications of labor, materials, and expense.

7.6. Labor Cost.—Labor cost was defined in Chap. 1 as the hire of any person to do whatever work he is skilled in doing. As thought of in connection with production, it is the cost of the labor that is used directly in the production of the commodity. It refers particularly to the labor doing the actual work on the product. It is called "direct" labor. It would include operators of machines and equipment that directly handle the material out of which the product is made. In the case of an enterprise in which the product is a service, it would include those persons whose labor could be intimately tied up with specific production of the service. For example, on a railroad, direct labor would include the train crew on a particular run.

The sure test of whether or not labor is direct is to determine by analysis whether or not the time at work can be charged specifically to some operation required in the production of the commodity. The work of foremen, clerks, and other assistants is generally classed as "indirect," because their labor during a day is spent on so many production operations that it would be quite expensive to keep a time record of their activity. It is, however, usually distributed over the production operations on the basis of a rule for expense distribution to be discussed later. Hence, the term "indirect" labor is obtained, meaning that such labor is indirectly charged to the production operations. Indirect labor is, therefore, classed with items of operating expense.

Labor for the distribution and sale of the product or service is generally classed as indirect because of the inability to tie the actual time spent with specific products. However, of late there has been awakened the need for closer analysis of labor costs in this field and attempts are being made to determine specific charges for labor cost.

7.7. Materials Cost.—Materials for production cost are the actual things used in making the product. They are called "direct" when they are actually the materials out of which the product is made. For the vacuum cleaner, they would be aluminum, metal tubing, rubber tubing, electric wire, and many others. Direct material is similar in determination to direct labor.

As there is indirect labor in labor cost, so there is indirect material in material cost. These materials are such things as coal, lubricating oil, brooms, stationery, and many others which are used in work coincident with production but which cannot be directly charged and allocated to specific operations.

7.8. Expenses.—This division of production costs includes expenses for things and services that are difficult to allocate directly to some part or operation.

Labor is represented as indirect in such groups as administration and management, foremen, draftsmen, time study men, inspectors, planning clerks, office force, engineers, and others.

Materials are represented as indirect in such things as office and drafting room supplies, stationery and others.

The cost of services and things that cannot readily be classified as labor or materials is also a part of this group. Such things are licenses, permits, charters, fees, franchises, rights of way, rents, taxes, insurance, lawyer's fees, consultant's fees, telephone and telegraph charges, depreciation, and many others.

Development cost is also an operating cost. It is neither labor nor material yet it is charged against the product in terms of so much per quantity of production. It is akin to royalty cost, which is the allocated proportional cost for patent rights charged against products that benefit from the patents.

As suggested in Sec. 7.6, the costs of distribution have generally been considered as indirect or in the class of expenses. As such they include traveling expenses for salesmen, salaries for salesmen, advertising, shipping, packing, and many others. Wherever these costs can be associated directly with specific products or service, it is worth while to consider them as direct costs.

7.9. Example of Operating Cost.—A manufacturing plant is installing a special piece of machinery to manufacture a special bolt to be used in its product. The items of operating cost are suggested by the following outline:

Supervision of Operation and Production.—The work of this item is done by the production division of the plant under the direction of the general manager.

Items of Cost.—Labor: Workmen directly employed to operate the machine.

Materials: The raw materials out of which the bolts are made.

Expense: Indirect labor: engineers to design the product, devise methods of manufacture, set up machines, and inspect the product; office clerks to order, receive, store, and issue materials of all kinds; clerks required to keep cost accounts and other office routines and supervise production. Indirect materials: materials such as lubricating and cutting oils and other materials of a service nature. Miscellaneous expense: transportation for materials; depreciation charges for machinery heat, light, power, insurance of various kinds, etc.; various other general expenses including legal fees associated with production.

Sale and Distribution of Product.—The work of this item is done by the sales organization.

Items of Cost.—Labor: salesmen and other labor used in planning and executing sales policy.

Materials: the usual supplies needed by such an office force. It may include advertising materials.

Expense: various forms of indirect labor not assignable to particular products. Indirect materials, same as indirect labor. Miscellaneous expense, similar to those required by production process.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following projects, proposals, or situations has been brought to your attention. *See problems for Chap.* 1.

You are asked to prepare an outline of the data that need to be collected, the estimates that must be made, and any pertinent information that seems necessary to enable you or your employers to evaluate the project, proposal, or situation and that will assist them in arriving at a decision and a plan of action.

Prepare a report on the operating costs for your assigned project.

CHAPTER 8

ESTIMATING OPERATING COST

8.1. Estimating Operating Costs.—The definition of the term "estimating" used in Sec. 6.1 is also to be applied to the estimation of operating costs. As was shown in Chap. 7, it is made up of three elements; direct labor, direct material, and expense.

8.2. Estimating Direct Labor Cost.—This estimate is made by determining the product of the hours of labor multiplied by the wage rate. Wage rates may be assumed to be those actually prevailing in the specific factory or those existing in the community for the specific type of labor required. The hours of labor are a measure of the accomplishment of average labor of the type required in terms of the number of hours per unit of accomplishment. This unit may be some easily understood and definable and measurable unit, such as piece or part, pound, foot, quantity, or other and is determined either from past records or from motion and time studies made in the plant.

$$\text{Direct labor cost} = (\text{hours})\left(\frac{\text{wage rate}}{\text{hour}}\right) \tag{8-1}$$

or

$$\text{Direct labor cost} = (\text{quantity})\left(\frac{1}{\text{quantity/hour}}\right)\left(\frac{\text{wage rate}}{\text{hour}}\right) \tag{8-2}$$

$$\text{Direct labor cost} = (\text{quantity})\left(\frac{\text{cost}}{\text{unit}}\right) \tag{8-3}$$

8.3. Estimating Direct Material Cost.—This estimate is made by determining the product of the amount of material required multiplied by its cost. The amount of material is obtained from the plans or prepared bills of materials. The cost of the material per some unit of measurement is obtained from current price lists or from direct quotations received from vendors or suppliers of the material in question. The unit for cost may be pieces, parts, pounds, feet, or other.

$$\text{Direct material cost} = (\text{quantity})\left(\frac{\text{cost}}{\text{unit}}\right) \qquad (8\text{-}4)$$

8.4. Estimating Expense.—The amount of expense for a production project is the prorata share of the total expense for the operating year. The prorata share is determined by applying the following formula:

$$\text{Prorata share or rate per base} = \frac{\text{total yearly expense}}{\text{unit of allocation or base}} \qquad (8\text{-}5)$$

Example: The items of expense for a certain enterprise for a typical year were as follows:

Rent	$1,600.00
Heat	500.00
Light	280.00
Fire insurance	176.00
Power	720.00
Foreman's salary	1,500.00
Repairs	850.45
Miscellaneous	271.13
Depreciation	2,733.33
Total	$8,630.91

The unit of allocation, generally called the "base," may be anything that is representative of the enterprise and that is easily measured for the project under consideration. Such bases as direct labor cost (the total wage paid to direct labor in the year), direct material cost (the total cost of all direct materials used in a year), prime cost (the sum of direct labor and direct material costs for the year), or direct labor hours (the total number of hours that direct labor works in a year) are used in industry.

Example: For the foregoing problem, assume,

1. Yearly direct labor cost: $10,080
2. Yearly direct material cost: $26,045
3. Yearly prime cost: $36,125
4. Yearly direct labor hours: 28,800 hr.

By applying these values in the formula, the following prorata shares are calculated:

1. Direct labor cost $\dfrac{\$8{,}630.91}{\$10{,}080} = \$0.856$ per direct labor dollar
2. Direct material cost $\dfrac{\$8{,}630.91}{\$26{,}045} = \$0.331$ per direct material dollar

3. Prime cost $\frac{\$8{,}630.91}{\$36{,}125} = \$0.238$ per dollar of prime cost
4. Direct labor hours $\frac{\$8{,}630.91}{28{,}800} = \$0.299$ per hour

The application of these prorata shares, or rates as they are called, may be made by using the following project or job. The enterprise, for which the data in the illustrative example have been given, manufactures school seats. It has received an order for 15 of them. The factory office has estimated the following labor and material cost data:

1. Direct labor cost: $98.10
2. Direct material cost: $230.49
3. Prime cost: $328.59
4. Direct labor hours: 255 hours

The formula for application is formula (8-5) rearranged:

$$\text{Expense for project} = (\text{unit of allocation or base}) \times (\text{rate}) \qquad (8\text{-}6)$$

1. Direct labor cost = (\$98.10) × (\$0.856) = \$83.97
2. Direct material cost = (\$230.49) × (\$0.331) = \$76.29
3. Prime cost = (\$328.59) × (\$0.238) = \$78.20
4. Direct labor hours = (255) × (\$0.299) = \$76.24

It is noted that there are four different values for the prorata expense for the seat job. This is because of the nature of the several methods.

The use of direct labor cost as a method of allocation may result in unjust distribution of expense. Jobs with high wage rates on inexpensive equipment would be charged a high expense, which would not be warranted; jobs with low wage rates on expensive equipment would be charged an unusually low expense, which would also not be warranted. There is difficulty in finding any correlation between wage rates and the expense of owning and operating a piece of equipment.

The same conclusion is also true of direct material cost. The fabricating of expensive material may be done on inexpensive equipment and that of inexpensive material on expensive equipment. Prime cost, which is the sum of direct labor cost and direct material cost, combines the characteristics of both and tends to give highly uncertain results.

Direct labor hours, on the other hand, are readily related to any piece of equipment. The expense rate that is determined by using it as a measure is roughly equivalent to a wage paid for the use of equipment.

8.5. Departmental Rate.—The method of calculating the prorata share of expense for a certain project explained in Sec. 8.4

is the simplest, easiest, and most applicable for very small enterprises.

As enterprises become larger, the characteristics of the equipment and machinery change. They become grouped in large departments with similar machines in the several departments. Some may even not have machines in them. For these reasons and others that are the special province of cost accounting, the single rate per hour as calculated in Sec. 8.4 is seriously in error. One department may have large machines requiring a great investment and another small machines with little investment. It is obvious that the department with large machines is more expensive to operate than the one with small machines. Hence, the use of the single rate would penalize products made largely in either department. If made in the department with large machines, the expense is underestimated; if made in the department with small machines, the expense is overestimated.

Further refinement is necessary. It is usually done on the basis of departmentalization. Example, The Harvey Seat Company has five departments; *A*, *B*, *C*, *D*, and *E*. It employs one man for each machine with a foreman in charge. Last year the factory expense items were as follows:

Rent, $1,600; heat, $500; light, $280; fire insurance, $176; power, $720; foreman's salary, $1,500.

Repairs (by departments):

A, $214.70; *B*, $345.75; *C*, $190.14; *D*, $82.27; *E*, $17.59.

Miscellaneous expense (by departments):

A, $118.72; *B*, $92.57; *C*, $29.43; *D*, $21.07; *E*, $9.34.

Value of equipment (by departments):

A, $15,500; *B*, $45,000; *C*, $8,000; *D*, $7,500; *E*, $6,000.

The factory operated 2,400 hours last year. The men in departments *A*, *B*, and *C* are paid at the rate of 40 cents per hour; in *D* and *E* at 30 cents per hour.

The company has just completed an order for 15 school seats for which the requisitions for direct material were as follows:

Department *A*	Department *B*	Department *C*	Department *D*	Department *E*
$27.30	$19.22	$42.15	$7.31	$5.07
19.43	3.47	9.21	9.24	6.92
23.18	11.19	1.37	3.17	1.41
7.94	13.41	16.25	4.25	

The direct labor hours on the order were as follows:

Department *A*	Department *B*	Department *C*	Department *D*	Department *E*
24	14	32	21	18
22	19	30		
18	21	19		
	17			
Totals 64	71	81	21	18

It is desired to distribute the expense, make a summary of the factory cost of the order, and compute the cost per chair.

Since the three elements of cost are direct labor, direct materials, and expense, it will be well to determine them in turn.

DIRECT LABOR

Department *A*		Department *B*		Department *C*		Department *D*		Department *E*	
Hours	Cost	Hours	Cost	Hours	Cost	Hours	Cost	Hours	Cost
24	$ 9.60	14	$ 5.60	32	$12.80	21	$6.30	18	$5.40
22	8.80	19	7.60	30	12.00				
18	7.20	21	8.40	19	7.60				
		17	6.80						
Totals 64	$25.60	71	$28.40	81	$32.40	21	$6.30	18	$5.40

Grand total for direct labor is $98.10.

DIRECT MATERIAL

Department *A*	Department *B*	Department *C*	Department *D*	Department *E*
$27.30	$19.22	$42.15	$ 7.31	$ 5.07
19.43	3.47	9.21	9.24	6.92
23.18	11.19	1.37	3.17	1.41
7.94	13.41	15.25	4.25	
Totals $77.85	$47.29	$67.98	$23.97	$13.40

Grand total for direct material is $230.49.

The expense figures, as given, were not for the order of 15 school seats but for the entire year. The problem is to determine how much of these expenses should be charged against this particular order so that the cost of each seat may be determined.

An examination of the expense figures shows that some part of each should be charged to each department in the plant.

First, there is prepared the Expense Analysis Sheet. On it are indicated the expense item, the selected base, the amount of the expense, the amount of the base, and lastly the rate per base. This last is determined by applying the equation (8-5).

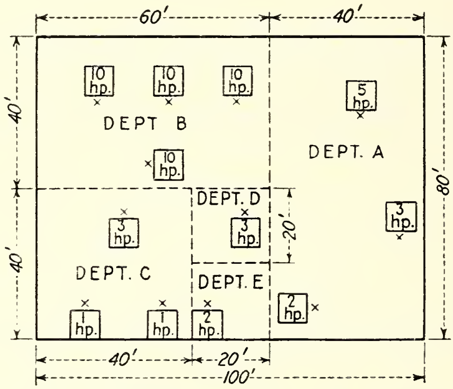

Fig. 8.1.—Floor plan and machine layout of the Harvey Seat Company.

The type of base to be used depends upon the character of the expense item. For example, rent expense is associated with the area or floor space of the plant; power expense is associated with the horsepower demands of each department. The quality selected as a base is one that is easily measured. The following list indicates some of the common bases:

Rent: Floor space

Taxes: Floor space

Heat: Floor space where heating requirements are similar; otherwise, square feet of radiation multiplied by a measure of heat loss

Light: Floor space where lighting requirements are similar; otherwise, on the basis of candle power or wattage of light source

Workman's compensation insurance: \$100 units of pay roll multiplied by the risk classification

Fire insurance: Floor space where risk is similar; otherwise, on floor space multiplied by rate per class of risk

Indirect labor and supervision: Number of direct labor hours or total number of labor hours (both direct and indirect)

Supplies and miscellaneous: Estimated from an analysis

Power: Horsepower connected. In the case of alternating current the power factor must be considered.

Depreciation: As previously discussed

Development charge: Prorated against the estimated amount of production

The method estimating operating expense can best be illustrated as follows (see Fig. 8.1):

EXPENSE ANALYSIS SHEET

Expense item	Base	Amount of expense	Amount of base	Rate per base
Rent............	Floor space	$1,600	8,000 sq. ft.	$ 0.20
Heat............	Floor space	500	8,000 sq. ft.	0.0625
Light............	Floor space	280	8,000 sq. ft.	0.035
Fire insurance....	Floor space	176	8,000 sq. ft.	0.022
Power...........	Horsepower	720	60 hp.	12.00
Foreman's salary..	Number of men	1,500	12 men	125.00
Depreciation.....	Life, no salvage value	Various amounts per department	30 years	

Second, there is prepared the Base Analysis Sheet. On it are indicated the selected bases and their distribution among the several departments.

BASE ANALYSIS SHEET

Base item	Department *A*	Department *B*	Department *C*	Department *D*	Department *E*	Total
Floor space.......	3,200	2,400	1,600	400	400	8,000
Horsepower......	10	40	5	3	2	60
Number of men...	3	4	3	1	1	12
Depreciation......	30 years	30 years	30 years	30 years	30 years	30 years

Third, there is prepared the Expense Distribution Sheet. On it are indicated the expense items and their distribution among the several departments. These amounts are determined by applying the equation

Amount of expense per department
= (amount of base) (rate per base)

Example: Distribution of rent.

Rate per base = \$0.20 per square feet (from Expense Analysis Sheet)

Area of Department *A* = 3,200 sq. ft. (from Base Analysis Sheet)

Amount of expense Department *A* = (\$0.20)(3,200) = \$640

EXPENSE DISTRIBUTION SHEET

Expense item	Total amount	Department *A*	Department *B*	Department *C*	Department *D*	Department *E*
Rent	\$1,600.00	\$ 640.00	\$ 480.00	\$ 320.00	\$ 80.00	\$ 80.00
Heat	500.00	200.00	150.00	100.00	25.00	25.00
Light	280.00	112.00	84.00	56.00	14.00	14.00
Fire insurance	176.00	70.40	52.80	35.20	8.80	8.80
Power	720.00	120.00	480.00	60.00	36.00	24.00
Foreman's salary	1,500.00	375.00	500.00	375.00	125.00	125.00
Repairs	850.45	214.70	345.75	190.14	82.27	17.59
Miscellaneous expense	271.13	118.72	92.57	29.43	21.07	9.34
Depreciation	2,733.33	516.66	1,500.00	266.67	250.00	200.00
Totals	\$8,630.91	\$2,367.48	\$3,685.12	\$1,432.44	\$642.14	\$503.73

Fourth, there is prepared the Rate per Department Sheet. This shows the determination of the departmental rate. The base used is the hours of machine time which corresponds to the hours of direct labor. Department *A* has three men who work 2,400 hr. per year. Hence, the total direct labor hours per year for Department *A* is 7,200. The rate per hour is then equal to the expense per year from the Expense Distribution Sheet of \$2,367.48, divided by 7,200 machine hours per year or \$0.329 per hour.

RATE PER DEPARTMENT SHEET

	Department *A*	Department *B*	Department *C*	Department *D*	Department *E*
Expense amount	\$2,367.48	\$3,685.12	\$1,432.44	\$642.14	\$503.73
	7,200	9,600	7,200	2,400	2,400
Rate per hour	\$0.329	\$0.384	\$0.199	\$0.267	\$0.210

Since it is known from the cost data how many direct labor hours were spent in each department, it is easy to get the expense by applying the equation. The results are shown in the following tabulation:

ACTUAL EXPENSE PER DEPARTMENT FOR ORDER OF 15 SCHOOL SEATS

Department	Direct Labor Hours	×	Rate per Hour	=	Departmental Expense
A	64	×	$0.329	=	$21.06
B	71	×	0.384	=	27.26
C	81	×	0.199	=	16.12
D	21	×	0.267	=	5.61
E	18	×	0.210	=	3.78
					$73.83

RECAPITULATION OF COST FOR 15 SCHOOL SEATS

Direct labor	$ 98.10
Direct material	230.49
Expense	73.83
Total	$402.42

$$\text{Cost per unit} = \frac{\$402.42}{15} = \$26.83$$

8.6. Machine Hour Rate.—Should the results of Sec. 8.5 be considered not accurate enough, the method can be extended to the individual machine in each department.

8.7. Equivalent Uniform Operating Cost.—In making the analysis of operating cost for an economy study, the following data may sometimes be obtained:

Year	Operating Cost
1	$500
2	510
3	530
4	560
5	600

The foregoing amounts for operating cost may represent the total operating cost or one element of operating cost. Since an economy study is ordinarily made not for each year of operation but for a typical year, it is necessary to reduce these various amounts to one amount, which may be called an "equivalent uniform operating cost."

The problem may be solved by using the method discussed under Interest known as Equivalence.[1] Assume that the rate of interest is that rate at which money could be borrowed, say 6 per cent. Then by its application:

---

[1] See Appendix A, p. 317.

Year	Operating Cost	×	Interest Factor	=	Total
1	$500	×	0.9434	=	$ 471.70
2	510	×	0.8900	=	453.90
3	530	×	0.8396	=	444.98
4	560	×	0.7921	=	443.59
5	600	×	0.7473	=	448.38
					$2,262.55

This total represents the present worth of the series of payments. The equivalent uniform amount will be equal to the annuity payment from this total. This amount can be found by applying the principles of uniform series of payments, as discussed in Appendix A.

The equation given on page 327 is

$$D = R\left[\frac{i(1+r)^n}{(1+i)^n - 1}\right] \tag{8-7}$$

Since $R = \$2{,}262.55$; $i = 6$ per cent; $n = 5$,

$$D = (\$2{,}262.55)\left[\frac{(0.06)(1.06)^5}{(1.06)^5 - 1}\right]$$

$$D = (\$2.262.55)\left[\frac{(0.06)(1.338)}{1.338 - 1}\right] = \$537.38$$

This figure, then, is the equivalent uniform annual operating cost. Suppose, however, that the data are as follows:

Year	Operating Cost
1	$600
2	560
3	530
4	510
5	500

The application of the principle of equivalence will give the following:

Year	Operating Cost	×	Present Worth Factor	=	Total
1	$600	×	0.9434	=	$ 566.04
2	560	×	0.8900	=	498.40
3	530	×	0.8396	=	444.98
4	510	×	0.7921	=	403.97
5	500	×	0.7473	=	373.65
			Total	=	$2,287.04

Substituting in the proper equation;

$$D = (\$2{,}287.04)\left[\frac{(0.06)(1.06)^5}{(1.06)^5 - 1}\right]$$

$$D = \$542.94$$

Another method of obtaining the equivalent uniform annual operating cost is to assume that the interest rate is zero. The operating cost for each year is multiplied by an interest factor of one.

Year	Operating cost	
	First series	Second series
1	\$500 × 1 = \$ 500	\$600 × 1 = \$ 600
2	510 × 1 = 510	560 × 1 = 560
3	530 × 1 = 530	530 × 1 = 530
4	560 × 1 = 560	510 × 1 = 510
5	600 × 1 = 600	500 × 1 = 500
	Total \$2,700	Total \$2,700

Each of these totals is to be divided by the number of years.

First series, $\frac{\$2{,}700}{5} = \$540$

Second series, $\frac{\$2{,}700}{5} = \$540$

Thus it is seen that, when the principle of equivalence is used with an ascending series of operating costs, the equivalent uniform annual operating cost is less than the simple arithmetic average, \$537.38 as compared with \$540.

However, when the series of operating costs is a descending one, the equivalent uniform annual operating cost is greater than the arithmetic average, or \$542.94 as compared with \$540.00.

It would, therefore, seem proper to use whichever method will give the more conservative value. If we assume that the larger amount will be more conservative, then, in the case of the ascending series, the value arrived at by the arithmetic average will be better; in a descending series, the value arrived at by using the principle of equivalence will be the more conservative. In a mixed series, it would be difficult to determine by inspection which was more conservative.

8.8. Equivalent Uniform Income.—Problems in which income varies may also be brought into equivalence by applying the methods applied to operating cost.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following *lettered* projects, proposals, or situations has been brought to your attention. You are to determine the total yearly operating cost for your assigned project. It is assumed that operation is at full capacity. For additional data, refer to the same lettered problems in Chaps. 1 and 6.

A. The first cost of a proposed 250-room commercial hotel has been estimated as follows:

Land		$ 300,000
Building construction	$680,000	
Architect's fee	34,000	
Taxes during construction	8,000	
Carrying charges during construction	115,500	
Contingencies	32,500	
Total first cost of building		870,000
Furniture and fixtures		100,000
Total first cost		$1,270,000
Working capital		$?,???,???

An analysis of yearly operating costs shows the following:

Taxes at $2.60 per $100 of assessed valuation. Valuation for taxes to be 75 per cent of total investment in land and buildings.

Depreciation, on building cost, which is total investment minus land. Life estimated at 40 years. Depreciation on furnishings is calculated for a life of 5 years.

Operating costs to be at the rate of $350 per room per year, of which $100 is fixed and $250 is variable.

B. Central concrete-mixing plant.

The first cost for such a plant has been determined as follows:

Storage, scales, and building	$ 3,500
Feeder truck	2,000
Mixer truck	4,500
Heater	500
Total	$10,500

An analysis of the annual operating costs for this plant shows the following:

Storage plant:
  Depreciation: Life, 10 years
  Labor: 2 men at $5 each per day
  Maintenance: $10 per month
  Insurance: $25 per year
Feeder truck:
  Depreciation: Life, 4 years
  Labor: 1 man at $5 per day
  Maintenance: $1 per day
  Fuel: 25 gal. gasoline per day at 18 cents and 2 qt. oil per day at 25 cents per quart
  License and insurance: $135 per year
Mixer truck:
  Depreciation: Life, 3 years
  Labor: 1 man at $6 per day
  Maintenance: $2.80 per day
  Fuel: 40 gal. gasoline per day at 18 cents and 4 qt. lubricating oil per day at 25 cents per quart
  License and insurance: $185 per year
General office:
  Salaries: $2,500 per year
  General office expense: 25 cents per cubic yard
Heater in storage plant:
  $2 per day for 75 days during the cold weather to be considered as a fixed operating cost.
  Depreciation: Life, 10 years
Concrete materials:
  Amounts for a batch of $1\frac{1}{2}$ cu. yd. (1—2.4—3.6 mix):
    Cement: 768 lb. at $2.40 per barrel of 376 lb.
    Sand: 2,040 lb. at $2.40 per ton of 2,000 lb.
    Stone: 2,780 lb. at $1.50 per ton of 2,000 lb.
    Water: 61.4 gal. at $0.25 per 1,000 gal.
  Normal operation: 150 days, 10 hr. per day, two batches per hour
Taxes: on storage building, $75

Determine the operating cost for the plant assuming that it operates at full capacity as indicated for normal operation.

C. The first cost for a power plant having Diesel or uniflow engines for prime mover has been estimated to be as follows:

Item	Uniflow	Diesel
Land	$ 8,000	$ 8,000
Buildings	187,000	107,000
Boilers and settings	37,000	
Stokers	10,850	
Condensers	11,900	
Piping	58,000	18,990
Generating units	135,700	322,000
Switchboard	17,500	17,500
Wiring	13,175	13,175
Chimney and flue	17,950	
Forced draft fans	22,700	
Coal and ash handling	34,300	
Engine-room crane	2,540	3,020
Feed-water heater	2,560	
Feed pumps	4,000	
Equipment foundations	22,000	13,700
Oil filters and tanks	2,510	
Cooling water supply	6,500	16,100
Fuel oil storage	....	9,680
Railroad siding	19,640	19,640
Totals	$613,825	$548,805

An analysis of the operating costs for these power plants shows them to be as follows (costs per year for 1,500-kw. peak load 100 per cent capacity for 8,760 hr.):

Item	Uniflow	Diesel
Fuel:		
Coal	2.8 lb./kw.-hr.	
Oil	....	0.0026 bbl./kw.-hr.
Labor	$12,660	$13,680
Lubricants	650	1,600
Supplies	3,900	8,000
Cooling water	....	1,400
Variable maintenance	12,800	13,200
Depreciation on all first cost except land	5% composite	5% composite
Taxes and insurance	2½%	2½%
Fixed maintenance	2%	2%

Coal costs $4 per ton of 2,000 lb.
Oil costs $3 per barrel

D. The first cost for a plant to manufacture milk-bottle caps has been estimated to be as follows:

Blanking press	$ 3,400
Blanking press	2,300
Printing press	1,200
Forming presses	21,420
Tubing machine	500
Dies	3,700
Building	22,400
Working capital	?????
Total	$54,920

The plant operates 8 hr. per day, 25 days per month, 12 months per year. The production is 1,000,000 caps per day. An analysis of operating costs shows the following:

DIRECT LABOR

Operation	Labor
Blanking outer cap	1 man at $4/day
Blanking inner cap	1 man at $3.50/day
Printing inner disk	1 man at $4/day
	1 man at $3/day
Forming outer disk	7 men at $3/shift for three shifts
Tubing and packing	2 men at $3.50/day

Material	Cost per 1,000,000 caps
Large disks, $3\frac{5}{8}$ in. diameter	$600
Small disks $1\frac{3}{4}$ in. diameter	125
Wax	55
Tubes and cartons	105

Production expense	Per year	Per day or per 1,000,000 caps
Indirect labor	$7,500	
Power and light	600	and $22.00
Heat	420	
Water	60	
Taxes	2% on full investment, less working capital	
Supplies	300	and 3.00
Repairs to machinery		and 2.00
Dies and tool upkeep		and 8.00
Building maintenance	1,200	
Insurance	2,400	
Depreciation:		
Building, 40 years		
Machinery, 10 years		
Dies, 1 year		

ADMINISTRATION AND SELLING EXPENSE

Office personnel	$ 3,900	
Sales force and expenses	24,000	
Executive salaries	24,000	
Supplies	300	
Legal costs	1,200	
Telephone	480	
Miscellaneous	1,200	

E. The first cost of a proposed cold-storage locker plant having 320 lockers has been estimated to be

Building	$ 5,220
Lockers and insulation	2,288
Refrigerating machinery	5,296
Land	1,000
Working capital	1,200
Total	$15,004

An analysis of yearly operating costs shows the following:

Depreciation:

Building: life, 40 years

Equipment: life, 10 years

Taxes on building and equipment: \$4 per \$100 on a 50 per cent assessment
Insurance: $1\frac{1}{2}$ per cent of total first cost of buildings and equipment
Power, light, and water: \$0.375 per locker per year
Ammonia and oil: \$0.15 per locker per year
Wrapping paper and twine: \$0.075 per locker per year
Miscellaneous: \$48 per year
State inspection: \$25 per year
Telephone: \$52 per year
Labor, manager and helper: \$3,600 per year

1. On Mar. 31, 1937, the debit balance of the power department expense account of a certain manufacturing company was \$1,897.05. Three-quarters of the power is used to operate machinery, the balance to produce electric current for lighting. Electric lights of equal capacity are spaced equally over the shop. The area of the shop is 100,000 sq. ft. and the total number of horsepower-hours used during the month of March was 150,000. Department *B* occupies 1,200 sq. ft. and its machines by test, require 25 hp., 8 hr. a day, and 26 days a month. What will Department *B* be charged for light? What will Department *B* be charged for power? (From examination for Professional Engineer's license, New York State, July 8, 1937.)

2. The total expense charged against Department *B* of a manufacturing plant during the month of September was \$1,240. This department employs 20 men working 8 hr. a day, 25 days a month. Order 236 passes through this department and requires the services of 3 men for 4 days. How much Department *B* expense is chargeable against this order? (From examination for Professional Engineer's license, New York State, June 28, 1933.)

3. The J. G. Williams Chair Company has a small factory employing 20 men and a superintendent. The departments are as follows:

Department	Floor space, square feet	Horsepower	Men	First cost machinery	Life, years
A	2,000	70	8	\$10,700	10
B	1,500	25	6	7,750	8
C	1,200	15	3	6,150	12
D	800	10	2	4,100	15
E	500	. .	1	2,500	20

Last year the factory expense items were: rent, \$2400; heat, \$300; light, \$120; fire insurance, \$150; power, \$960; superintendent's salary, \$2,000. Repairs (by departments): *A*, \$315.60; *B*, \$210.75; *C*, \$185.17; *D*, \$91.27; *E*, \$27.21. Miscellaneous expense (by departments): *A*, \$98.32; *B*, \$61.26; *C*, \$43.18; *D*, \$17.86; *E*, \$15.38. Departments *A*, *B*, and *C* pay a wage scale of 40 cents per hour; *D* and *E* pay 50 cents per hour.

The factory operated 2,000 hr. last year. The company has just completed an order of 200 porch chairs, for which the requisitions show the following items as cost of direct materials:

Department *A*	Department *B*	Department *C*	Department *D*	Department *E*
$18.75	$6.32	$10.18	$6.23	$7.14
7.86	4.13	9.71	4.06	8.50
9.08	3.21	6.32	5.27	3.25
4.52	0.17	1.19	3.72	
1.76	.....	0.42		

The work done on the order as shown by the time cards was as follows:

Department *A*, hours	Department *B*, hours	Department *C*, hours	Department *D*, hours	Department *E*, hours
17	9	$3\frac{1}{2}$	5	1
14	6	7	12	2
21	13	$11\frac{1}{2}$	9	2
37	8	18	3	3
23	5	6	7	
16	..	21		

Distribute the expense by each of the following methods:

a. Single yearly rate based on direct labor cost
Total direct labor cost for preceding year was $15,000

b. Single yearly rate based on direct material cost
Total direct material cost for preceding year was $17,000

c. Single yearly rate based on prime cost

d. Single yearly rate based on direct labor hours
Total direct labor hours for last year was 36,000 hours.

e. Departmental rate

Then make a cost summary of the order and find the factory cost per order and chair.

4. In the preparation of powdered coal, an engineer states his labor item to be 15 cents per ton of coal produced. He says the cost of labor, of power, and of repairs is as 3:5:2. If 50 tons of coal are powdered per day, 25 days per month each year, what is the annual cost of labor, of power, and of repairs? What is the total cost per year? What is the cost of preparing a ton of coal?

CHAPTER 9

OPERATING-COST PATTERNS

9.1. Operating-cost Patterns.—An operating-cost pattern is a chart or diagram that shows how the operating cost varies with capacity. Many pieces of engineering equipment and machinery are operated at varying capacities. Likewise, industrial enterprises are operated at varying rates of production. The operating-cost pattern shows the relationship between the several elements of operating cost and the varying rates of production.

In the illustrations that follow, it is assumed that the normal working capacity of most industrial plants is at 80 per cent of the theoretical capacity. This will correspond to, for illustrative purposes, 40 hr. of labor per week and 400 units of production for the same period.

9.2. Variations of the Elements of Operating Cost with Capacity or Rate of Production.—From Chap. 8, it was learned that the elements of operating cost are direct labor, direct material, and expense. Under various conditions, these elements vary in different ways with capacity or rate of production.

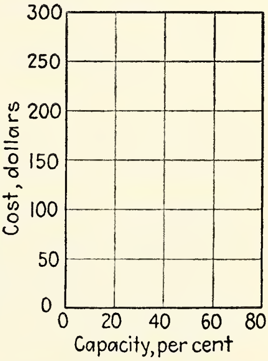

Fig. 9.1.—Basic coordinate chart.

The simplest method of studying the interrelationships among the various factors in an economy study is the graphical chart. The basic chart for studying these factors consists of a coordinate ruled background (Fig. 9.1). The ordinate or y axis is in terms of dollars; the abscissa or x axis is in terms of capacity. Capacity is usually expressed in the following terms:

1. Units of product
2. Hours worked
3. Per cent of theoretical capacity

Direct Labor.—The variation of direct labor with capacity will depend upon the conditions of work. A first condition is that which exists where labor is hired and laid off as the requirements of production dictate. It is represented by the diagram or pattern shown in Fig. 9.2. The total labor cost (ordinate) at any capacity is equal to the number of hours worked multiplied by the cost per hour. The relationship between labor cost and capacity is directly proportional or is a straight-line one.

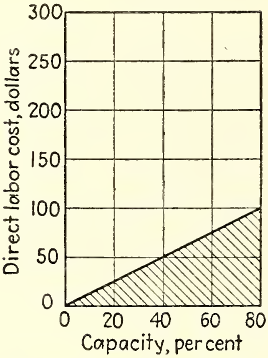

FIG. 9.2.—Direct labor cost versus capacity.

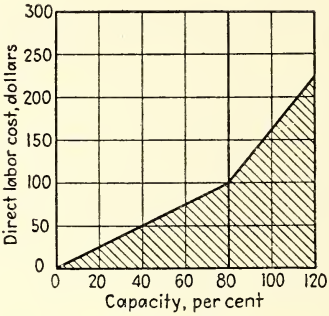

FIG. 9.3.—Direct labor cost plus overtime versus capacity.

A second condition exists when overtime is worked for which the cost per hour of labor has increased, usually to one and one-half times the normal rate. The payment of overtime begins at the 80 per cent capacity point or 40 hr. per week. From that point on, the total labor cost is also a straight-line relationship and is shown added to the pattern in Fig. 9.2, which results in the pattern shown in Fig. 9.3. Thus the total curve for labor is a straight line from 0 to 80 per cent where overtime begins, and another straight line from 80 to 120 per cent which is the zone of overtime payment.

Direct Material.—The variation of direct material cost with capacity depends largely upon the efficiency of usage of the material. If the amount of wasted material, which is inherent in the process, is of a uniform nature and does not vary with the capacity, the curve for material cost is the straight-line relationship, shown in Fig. 9.4.

If, however, the efficiency of usage is such that at low capacities the wastage is greater than at higher capacities, then the curve for material costs is not a straight-line relationship but a curved one in which the slope decreases as the capacity or rate of production is increased. This is shown in Fig. 9.5. This condition may also be brought about by discounts from purchase price for buying large quantities of the material in question.

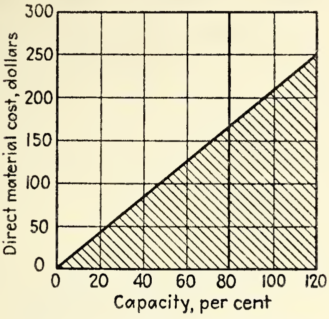

FIG. 9.4.—Direct material vs. capacity or use.

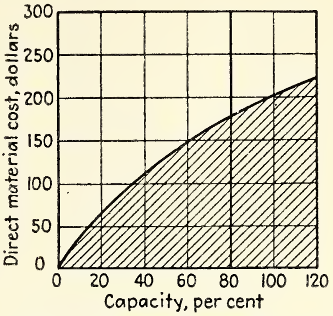

FIG. 9.5.—Direct material vs. capacity or use. Increasing efficiency in usage.

Expense.—As was seen in Chap. 8, expense is made up of a number of factors. The variation of these factors with capacity does not follow the same type for all. Such items as rent, depreciation, heat, and fire insurance are fixed in amount for any capacity. The variation is shown in Fig. 9.6.

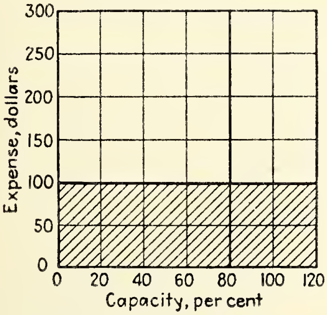

FIG. 9.6.—Expense vs. capacity.

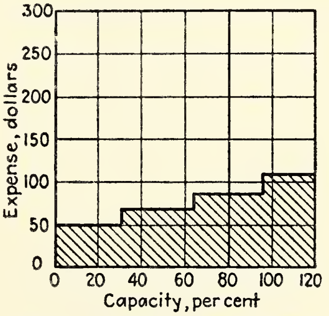

FIG. 9.7.—Step-by-step variation of indirect labor cost vs. capacity.

Other items of expense may vary with capacity in a manner similar to direct material, as shown in Figs. 9.4 and 9.5. Indirect labor, for example, may vary with capacity, as in Fig. 9.5. In the low capacity end a minimum of such labor will have to be employed even though they may not be working at full capacity. Or the situation shown in Fig. 9.7 may exist. In this situation, as certain capacities or rates of production are reached, additional indirect labor is added. This is also true of the variation in supervisors and foremen.

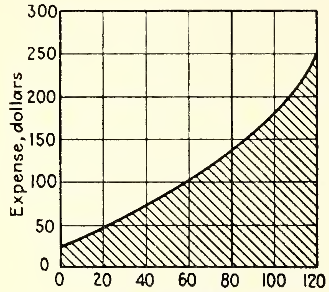

Fig. 9.8.—Variation of power cost with capacity.

Charges for power expense may vary, as shown in Fig. 9.8. It is a characteristic of power-using and -generating apparatus, as will be shown later, that the variation should not be a straight line but a curve, primarily because of the effect of efficiency.

Figures 9.9 and 9.10 show how storage charges and interest charges vary when a fixed quantity per year is to be purchased or manufactured in varying numbers of lots per year. As the number of lots increases, the size of the lots decreases with a consequent decrease in these charges.

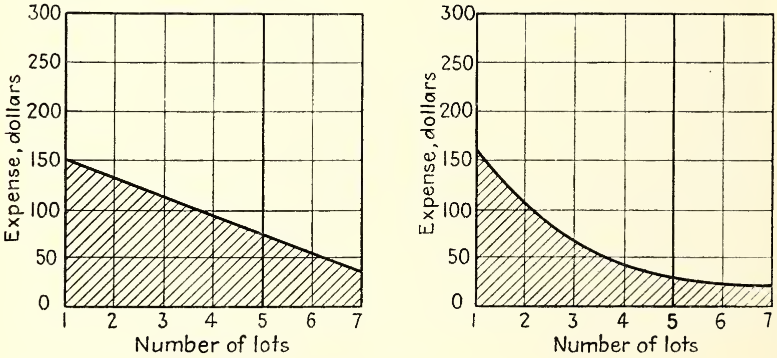

Fig. 9.9. Fig. 9.10.

Figs. 9.9 and 9.10.—Decreasing expense with increasing number of lots or runs.

9.3. Total Operating-cost Patterns.—The preceding section discussed the way in which the elements of operating costs vary with capacity or rates of production. These elements, when combined in various forms, produce total operating-cost patterns. The accompanying diagrams show total operating-cost patterns so derived.

It will be observed that the shape and character of the total cost line are different for the several combinations. Of particular

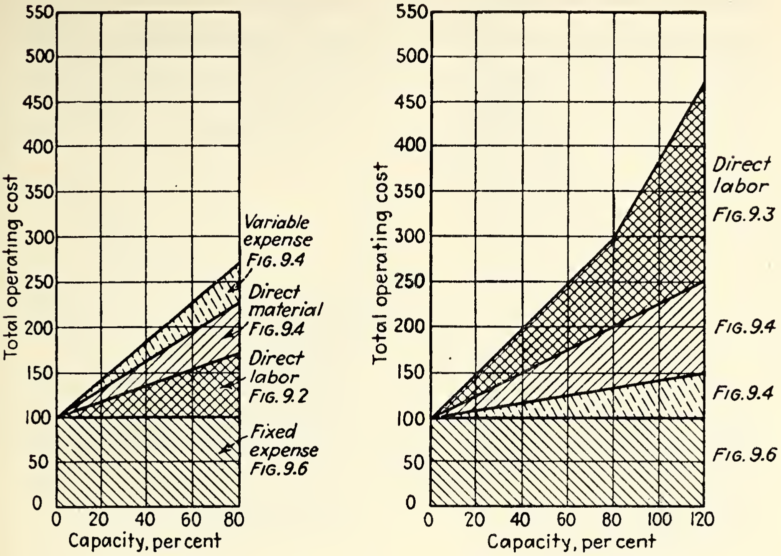

Fig. 9.11. Fig. 9.12.

Figs. 9.11 and 9.12.—Total operating cost curves.

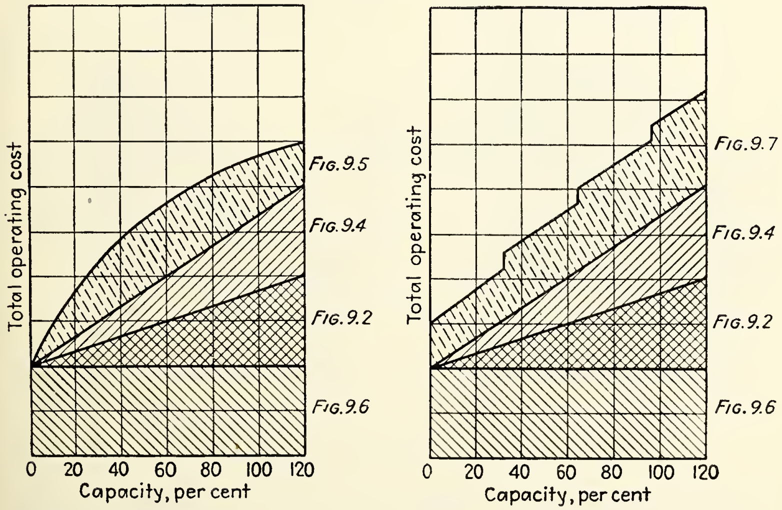

Fig. 9.13. Fig. 9.14.

Figs. 9.13 and 9.14.—Total operating cost curves.

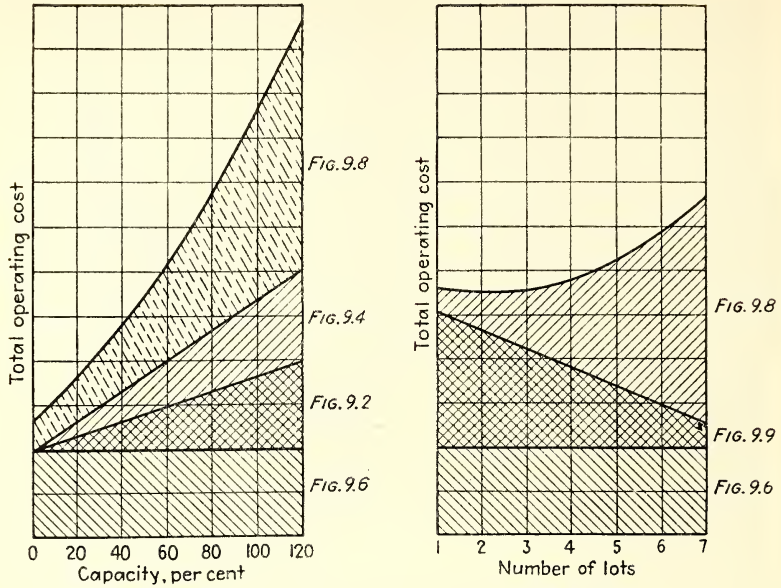

Fig. 9.15. Fig. 9.16.

Figs. 9.15 and 9.16.—Total operating cost curves.

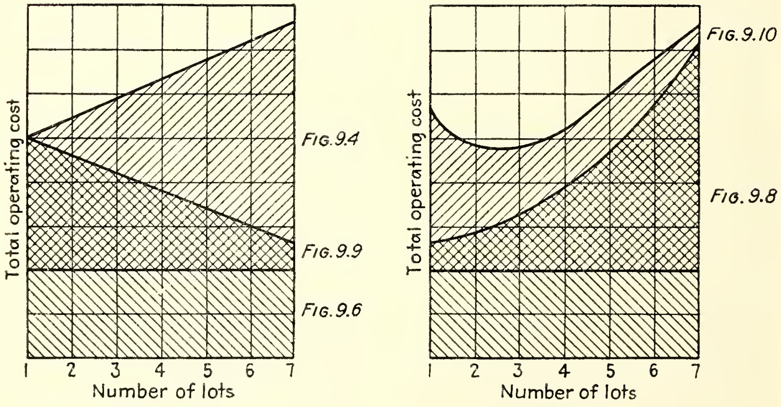

Fig. 9.17. Fig. 9.18.

Figs. 9.17 and 9.18.—Total operating cost curves.

interest is the pattern in Figs. 9.16 and 9.18. In each case, the total operating-cost line decreases to a minimum and then increases.

9.4. Unit Operating-cost Patterns.—In setting the selling prices for a product or service, it is necessary to know whether the selling price is greater or lower than the cost of production. The unit operating cost, therefore, becomes of prime importance. For each of the several combinations of the elements of operating cost depicted in Figs. 9.11 to 9.18, a unit-cost curve may be derived by dividing the total operating cost at any certain capacity or rate of production by a corresponding measure of capacity or rate of production. The unit operating-cost curves together with their corresponding total operating-cost curves are shown in Figs. 9.19 to 9.26.

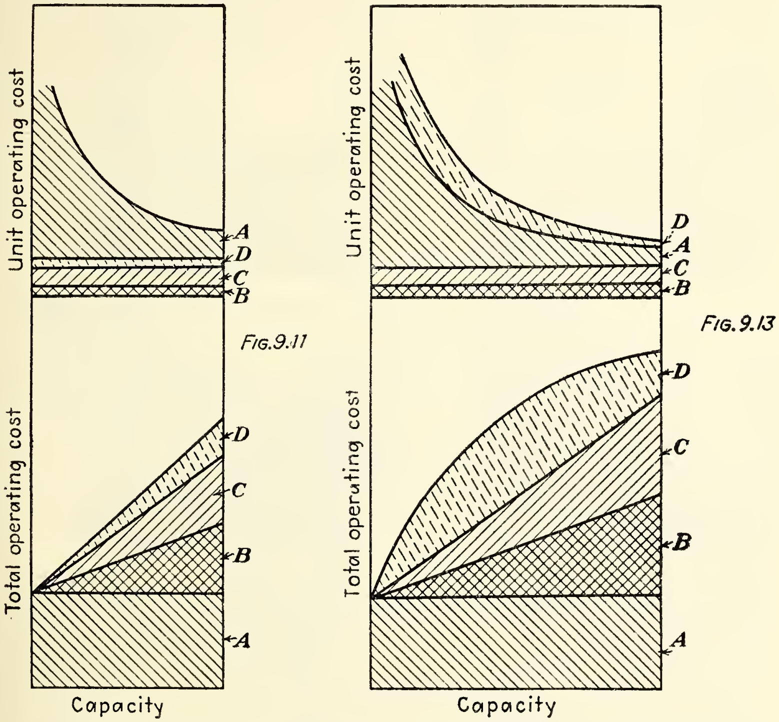

Fig. 9.19. Fig. 9.20.

Figs. 9.19 and 9.20.—Combined total and unit operating cost curves.

It will be observed that, in general, the shape of the unit operating-cost curve slopes downward to the right, the unit value becoming less and less. For the combinations shown in Figs. 9.16, 9.15, and 9.18, the corresponding unit operating-cost curves are shown in Figs. 9.22, 9.24, and 9.26. It will be observed that each of these unit curves slopes downward to the right, reaches a minimum, and then curves upward to the right. It should be noted that the areas noted by letters in the lower part of the figure are parts of the total operating cost. In the upper figure of unit operating cost the letters correspond to the same areas in the lower part. An element of cost that varies from zero at zero capacity with a straight-line relationship appears as a triangle in the lower diagram and as a uniform horizontal line in the upper diagram because of the direct variation with capacity. Where in the lower diagram the expense is uniform, in the upper diagram it will decrease from the left to the right in size because it is spread over more and more capacity or units of product.

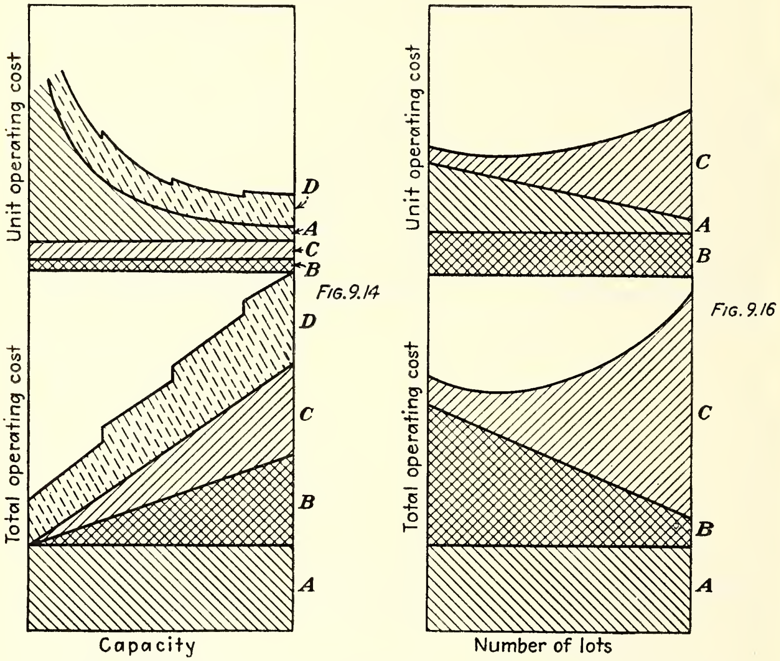

Fig. 9.21. Fig. 9.22.

Figs. 9.21 and 9.22.—Combined total and unit operating cost curves.

9.5. An Industrial Plant Problem.—For purposes of illustration and development of these interrelationships, the following data concerning the Atlas Manufacturing Company are presented:

First cost or plant investment			$82,000
Operating cost—production:			
Expense	Fixed	Variable	
Rent	$1,600		
Heat	500		
Light	280		
Fire insurance	184		
Power		$ 720	
Foreman's salary	1,500		
Repairs		850	
Miscellaneous		270	
Depreciation	3,280		
Totals	$7,344	$1,840	$ 9,184
Direct material cost:			
3,200 units per year at $7.70 per unit			24,640
Direct labor cost:			
3,200 units per year at $3.20 per unit			10,240

9.6. Statistical Analysis.—This is a convenient method of bringing together the various factors of cost and of making further calculations. The data developed in this manner can then be transferred to a chart of operating costs.

TABLE 9.1.—STATISTICAL ANALYSIS

	Per cent of theoretical capacity				
	4	20	40	60	80
Fixed expense	$ 7,344	$ 7,344	$ 7,344	$ 7,344	$ 7,344
Variable expense	92	460	920	1,380	1,840
Direct labor	512	2,560	5,120	7,680	10,240
Direct material	1,232	6,160	12,320	18,480	24,640
Totals	$ 9,180	$16,524	$25,704	$34,884	$44,064
Units produced	160	800	1,600	2,400	3,200
Unit operating cost	$ 57.37	$ 20.65	$ 16.06	$ 14.53	$ 13.77

9.7. Cost of Production.—The original data are superimposed on the basic chart (Fig. 9.27, page 116) in the following order:

1. *Fixed Expense.*—This amount, $7,344, is scaled off on the ordinate. A horizontal line is drawn from this point between the vertical ordinates for zero capacity and for 80 per cent capacity. The reason why this line is horizontal is that a fixed expense is applicable to the cost of operation of a business for all capacities.

2. *Variable Expense.*—This amount, $1,840, is scaled off above the amount for fixed expense, or a total of $7,344 plus $1,840 or $9,184 at the 80 per cent capacity point. From this point a line is drawn to where the fixed expense line and the zero capacity line meet. This expense varies with the amount of capacity operated. A straight line is used because on the average this expense varies directly with productive capacity.

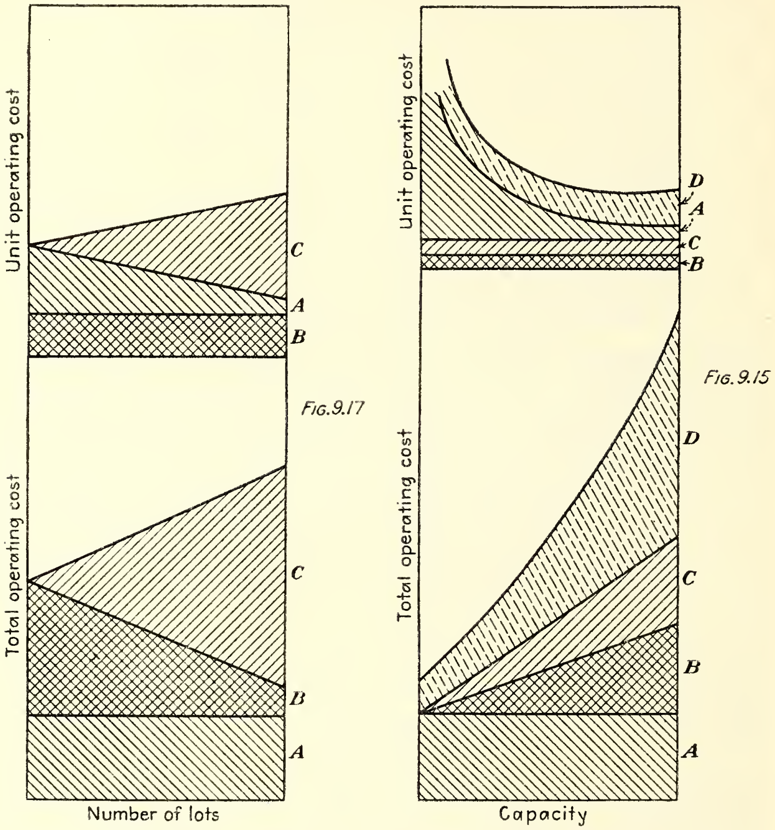

Fig. 9.23. Fig. 9.24.

Figs. 9.23 and 9.24.—Combined total and unit operating cost curves.

3. *Direct Labor.*—This amount, $10,240, is scaled off above the amount for fixed expense, or a total of $7,344 plus $1,840 plus $10,240 or $19,424. From this point a line is drawn to where the

fixed expense line and the zero capacity line meet and for the same reasons as for variable expense.

4. *Direct Material.*—This amount, \$24,640, is scaled off above the amount for direct labor or a total of \$7,344 plus \$1,840 plus \$10,240 plus \$24,640 or \$44,064. As in the case of variable expense and direct labor, a diagonal line is drawn to where the fixed expense line and the zero capacity line meet and for the same reason.

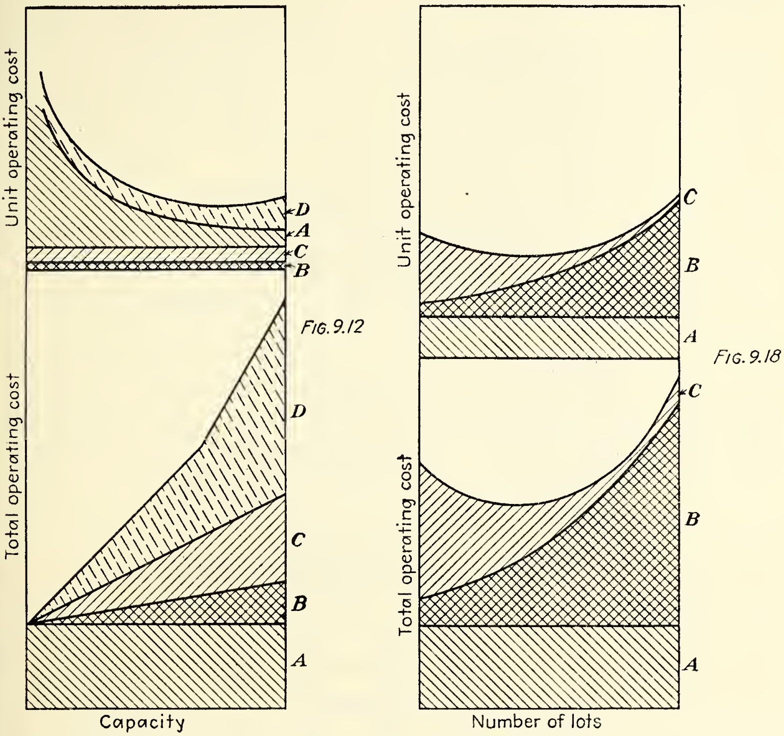

Fig. 9.25. Fig. 9.26.

Figs. 9.25 and 9.26.—Combined total and unit operating cost curves.

5. *Total Operating Cost.*—The area under this last line may be thought of as representing the total cost of production. The height of this line above the base line at any point represents the total cost of production at that capacity point.

The value for this height may also be found by adding together the fixed expense and the proportional part of the variable expense, direct labor, and direct material, as shown on the statistical analysis sheet.

6. *Unit Operating Cost.*—When the total cost of production at any capacity point is divided by the amount produced at that point, the unit cost for that point is found. This was done in the preceding tabulation.

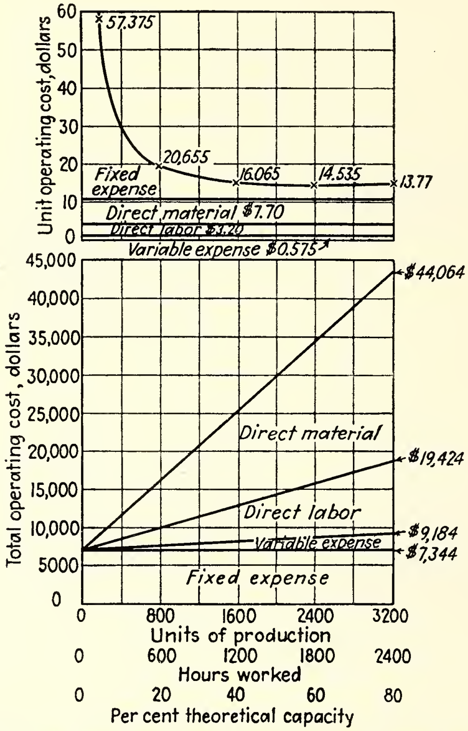

Fig. 9.27.—Combined total and unit operating cost chart for an industrial enterprise.

When these values are plotted in the upper part of Fig. 9.27, a curved line is drawn between the several points. This curve represents the changes that occur in unit costs as the amount produced or capacity is increased. It is to be noted that the greater the production rate the lower the unit cost.

When the diagram showing the various costs of production and the foregoing tabulation is examined, it is seen that there are two groups of costs:

1. Those that vary directly with productive capacity; namely, variable expense, direct labor, and direct material

2. Those that are the same at all capacities; namely, fixed expense

These may also be shown in the upper part of Fig. 9.27. Each item of cost is divided by the number of units produced. It will be observed that those items of cost that were variable in the lower diagrams now become uniform. This must be true because of the manner in which the lower chart was constructed. It should be noted that fixed expense becomes larger and larger as the number of units produced becomes smaller. This is because the smaller number of units must absorb all of the fixed expense, which is, therefore, more per unit at lower production rates than at higher rates.

The obvious conclusion is, therefore, that it is best to operate an industrial plant or any other piece of equipment as close to its rated capacity as possible.

9.8. An Automobile Problem.—It is desired to study the operating costs of driving an automobile. The following information has been secured in an analysis of the operating costs:

	Variable	Fixed
Depreciation	....	$150.00
Insurance	....	52.33
License fee	....	12.50
Repairs	$ 25	
Servicing	40	
Motor tune-up	14	
Complete overhauling	18	
Antifreeze	3	
Tires	116	
Lubricating oil:		
Crankcase	40	
Differential	16	
Totals	$272.00	$214.83

These costs were based on operating the automobile for 40,000 miles per year. It was found that the cost of gasoline depended upon the speed at which the automobile was operated. Information on gasoline consumption shows the following table:

GASOLINE CONSUMPTION

Nominal speed in miles per hour	Miles per gallon of gas
10	16.8
20	19.3
30	19.4
40	18.6
50	17.5
60	15.6

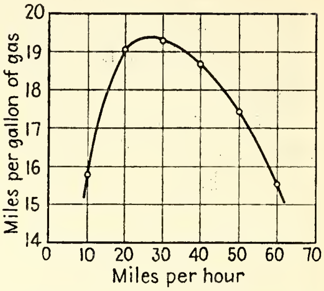

FIG. 9.28.—Typical curve of gasoline consumption for a medium-priced automobile.

This information is shown in graphical form in Fig. 9.28.

The cost of gasoline at various rates of speed will then be as follows:

Nominal speed in miles per hour	Miles per gallon of gas	Gallons of gas per 40,000 miles	Cost of gas at 18 cents per gallon
10	16.8	2,380.9	$428.56
20	19.3	2,072.5	373.05
30	19.4	2,061.8	371.12
40	18.6	2,150.5	387.09
50	17.5	2,285.7	411.42
60	15.6	2,564.1	461.53

These operating costs may be charted as was done for the industrial plant. It is assumed that the fixed expenses will be depreciation, insurance, and license fee, and that the others will be variable. Figure 9.29 is the chart of these costs.

It will be observed that in the chart there has been indicated the cost of gasoline for operating the car at various speeds from 10 to 60 m.p.h. The unit cost curve in the diagram is for 60 m.p.h. Additional lines could be drawn for other speeds. Another diagram (Fig. 9.30) may be drawn to show more clearly the effect of rate of speed on unit operating cost. This diagram will be based on various mileages per year. Note that its base scale is in miles per hour.

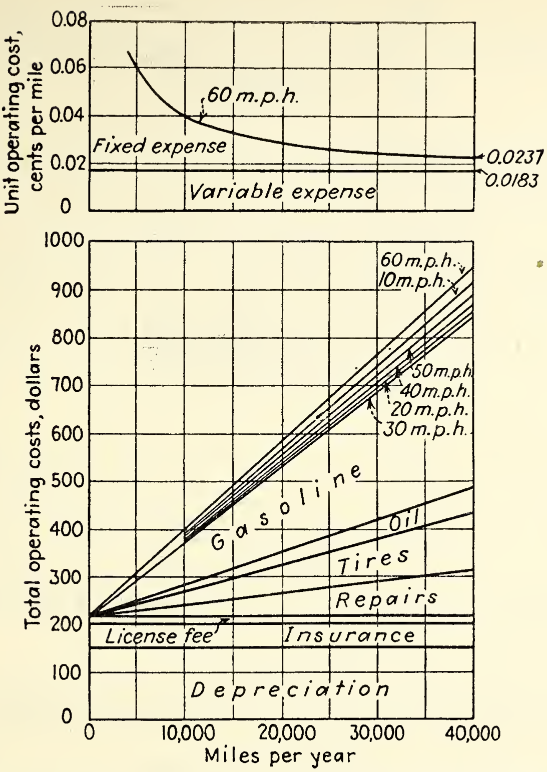

FIG. 9.29.—Combined total and unit operating-cost chart for an automobile.

OPERATING COSTS AT 40,000 MILES PER YEAR

Cost item	10 m.p.h.	20 m.p.h.	30 m.p.h.	40 m.p.h.	50 m.p.h.	60 m.p.h.
Fixed.....	$214.83	$214.83	$214.83	$214.83	$214.83	$214.83
Variable..	272.00	272.00	272.00	272.00	272.00	272.00
Gas......	428.56	373.05	371.12	387.09	411.42	461.53
Totals....	$915.39	$859.88	$857.95	$873.92	$898.25	$948.36
Unit......	0.0288	0.0214	0.0214	0.0218	0.0224	0.0237

At any other total mileage per year, it will be necessary to determine the variable costs and gas cost for the new total mileage.

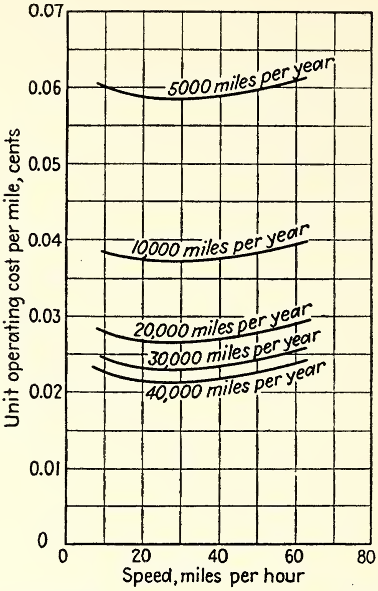

FIG. 9.30.—Unit operating costs for a medium-priced automobile.

OPERATING COSTS

Cost item	10 m.p.h.	20 m.p.h.	30 m.p.h.	40 m.p.h.	50 m.p.h.	60 m.p.h.
At 30,000 miles per year						
Fixed.....	$214.83	$214.83	$214.83	$214.83	$214.83	$214.83
Variable..	204.00	204.00	204.00	204.00	204.00	204.00
Gas......	321.42	279.78	278.34	290.31	308.56	546.15
Unit......	0.0247	0.0233	0.0232	0.0236	0.0242	0.0255
At 20,000 miles per year						
Unit......	$ 0.0282	$ 0.0268	$ 0.0268	$ 0.0272	$ 0.0278	$ 0.0290
At 10,000 miles per year						
Unit......	$ 0.0389	$ 0.0376	$ 0.0375	$ 0.0379	$ 0.0385	$ 0.0398
At 5,000 miles per year						
Unit......	$ 0.0604	$ 0.0590	$ 0.0590	$ 0.0594	$ 0.0600	$ 0.0613

The shape of the curves indicates that the most economical speed at which to operate the car is between 20 and 30 m.p.h.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following lettered projects, proposals, or situations has been brought to your attention. You are to prepare a statistical analysis and operating cost pattern chart for your assigned project. Show fixed and variable costs as they vary with capacity or rate of production. Also show unit operating costs. For additional data, refer to the same lettered problems in Chaps. 1, 6, and 8.

A. The yearly operating costs at full capacity of a 250-room commercial hotel have been estimated to be as follows:

Taxes	$ 22,815
Depreciation:	
Building	21,750
Furnishings	20,000
Operating costs:	
Fixed	25,000
Variable	62,500
Total	$152,065

The statistical tabulation is to be for 0, 25, 50, 75, and 100 per cent capacity. The chart base scales are to be in (*a*) per cent capacity and (*b*) room-nights. The unit operating costs are to be in dollars per room-night.

B. The yearly operating costs for a central concrete-mixing plant operating for 150 days per year and producing 4,500 cu. yd. of mixed concrete have been determined to be

Depreciation	$ 2,400
Direct labor	$ 3,150
Maintenance:	
Fixed	120
Variable	570
Insurance and licenses	345
Gasoline for trucks	1,755
Lubricating oil for trucks	225
General office expense:	
Fixed	2,500
Variable	1,125
Heater, fixed	150
Direct materials	28,351
Taxes	75
Total	$40,766

The statistical tabulation is to be for 25, 50, 75, 100, 125, and 150 days. The chart base scales are to be in (*a*) number of days and (*b*) cubic yards of mix. The unit operating costs are to be in dollars per cubic yard.

C. The yearly operating costs for a power plant having either Diesel or uniflow engines for the prime mover have been estimated to be as follows (costs per year for 1,500-kw. peak load, 100 per cent capacity for 8,760 hr.):

	Diesel	Uniflow
Fuel	$102,492	$73,584
Labor	13,680	12,660
Lubricants	1,600	650
Supplies	8,000	3,900
Cooling water	1,400	
Variable maintenance	13,200	12,800
Depreciation	27,042	30,291
Taxes and insurance	13,720	15,345
Fixed maintenance	10,976	12,276

The operating costs for capacities other than 100 per cent are given below. Depreciation, taxes and insurance, and fixed maintenance are the same for all capacities. Coal costs $4 per 2,000 lb. Oil costs $3 a barrel.

	Diesel	Uniflow
At 25 per cent capacity:		
Fuel:		
Oil	0.0031 bbl./kw.-hr.	
Coal	....	4.43 lb./kw.-hr.
Labor	$12,600	$11,580
Lubricants	1,000	500
Supplies	5,800	3,300
Maintenance	8,000	8,000
Cooling water	800	
At 50 per cent capacity:		
Fuel, oil	0.0027 bbl./kw.-hr.	
Coal	....	3.41 lb./kw.-hr.
Labor	$12,600	$12,660
Lubricants	1,200	550
Supplies	7,200	3,500
Maintenance	10,500	10,800
Cooling water	1,000	
At 75 per cent capacity:		
Fuel, oil	0.0026 bbl./kw.-hr.	
Coal	....	2.90 lb./kw.-hr.
Labor	$13,680	$12,660
Lubricants	1,400	600
Supplies	7,600	3,700
Maintenance	12,000	12,000
Cooling water	1,200	
At 125 per cent capacity:		
Fuel, oil	0.0028 bbl./kw.-hr.	
Coal	....	3.07 lb./kw.-hr.
Labor	$14,300	$13,200
Lubricants	1,900	730
Supplies	8,600	4,150
Maintenance	14,600	13,750
Cooling water	1,650	

For the chart, the base scales are to be in (*a*) percentage of capacity and (*b*) kilowatt-hours. The unit operating cost is to be per kilowatt-hour.

D. The yearly operating costs for a factory manufacturing milk-bottle caps at the rate of 1,000,000 per day have been estimated at (plant operates 300 days per year at 8 hours per day):

	Per year	Per 1,000,000 caps per day, per year
Labor	....	$ 25,350
Material	....	265,500
Tools and dies	....	3,700
Expense:		
Labor	$ 7,500	
Power and light	600	and 6,600
Heat	420	
Water	60	
Taxes	1,098	
Supplies	300	and 900
Repairs in machinery	....	600
Upkeep of dies and tools	....	2,400
Building maintenance	1,200	
Insurance	2,400	
Depreciation	3,442	
Administrative and selling	55,080	

The statistical tabulation is to be for each 20 per cent of capacity assuming that a production of 1,000,000 caps per day per year of 300 days represents 80 per cent of theoretical total capacity. The chart scales are to be (*a*) in percentage of capacity, (*b*) in days per year, and (*c*) in units of 1,000 caps.

E. The yearly operating costs for a proposed cold-storage locker plant having 320 lockers have been estimated to be as follows:

Depreciation	$ 888
Taxes and insurance	468
Power, light, water	120
Ammonia and oil	48
Wrapping paper and twine	24
Miscellaneous (fixed)	48
State inspection (fixed)	25
Telephone	52
Manager	3,600

The statistical tabulation is to be for each 80 lockers. The chart scales are to be in (*a*) Percentage of capacity and (*b*) number of lockers. The unit operating costs are to be in dollars per locker.

CHAPTER 10

INCOME, RETURN AND YIELD

10.1 Income.—Income is the opposite of expense. It represents the total of money values from all sources which can be credited to the project. It may be obtained from the sale of commodities, machinery, or equipment. It may be dividends from other investments, interest received on bonds or notes receivable, rents, licenses, or royalties. It is the product of price multiplied by quantity of product sold.

10.2. Price.—Price is the amount paid for a unit of sale. It is a measure of the valuation that an individual puts upon a thing in terms of his own peculiar notions about its importance to him. Since people vary in their notions concerning the importance of things, there may be many valuations put upon things. These are reflected in many ideas of price.

Price is also affected by the relative supply and demand for the particular thing. When the supply is high and demand low, prices tend to go down. When the supply is low and demand high, prices tend to go up. Free competition plays a great part in the fluctuations of prices.

10.3. Economic Return.—Return is the net income that is left after all operating costs have been subtracted from the total income.

Example: A project requires a total investment of $500,000. The income for the year is $350,000 and the operating cost is $300,000.

Income	$350,000
Operating cost	300,000
Economic return	$ 50,000

10.4. Economic Yield.—Economic yield is that per cent of the capital investment which net economic return bears to it.

Example:

Investment	$500,000
Net economic return	50,000

$$\text{Economic yield} = \frac{\text{net economic return}}{\text{investment}} \times 100 \text{ per cent}$$

$$\text{Economic yield} = \frac{50{,}000}{500{,}000} \times 100 \text{ per cent} = 10 \text{ per cent}$$

The economic yield is a measure of the earning power of the project under the conditions surrounding it, but uninfluenced by the method of financing it.

For the effect of the method of financing on yield, refer to Sec. 14.3.

10.5. Fair Return.—After the yield on an investment has been determined, the question arises as to what should be considered a fair return. How much should an investment earn? What are the factors that should help determine the desirable yield on an investment?

10.6. Interest Rates.—One of the easiest comparisons to make is to compare the actual yield to interest rates. Interest rates vary according to the kind of investment or transaction on which they are paid.

Interest rates on money on deposit in a savings bank vary from 1½ to 3 per cent. Money in such an account cannot be withdrawn in large amounts without at least a month's notice.

Interest rates on loans by a bank depend upon the time for which and the manner in which the money is lent. The interest rate on ordinary bank loans for 30 days, 60 days, and even longer, is in many states 6 per cent. The interest rates for first mortgages vary from 4½ to 6 per cent. The interest rates charged by building and loan associations vary from 5 to as high as 8 per cent depending upon the regulations concerning fines and fees for loans.

The following quotation from the financial section of *The New York Times* for Mar. 26, 1939, is part of an article by Elliot V. Bell entitled, Menace to Thrift Seen in Low Yields.

> The average yields of government bonds and the highest grade corporation issues are now around the lowest levels on record. According to the current Federal Reserve Bulletin, the average rate of return on United States Government bonds toward the end of last month was 2.43 per cent, the yield on high-grade municipals was 2.70 per cent, the yield on triple-A corporate bonds 3 per cent, and the yield on all grades of corporate bonds 3.81 per cent. Ten years ago, in 1929, the yield on government bonds was 3.60 per cent, on municipals 4.27 per cent,

on triple-A corporate 4.73 per cent, and on all corporates 5.21 per cent. Twenty years ago the average yields were: government bonds, 4.62 per cent; municipals, 4.46 per cent; triple-A corporates, 5.48 per cent; and all grades of corporates, 6.26 per cent.

Another measure of fair return is the rates set up by state commissions of public service, which either determine fair return rates or enforce rates set by legislative enactment. The Federal government has also set up rates of return which apply to certain government contracts.

10.7. Elements of Fair Rate of Return or Yield.—Professor Grant suggests in "Principles of Engineering Economy" that the fair rate of return should be made up of three elements:[1]

1. The current rate of interest obtainable on a conservative investment. One would consider the investment in a first mortgage as a conservative.
2. An element which would represent the hazards which would appear to be inherent in the project, giving consideration to such matters as competition, risks of changing market conditions, and dangers of sudden obsolescence of machinery and methods.
3. A factor of safety which considers the uncertainties involved in cost estimates and valuations of pertinent factors in the project or situation.

It has been suggested that the fair rate of return should be such that the project will earn it as an average over the natural business cycle for its particular kind of product. The argument is that for various businesses there are different cycles. In other words, the cycle of change from expansion to peak to decline up to expansion again requires different lengths of time for different businesses.

If a business expected to earn 10 per cent on the average over a life cycle of 3 years, it might earn as follows: first year, 15 per cent; second year, 10 per cent; and third year, 5 per cent. The average is 10 per cent.

C. E. Knoeppel, a consulting engineer in the field of management, suggests the following scale as a gauge of net profit requirements:

6 per cent as a "deadline" below which interest on investment is not earned

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[1] GRANT, E. L., "Principles of Engineering Economy," p. 31, The Ronald Press Company, New York, 1930.

9 per cent as the minimum requirement
12 per cent as the standard requirement
15 per cent as the desirable requirement
18 per cent as the maximum requirement

Before the Federal Trade Commission several years ago, in a then famous rate case, Mr. Knoeppel advanced the following principle:

> . . . a return to the investor in an industrial can be said to reconcile the difference between a safe and sure return, and a possible but unguaranteed return, if the rate is two to three times the guaranteed return as an average over a period of years.[1]

The foregoing scale embodies this principle.

A further statement from Mr. Knoeppel suggests that profit constitutes,

> *Reward* for skill, ability, experience, and knowledge
> *Incentive* for effort and endeavor
> *Protection* against chances which must be taken
> *Insurance* against losses that are entirely unforeseen
> *Assurance* of business success and perpetuation

10.8. Return per Unit.—Return in Sec. 10.3 was thought of as return for the total business of a year. There is a relationship between total return and return per unit which should be understood.

If a single unit only is produced, the return for that unit is the same as the return for the year. If, however, two units have been produced, the return per unit is just one-half of the total return for the year. Another way to look at this question is to assume that it took just half a year to produce one unit. Therefore, the return per unit should be the return for one-half year, which is the same as half of the return for a whole year.

This relationship is usually called by the term "turnover." Another meaning is the rate at which goods are sold in a store. Turnover of goods in a grocery store is very fast, say once a week for most items; in a clothing store it is not very fast, say once a quarter year. Slow-moving and slowly produced goods must, therefore, receive a greater return per unit than fast-moving or faster produced goods.

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[1] Quoted from *Factory and Industrial Management*, April–May, 1932.

The easiest way to determine return per unit is to determine the return per year and then divide by the number of units. Of course, the return per any unit of time could be divided by the production for the same unit of time in order to get the return per unit.

Thus, for fast-moving goods or those having a high turnover, the spread between cost of production and selling price may be smaller than the corresponding spread for slow-moving goods or those having a low turnover.

10.9. Expected Economic Return and Yield on the Investment.—The expected economic return and yield on the investment is that return and yield which corresponds to Fair Return discussed in Secs. 10.5, 10.6, and 10.7. It is that return and yield on the investment which it is believed will attract persons to invest their funds in it. It is, of course, an individual's own interpretation of what the fair return to him should be.

In order that a business or project may realize the level of return and yield that is believed satisfactory, the price which it charges for its products or services must be such as will include the cost of production, cost of distribution, and the expected return on the investment. It is useful for the promoter to be able to forecast what the minimum total for those three factors will be. Production and distribution costs are obtained from estimates of operating cost. To their total is added the expected return. This new total is called "total operating cost plus expected return." This amount is used by the promoter to check the minimum requirements as to selling price.

It is to be understood that this expected return is not to be added to the cost of production on the accounting books of the business. It is to be used only for the purpose of getting a "line" upon what the appropriate selling price ought to be. If the market price is higher than this minimum, the promoter can either sell at the market and make more return on his investment, or sell at lower than the market and make at least the expected return. If the market price is lower than this minimum, he will have to take less return on the investment or reduce his operating costs so as to meet the market price.

In the analysis of most engineering projects, the income from them cannot always be determined in advance. It becomes necessary to assume an income figure for comparative purposes.

This is best done by assuming an expected return or yield on the investment and adding it to the total of operating costs and calling it "total operating cost plus expected return."

Example: Using the data given in Sec. 10.3 and assuming that the promoter wishes to receive an expected return of 12 per cent on his total investment,

Operating cost	$300,000	
Expected return	60,000	(0.12 of $500,000)
Total operating cost plus expected return	$360,000	

10.10. Determination of Expected Return on Investment.—The investment in a project has been defined as the sum of first cost and working capital. This is the amount on which the investor wishes to make his expected return. The investor wishes to maintain this investment in the project throughout its life. This he does by making a so-called charge for depreciation which is really for the maintenance of capital. Thus, while the asset, such as a piece of equipment or building, actually depreciates in value for every year of service, the investor offsets this depreciation by either one of two processes.

In the first, he sets aside the charge for depreciation in a fund, which may or may not be placed at compound interest. In either case the sum of the depreciated value of the equipment, which is book value, plus the total in the depreciation fund plus working capital equals at all times during the life of the project the original net worth. The original investment has thus been maintained.

The second method does not set aside the charge for depreciation in a separate fund but keeps it within the business. As the asset depreciates in value, there is an equal offsetting value kept in the business which when added to the book value of the equipment and the original working capital equals at all times during the life of the project the original net worth. The original investment has thus been maintained.

Since the net worth of the project has been maintained at its original value during the life of the project, the investor is justified in expecting to receive an expected return based upon the full and original value of the investment.

If, for some reason in the operation of the project, as in a mine, the charge for depletion is returned to the investor together with the expected return, then the net worth is not maintained but is decreasing all the time. The investor, in this case, can expect to receive only an expected return on the decreasing net worth that is in the project at any time.

10.11. Unit Operating Cost plus Expected Return on Investment.—This term is similar to the unit operating cost discussed in Sec. 9.7. It is equal to the total operating cost plus expected return divided by the units of production at the specified capacity. This unit operating cost may be directly compared with the selling price because it is the selling price that will give an income equal to the total of production and distribution costs and the expected return. It is a unit that is not found from the accounting books but is calculated solely for assistance in making decisions. It was explained in Sec. 10.9 what happens when market price is more or less than this unit operating cost plus expected return.

10.12. Curve of Unit Cost plus Expected Return on Investment.—In Chap. 9, a unit cost curve was developed which showed the relationship between the cost of production and varying amounts of production or rates of capacity.

A similar curve may be drawn for unit cost plus expected return on investment. The amount of the expected return is added to the lower curve of total yearly costs. It is then divided by the suitable capacity figure and added to the upper curve which is the unit cost curve. The resulting total curve in each case is respectively total yearly cost plus expected return on investment and unit cost plus expected return on investment.

This addition may be illustrated by considering the examples used in Chap. 9.

Example 1: The industrial plant.

Assume that the total investment in the plant, including working capital, is $50,000 and that the investor expects to earn a return at a yield rate of 10 per cent.

Expected return then equals 10 per cent of $50,000 or $5,000. This is added to the total cost section as shown by dotted lines in Fig. 10.1.

In Table 9.1 is found statistical calculation of the unit cost. Unit cost plus expected return on investment may be found readily by adding to the data given there the amount of the expected return.

Table 10.1.—Statistical Analysis

	Per cent of capacity				
	4	20	40	60	80
Total production cost	$ 9,180	$16,524	$25,704	$34,884	$44,064
Expected return on investment	5,000	5,000	5,000	5,000	5,000
Total production cost plus expected return on investment	$14,180	$21,524	$30,704	$39,884	$49,064
Production, units	160	800	1,600	2,400	3,200
Unit cost plus expected return on investment	$ 88.62	$ 26.90	$ 19.19	$ 16.61	$ 15.33

The curve may now be drawn in the top part in dotted line (see Fig. 10.1).

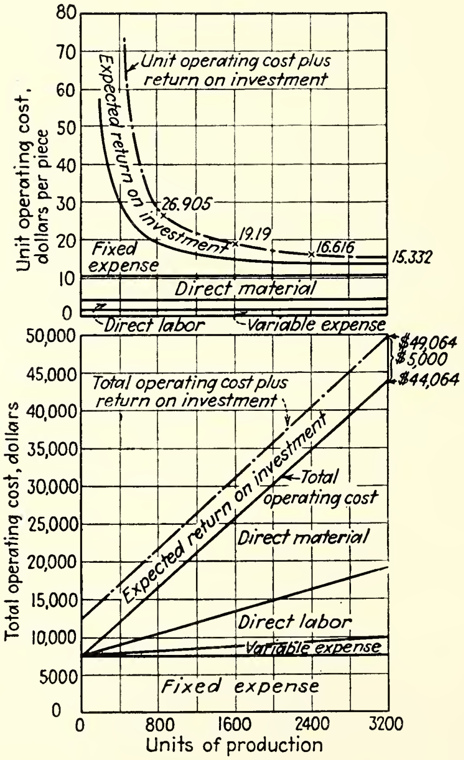

Fig. 10.1.—Operating-cost chart for an industrial enterprise with expected return on investment added. See Fig. 9.27.

It should be noted that expected return on investment is similar to fixed expense in that it is constant for all rates of production or capacities. Therefore the increment added above the present unit cost curve becomes smaller as the rate of production increases.

Example 2: The automobile.

If the automobile is privately owned, the chances are that the owner will not consider or think about earning a return on his investment. If, however, it is a part of a fleet of cars owned by an industrial enterprise, the owner will wish to earn a return on it just as he would do for machinery.

Assume that the investment in a car is $850 and the owner wishes to earn at a yield rate of 12 per cent. Expected return then equals 12 per cent of $850 or $102. For the revised chart, see Fig. 10.2.

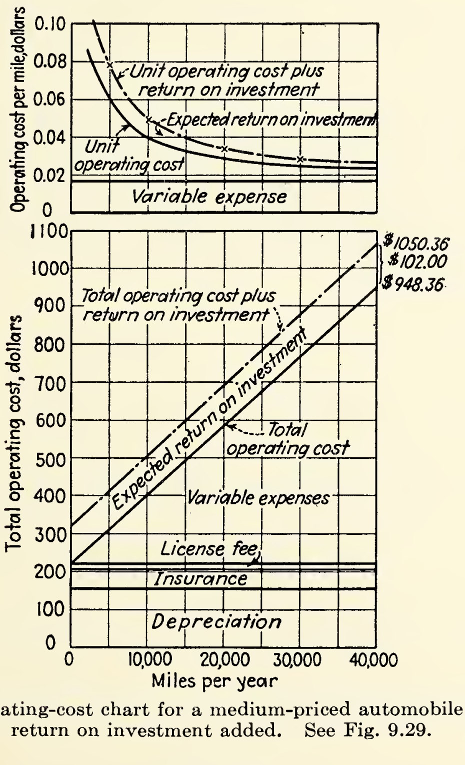

FIG. 10.2.—Operating-cost chart for a medium-priced automobile with expected return on investment added. See Fig. 9.29.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following lettered projects, proposals, or situations has been brought to your attention. You are to determine working capital and then total investment. Calculate the expected return on the investment, total operating cost plus expected return, and unit operating cost plus expected return on investment. Make an operating-cost pattern chart. For additional data refer to the same lettered problems in Chapts. 1, 6, 8, and 9.

A. The yearly operating costs for a 250-room commercial hotel have been estimated to be as follows:

	Percentage of capacity			
	25	50	75	100
Total fixed operating costs.....	$89,565	$89,565	$89,565	$89,565
Total variable operating costs..	15,625	31,250	46,875	62,500

Unit operating costs are to be in dollars per room-night.

First cost has been estimated at $1,270,000; working capital is to be equal to one month's income at 100 per cent capacity. Income is to be 365 days at $4 per day. The expected yield on the total investment is to be 8 per cent.

B. The yearly operating costs for a central concrete-mixing plant operating 150 days a year and producing 4,500 cu. yd. of concrete mix have been determined as follows:

	Number of days					
	25	50	75	100	125	150
Total fixed operating costs................	$5,590	$ 5,590	$ 5,590	$ 5,590	$ 5,590	$ 5,590
Total variable operating costs................	5,862	11,725	17,588	23,450	29,312	35,176

Unit operating costs are to be in dollars per cubic yard. First cost of plant is $10,500. Working capital is to be equal to deliveries for 30 days at $9 per cubic yard. Expected return is to be 15 per cent.

C. The yearly operating costs for a 1,500-kw. power plant having uniflow engines as prime mover have been estimated to be as follows:

UNIFLOW ENGINES

	Percentage of capacity				
	25	50	75	100	125
Total fixed operating costs....	$57,912	$57,912	$57,912	$ 57,912	$ 57,912
Total variable operating costs.	52,485	72,317	86,119	103,594	132,679

First cost has been estimated at $613,825. Working capital is to be equal to operating costs for one month at 50 per cent capacity.

Unit operating costs are to be in cents per kilowatt-hour. Expected return on the investment is to be 7 per cent.

D. The yearly operating cost for a factory making milk-bottle caps at the rate of 1,000,000 per day for 300 days has been found to be as follows:

	Percentage of capacity			
	20	40	60	80
Total fixed operating costs.....	$72,100	$ 72,100	$ 72,100	$ 72,100
Total variable operating costs..	76,262	152,525	228,787	305,050

Unit operating costs are to be in units of 1,000 caps. First cost for building and machinery has been estimated at $54,920. Working capital is to be cost of one month's production. Expected return is to be 15 per cent.

E. The yearly operating costs for a proposed cold-storage locker plant having 320 lockers has been estimated to be as follows:

	Number of lockers			
	80	160	240	320
Total fixed operating costs.........	$5,081	$5,081	$5,081	$5,081
Total variable operating costs......	48	96	144	192

Unit operating costs are to be in dollars per locker per year. First cost has been estimated at $15,004 including working capital. Expected return on the investment is to be 12 per cent.

CHAPTER 11

CRITERIA FOR MAKING A DECISION

11.1. Economy Analysis.—An economy analysis is a study of the factors of cost that pertain to a specific project in order to be able to determine whether or not it should be undertaken. These factors of cost and certain others derived from them together with other devices are the criteria with which the promoter or supplier of funds evaluates projects.

The factors of cost are those which have been developed in previous chapters: first cost and investment, yearly operating cost, and income. Factors derived from these original-cost factors are return, economic yield, yearly operating cost, yearly operating cost plus expected return, unit operating cost, and unit operating cost plus expected return.

Three devices are useful in coordinating these criteria: the operating-cost pattern curve, the break-even chart, and the profit and loss statement.

11.2. Profit and Loss Statement.—A profit and loss statement is a form of report used by businessmen to show the values in a project that have changed from one date to another date. It also shows whether or not the net resulting change is negative or positive; namely, a loss or a profit. This same technique may be used as a fundamental method for determining whether or not a project is worth investing in.

In its simplest form it is as follows:

Income from the project....................	$xxx,xxx.xx
Operating cost of the project.................	xxx,xxx.xx
Net return from the project..................	$ xx,xxx.xx

A more detailed form is the following one which is typical for a business enterprise:

TYPICAL PROFIT AND LOSS STATEMENT

Gross sales billed		$xxx,xxx.xx
Deduct:		
Cash discounts allowed customers	$ x,xxx.xx	
Quantity discounts	x,xxx.xx	
Returned goods	x,xxx.xx	xx,xxx.xx
Net sales billed		$xxx,xxx.xx
Deduct:		
Cost of net sales billed or manufacturing cost of sales billed:		
Direct labor	$xx,xxx.xx	
Direct material (less discounts on purchases)	xx,xxx.xx	
Departmental expense	xx,xxx.xx	xxx,xxx.xx
Gross margin		$xxx,xxx.xx
Deduct:		
General expense:		
Distribution expense	$xx,xxx.xx	
Administrative expense	xx,xxx.xx	xx,xxx.xx
Income from operations or net operating return		$ xx,xxx.xx
Additions to income:		
Income from governmental and other readily marketable securities		x,xxx.xx
Total return (gain or loss)		$ xx,xxx.xx

It is not the intention to discuss the accounting problems associated with the profit and loss statement, but to adapt it to the determination of whether or not a project is worth while from the standpoint of the investor. The various elements that make up the profit and loss statement have been discussed in previous chapters. It remains to show how they may be woven into this business form. An illustrative problem will be used to show the method of gathering the information and its use.

11.3. The Situation.—A group of people are talking at a social gathering in a small college town. They are bemoaning the fact that it is so difficult to obtain satisfactory places to live at a reasonable rent. One man is quite impressed with the question that was raised. He goes home, discusses the problem with his wife, and decides that the answer to it may provide a place for investing the extra funds that have come into his possession recently.

Note that the first step of the *engineering process* has begun; namely, "Research on the problem of satisfying the need." Since the problem is laid in a small college town, it is logical that this man, call him the promoter, should study the situation as it exists in other college towns of about the same size and location. His survey will include an analysis of the kinds of living accommodations that exist in those other towns and the number of them. He will be interested in the size of accommodations, the number of rooms, their size, garages, etc. He will find people living in nicely constructed and arranged apartment buildings. There will be hotels and other forms of rooming accommodations.

He will investigate the ability of people in this town to pay rent. He will wish to know what they can afford to pay from the salary that they receive. This will call for a study of the budgets of the local people.

He will establish the relationship between supply and demand for living accommodations.

The next step in the *engineering process* is "Invention or adaption of existing means and methods." His problem here is to select a few types of living accommodations which he believes will be satisfactory and, in the light of the information previously collected, modify and adapt them to suit his problem. As a result of his study, he comes to the conclusion that there are two alternative plans that he might use. One is to build a housing development consisting of small single houses. The other is to build an apartment building. He is quite satisfied that either of these plans will be satisfactory and feels quite sure that he could easily rent all of the houses or apartments.

The next step of the promoter is "Preliminary valuation of the project." Before he can present it to the public and before he is ready to invest in it, he wishes to find out definitely what the economic, intangible, and financial aspects of the project are. Accordingly, he proceeds to gather the information necessary to the several analyses that he wishes to make.

11.4. First-cost Analysis.—For this purpose the promoter hires an architect to prepare preliminary plans for the housing project and to make estimates of the cost of construction. Because the estimate desired is to be used in a preliminary way, the architect is to use the index figure method of figuring the first costs and submits the following data:

Housing development: 20 small houses consisting of five rooms and bath with attached garage located on a suitable plot of ground; contents 13,850 cu. ft.

From index numbers, it is found that for the type of construction under consideration each house will cost 31½ cents per cubic foot.

	For One House	For 20 Houses
Building cost will be (13,850) ($0.315)	$4,362.75	$ 87,255.00
Cost of land including improvements	1,100.00	22,000.00
Total estimated cost	$5,462.75	$109,255.00
(The architect rounded this figure to $5,500 per house.)		
Cost of development (20)($5,500)		$110,000

11.5. Income Analysis.—From his previous survey the promoter estimates that he will be able to rent his houses for $60 per month and the garage for $5, or a total of $65 per month. Realizing that college populations fluctuate greatly, especially at the close of the school year, but taking into account summer sessions, he believes that he can rent his house for 100 per cent of the time. His total income for the year may then be estimated as

$$(20)(\$60 + \$5)(12) = \$15{,}600$$

11.6. Operating Expense Analysis.—The costs of operating the houses is as follows:

Taxes, $0.05 on a 20 per cent valuation:	
Total ($110,000)(0.20)($0.05)	$1,100
Fire insurance, $0.50 per $100 on an 80 per cent valuation not including ground or development charges:	
Total $\frac{\$110{,}000 - \$22{,}000}{\$100}$ (0.80)($0.50)	352
Depreciation, 2 per cent on the buildings (straight-line):	
Total ($88,000)(0.02)	1,760
Repairs, rent for two months per house per year:	
Total (20)(2)($65)	2,600
Miscellaneous	440
Agent's commission, 4 per cent of the gross revenue:	
Total ($15,600)(0.04)	624
Total operating expense	$6,876

11.7. Return Analysis.

Income at 100 per cent capacity	$15,600
Operating expense at 100 per cent capacity	6,876
Net return	$ 8,724

It should be observed that this calculation is, in effect, an application of the simple profit and loss statement.

11.8. Application of Detailed Profit and Loss Statement.—For purposes of illustration, the problem of the industrial enterprise used in Secs. 9.5, 9.6, and 9.7 will be used.

The following additional data are assumed:

Operating cost, sales:	
Fixed	$2,560
Variable	$8,640 for 3,200 units
Income:	
Selling price	$20 per unit

Profit and Loss Statement

Gross sales billed		$64,000
Cash discounts	$ 000	
Quantity discounts	000	
Returned goods	000	000
Net sales billed		$64,000
Manufacturing cost of sales billed:		
Direct labor	$10,240	
Direct material	24,640	
Departmental expense:		
Fixed	7,344	
Variable	1,840	44,064
Gross margin		$19,936
General expense:		
Sales expense:		
Fixed	$ 2,560	
Variable	8,640	11,200
Net operating return		$ 8,736

11.9. Reversed Application of Profit and Loss Statement.—When it is impossible to estimate or determine the amount of income for a project, the application of the profit and loss statement analysis is reversed. In the usual procedure, income is known and return is sought as an answer. In the reversed procedure, return is assumed as known and income is sought as as answer. This method finds its use in a great majority of projects that come within the scope of engineering economics. This is so because rarely is it possible in an engineering project to know the amount of the income. In consequence thereof, the return from the project must be assumed and the income backfigured. Income in this case is the equivalent of operating cost plus expected return on the investment which was discussed in Secs. 10.9 and 10.10. An illustrative problem will be the automobile discussed in Sec. 9.8.

Profit and Loss Statement

Estimated service, 40,000 miles at 60 m.p.h.

Total yearly operating cost	$ 948.36
Expected return at 12 per cent yield on investment of $850	102.00
Total operating cost plus expected return on investment	$1,050.36

This total amount represents the income that would be necessary, if the automobile is a commercial venture, in order to earn through its operation a yield of 12 per cent on the original investment of $850.

11.10. Economic Yield Analysis.—The economic yield as defined in Chap. 10 is equal to

$$\frac{\text{Net return}}{\text{Investment}} \times 100 \text{ per cent}$$

Housing problem:

$$\text{Economic yield} = \frac{\$8,724}{\$110,000} \times 100 = 7.93 \text{ per cent}$$

Industrial enterprise problem:

$$\text{Economic yield} = \frac{\$8.736}{\$\ 82,000} \times 100 = 10.65 \text{ per cent}$$

The promoter of the project will compare this economic yield with yields from other similar projects in order to determine if it is a satisfactory yield.

11.11. Unit Operating Costs.—Two types of operating costs were discussed in Chaps. 9 and 10; unit operating cost in Sec. 9.4, and unit operating cost plus expected return on investment in Sec. 10.11.

It was shown that the term "unit operating cost" was equal to the total yearly cost divided by the quantity produced. It was also shown that the term was meaningless unless it was accompanied by a statement of specific capacity or rate production. The unit operating-cost pattern was such that the unit cost decreased as capacity or rate of production increased except in some cases where it again increased slightly.

Operating-cost patterns are developed in Fig. 11.1 for the housing problem and in Fig. 11.2 for the industrial enterprise problem.

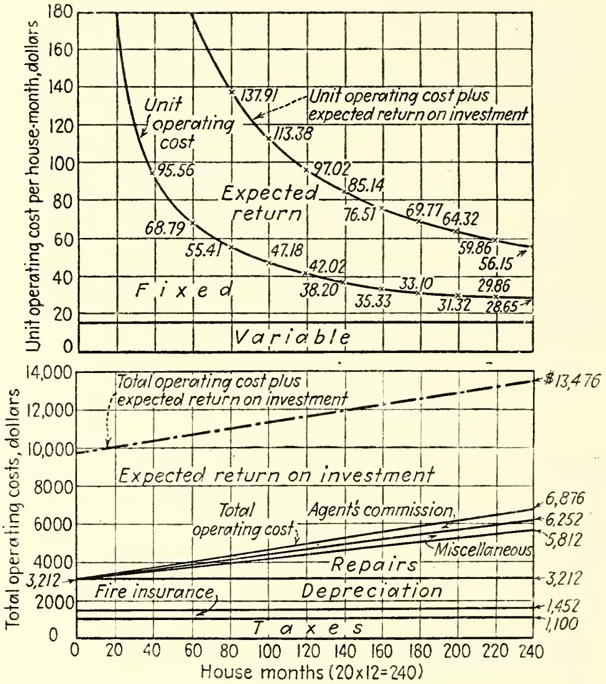

Fig. 11.1.—Operating-cost chart for a housing development.

The operating-cost curve for the automobile problem used in Sec. 11.9 is shown in Fig. 10.2.

11.12. Break-even Chart.—The break-even chart may be thought of as a pictorial profit and loss statement. It shows the relationship between the several factors of cost and income. It is made by adding to the chart that shows total operating cost and unit operating cost an income line and a unit selling price line.

Figure 11.3 is the break-even chart for the housing problem. The income is $15,600 yearly. The income line is drawn from the zero-zero intersection to $15,600 on the ordinate above 240 house-months. It will be observed that this income line crosses the total-operating-cost line at about 64 house-months. This is the point in terms of capacity where the total income and the total operating cost for that capacity are equal. It is, therefore, the point of "no gain–no loss." If operation is below this point, the business will be losing; if above this point, the business will operate at a gain.

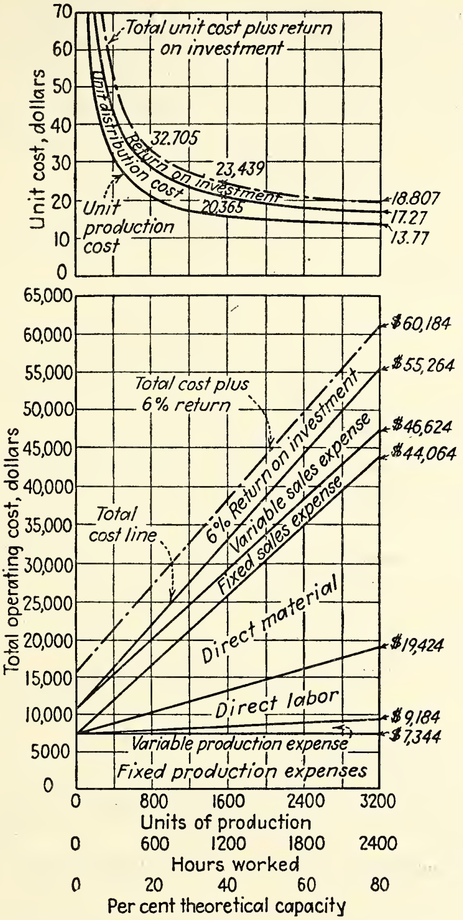

Fig. 11.2.—Operating-cost chart for an industrial enterprise. See Fig. 9.27 and 10.1.

The income line also crosses the total-operating-cost-plus-expected-return line at about 197 house-months. This is the point in terms of capacity where the relationship between income and operating cost produce an expected return of 6 per cent on the investment. If operated above this point, the project will produce a return in excess of a 6 per cent yield. If operated between this point and the point of no gain–no loss, the return will vary from zero yield to a yield of 6 per cent.

In the upper part of the chart, in the unit section, the unit selling price has been drawn in. It will be observed that this line, which is in reality a unit-income line, crosses the unit-operating-cost and the unit-operating-cost-plus-expected-return lines at the same points as in the lower diagram.

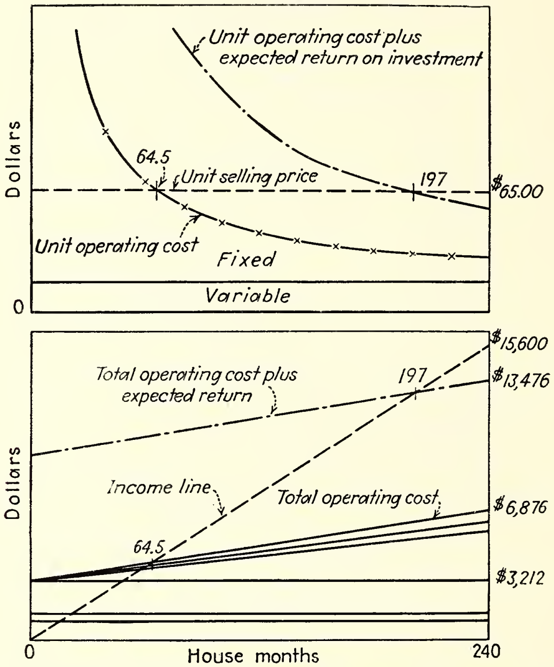

FIG. 11.3.—Break-even chart for a housing development. See Fig. 11.1.

Thus the break-even chart enables one to see the relationship between the several elements or factors in costs as compared with income and with capacity or rate of production. It also enables one to visualize the condition of the project in terms of capacity and selling price at that capacity.

Figure 11.4 is a break-even chart for the industrial enterprise. It shows information similar to that of the housing project.

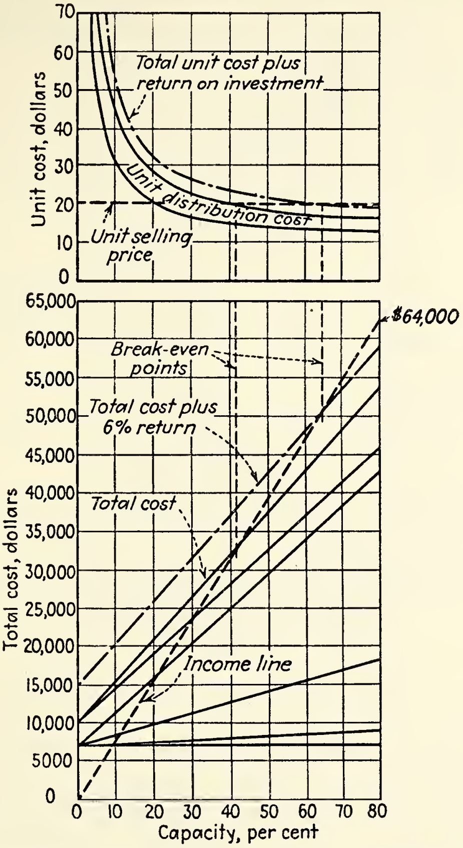

FIG. 11.4.—Break-even chart for an industrial enterprise. See Fig. 9.27, 10.1, and 11.2.

In Fig. 11.5, the chart of Fig. 10.2, the automobile problem, is reproduced. Income has been assumed to be the operating cost plus expected return at 30,000 miles. The income line is then drawn from zero-zero to the intersection of the 30,000-mile ordinate and the line of operating cost plus expected return. The implications of the relationships between the factors of cost, assumed income, and capacity or rate of production are evident.

11.13. Calculation of Break-even Point.—In Sec. 11.12 the break-even chart was explained. The break-even point on the chart was the point at which the income line crossed the total-operating-cost line. Another break-even point was where the

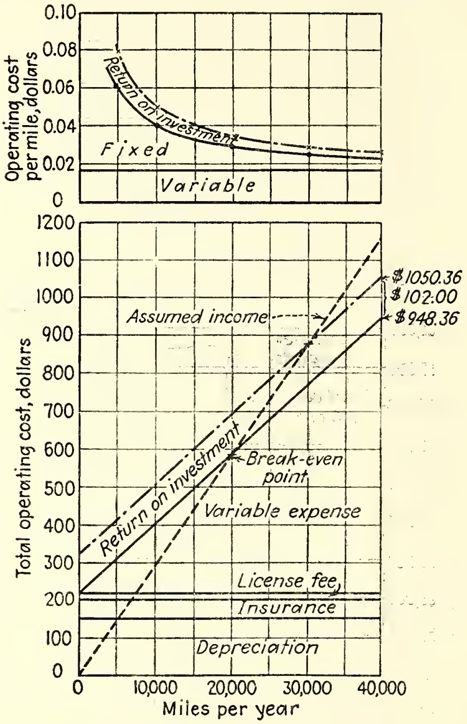

Fig. 11.5.—Break-even chart for a medium-priced automobile. See Figs. 9.29 and 10.2.

income line crossed the line of operating cost plus expected return on the investment. There may be other break-even points depending on the particular relationship between income and some other factor which the investigator wishes to study.

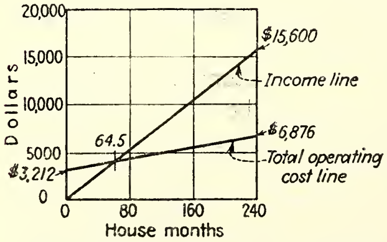

Fig. 11.6.—Basic data for calculation of break-even point for a housing development. See Figs. 11.1 and 11.3.

The calculation of this point may be accomplished by the application of the principles of analytical geometry. The geometrical relationships are shown in Fig. 11.6 for the housing problem.

The equations of these two lines may be found by inspection. They are

Income sales line,

$$y = m_1x$$
$$y = \frac{15{,}600x}{240}$$

Operating cost line,

$$y = m_2x + b$$
$$y = \left(\frac{6{,}876 - 3{,}212}{240}\right)x + 3{,}212$$

The break-even point occurs where these two lines cross. This is at the point where the ordinate at a particular capacity point is the same for each line. Therefore, since the y values are the same, the equation of the point may be written

$$\frac{15{,}600x}{240} = \left(\frac{6{,}876 - 3.212}{240}\right)x + 3{,}212$$
$$\left(\frac{15{,}600 - 6{,}876 + 3{,}212}{240}\right)x = 3{,}212$$
$$x = 64.5 \text{ house-months}$$

11.14. Minimum Unit-cost Point.—In Sec. 9.4, the development of a unit-operating-cost curve was presented. In Figs. 9.22, 9.24, and 9.26 the shape of the unit-operating-cost curve was such that after decreasing, as use or capacity increased to a certain point, it then increased. This is an example of the law of diminishing returns, which states that, in general, as factors of production are added, a less proportionate increase in amount produced occurs after a certain rate of production has been reached.

This property of the use of production factors is, therefore, very valuable.

An obvious property of a curve of diminishing returns, as shown in Figs. 11.7 and 11.8, is that there is a minimum point to the curve. It is at this point that the addition of greater amounts of the factors of production produces less increase in production. Therefore, this minimum point in the unit-cost curve may be used as a criterion in arriving at decisions, because it is at this point of rate of production or rate of capacity that the least operating cost occurs. There are two types of curves, as illustrated in Figs. 11.7 and 11.8.

In Fig. 11.7, the curve approaches the minimum point sharply. In Fig. 11.8, the curve approaches it in a broad and very gradual manner. The minimum point is easy to define in the first case but difficult in the second. In other words, the combination of cost factors in the first case is such that the minimum unit-cost point is within very narrow limits; in the second case the unit-cost point is within large limits. In other words, the selection of of a minimum cost point in the second case may be made over wider limits of rate production than in the first case. Therefore, machinery and equipment subject to this property are best designed when the unit-cost curve is of the second type because they are more flexible than the first.

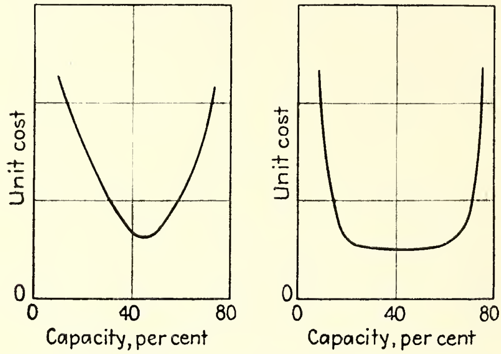

FIG. 11.7. FIG. 11.8.

FIGS. 11.7 and 11.8.—Typical minimum operating-cost curves.

11.15. An Insulation Problem. Law of Diminishing Returns. The architect is discussing with the promoter of the housing development the value of wall insulation. The following data have been gathered by the architect:

Wall construction: 2- × 4-in. studs

Siding or clapboards over sheathing with building paper between

¾-in. plaster on metal lath

Area of wall under consideration: 100 sq. ft.

First cost, 30 cents per square foot	$30.00	
Depreciation, 3 per cent		0.90
Return on investment		1.80
Total		$2.70

The architect proposes to add ½ in. of rigid insulation between the sheathing and the siding. He then proposes to add rigid insulation in units of ½ in. in thickness. In order to determine how much rigid insulation will give the best economy, he desires to calculate the cost of heat loss through the wall consisting of various thickness of rigid insulation.

This operating cost will equal the sum of the fixed operating costs and variable operating costs, or cost of heat losses. The cost of heat losses will depend upon the heat transmission of the wall section. The heat transmission factors for various thicknesses of rigid insulation are as follows:

Thickness of Rigid Insulation	Heat Transmission B.t.u. per Degree Difference in Temperature per Square Foot
. .	0.255
½	0.184
1	0.144
1½	0.118
2	0.100
2½	0.087
3	0.077
3½	0.069
4	0.062
4½	0.057
5	0.052
5½	0.048
6	0.045

The first cost of the rigid insulation is 4.2 cents per square foot, or $4.20 for the area under consideration. If return on investment and depreciation are assumed at the same rates as for the original wall, the fixed operating costs for each ½ in. of rigid insulation will be

Depreciation....................	$0.126
Return on investment...........	0.252
Total.........................	$0.378 or $0.38

The cost of heat loss is given by the following equation:

$$C_{hl} = C_c Z T U A$$

in which C_{hl} is cost of heat loss; C_c is cost of coal per 1,000,000

B.t.u. produced (assume at \$0.70); ZT is the measure of heat loss per year in terms of degree-hours (assume 150,000); U is the factor of heat transmission in B.t.u. per square foot per degree difference in temperature; A is the area of wall surface (in this case 100 sq. ft.).

Cost of heat loss for the original noninsulated was

$$C_{hl} = \frac{(\$0.70)(150{,}000)(0.255)(100)}{1{,}000{,}000} = \$2.67$$

Cost of heat loss for original wall plus $\frac{1}{2}$-in. rigid insulation

$$C_{hl} = \frac{(\$0.70)(150{,}000)(0.184)(100)}{1{,}000{,}000} = \$1.93$$

This computation carried through for all the various thicknesses of rigid insulation is shown in the accompanying tabulation.

If to this cost heat loss is added the fixed operating cost for the several thicknesses of rigid insulation, the total operating cost will be as follows:

Thickness of rigid insulation	Cost of heat loss	Reduction in heat loss	Fixed operating cost	Total operating cost
..	\$2.67	\$0.00	\$2.70	\$5.37
$\frac{1}{2}$	1.93	0.74	3.08	5.01
1	1.51	0.42	3.46	4.97
$1\frac{1}{2}$	1.24	0.27	3.84	5.08
2	1.05	0.19	4.22	5.27
$2\frac{1}{2}$	0.91	0.14	4.60	5.51
3	0.81	0.10	4.98	5.79
$3\frac{1}{2}$	0.72	0.09	5.36	6.08
4	0.65	0.07	5.74	6.39
$4\frac{1}{2}$	0.59	0.06	6.12	6.71
5	0.54	0.05	6.50	7.04
$5\frac{1}{2}$	0.50	0.04	6.88	7.38
6	0.47	0.03	7.26	7.73

The above data are shown in chart form in Fig. 11.9.

It will be observed that fixed operating costs increase with the increase of thickness of the rigid insulation and that the heat loss decreases as the thickness of the insulation increases. The sum of the respective values for the several thicknesses gives the curve of total operating costs. A definite minimum point will be seen at the 1-in. thickness. Any further thickness above this point is not justified from the standpoint of economy and money saving. It is very easy, however, to justify an increase above the minimum point, if the effect on the heating problem is to make the house more easily heated. This may be in the nature of intangible factors.

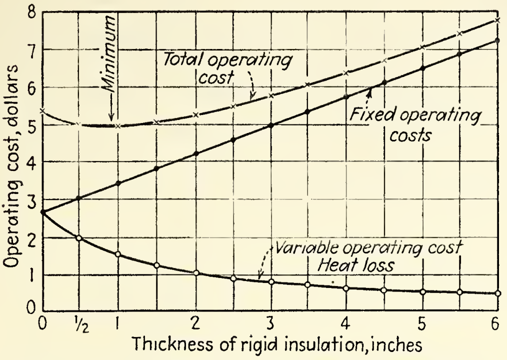

Fig. 11.9.—Operating-cost chart for wall insulation.

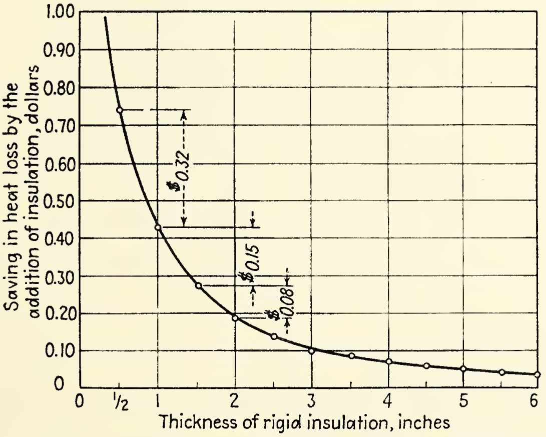

Fig. 11.10.—Curve of diminishing returns, showing diminishing savings of additional $\frac{1}{2}$ in. thicknesses of insulating material.

There is one other matter not to be forgotten. For a true picture of the economy of insulation taking the over-all problem of heating the house, the effect of insulation on the reduction in size of the heating plant should be taken into consideration.

If the differences in heat loss are plotted, Fig. 11.10 is secured.

The curve joining the heat loss savings for each additional $\frac{1}{2}$ in. thickness of rigid insulation shows a diminishing value with each addition.

11.16. Kelvin's Law.—Lord Kelvin showed that the most economical size of a conductor of electricity was one whose fixed investment charges just equaled the cost of lost energy.

This law is a statement of that property of the combination of the cost factors of production which was discussed in the last section. There is one difference. In this case, a specific quantity of electricity is delivered through the wire. The mechanism to do this consists of the conductor. The size of this conductor can be varied within certain limits. As the size of the wire is decreased the first cost of the wire is decreased and, therefore, the fixed investment charges are decreased. But as the size of the wire is decreased, the resistance to flow of current is increased with a consequently increased cost of power lost. The sum of these two types of costs, one increasing with change in size of wire and the other decreasing, will give a varying operating cost. The characteristic of this sum is that when plotted against size, the curve resulting therefrom is parabolic in nature, that is to say, it has a minimum point.

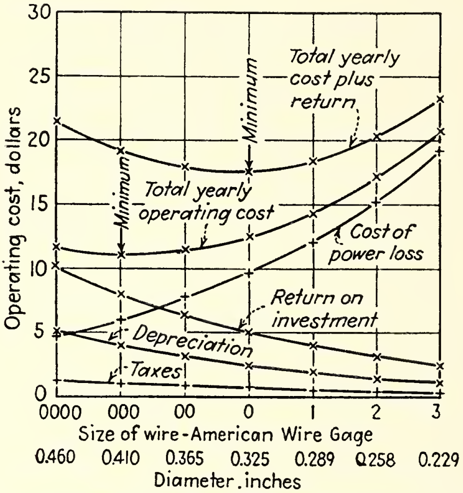

FIG. 11.11.—Operating-cost chart for the determination of the economical size of a conductor.

Figure 11.11 is a graphical representation of this problem. It will be observed that the minimum point for total yearly operating cost is at wire size "000." Since in order to judge a project it is necessary to consider earning a return on the investment in it, and since there are no income figures available, it becomes necessary to add an expected return to the total yearly operating costs. This return on the investment will also decrease as the size of the wire decreases since first cost decreases. When added to the curve for total yearly operating costs, there results another curve whose minimum point is at wire size "0," a shift of several sizes. Thus it is seen that in order to have the correct criterion for this type of situation, the minimum point must be that which is found on the curve of total yearly operating costs plus expected return on the investment.

This type of situation is similar to the first cost curve pattern discussed in Chap. 5, Sec. 5.3. In that section two examples were given, a bridge and a transmission line. The principal characteristic of those problems was that the length of the bridge and length of the transmission line were fixed. Within this fixed length, the number of spans could be varied, giving rise to differing first costs. Likewise, in this conductor problem, the amount of current to be delivered is fixed but the size of the wire is variable. The resulting relationship of cost factors is such that some cost factors decrease with the variable factors while others increase. The total resulting cost is parabolic in nature. Other similar situations may be found in industry, such as

1. The size of a pipe for the flow of a fixed quantity of water. As the pipe size increases, fixed operating costs as well as expected return on the investment increase, but cost of power loss because of friction decreases.

2. The number of effects for an evaporator system to evaporate a fixed quantity of liquid. As the number of effects increase, the fixed operating costs as well as the expected return on the investment increase but the cost of steam used decreases.

3. The surface of a condenser for condensing a fixed amount of vapor. As the surface of the condenser is increased, the fixed operating costs as well as the expected return on the investment increase while the costs for pumping the cooling water decrease.

4. The size of a purchase lot or production lot when purchasing or producing a fixed number of pieces per year. As the number

of lots is increased, the cost of repeated ordering or of machine setups increases but the inventory of pieces decreases with a consequent decrease in storage cost and expected return on investment in inventory. Figures 9.15, 9.16 to 9.18, 9.22, 9.24 and 9.26 are representations of production cost patterns for this type of problem.

11.17. Recapitulation.—The information that results from making the economy analysis may be listed as follows:

Housing Problem.—Data for 100 per cent capacity:[1]

1.	First cost of investment	$110,000
2.	Income	15,600
3.	Operating cost	6,876
4.	Return	8,724
5.	Economic yield	7.93%
6.	Total operating cost plus expected return of 6 per cent	$ 13,476
7.	Unit operating cost	28.65 per house per month
8.	Unit operating cost plus expected return of 6 per cent	56.15 per house per month
9.	Rental or selling price	65.00 per house per month
10.	Break-even point for no gain—no loss	64.5 house-months
11.	Break-even point for 6 per cent yield on investment	210 house-months

Industrial Enterprise.—Data for 100 per cent capacity:[2]

1.	First cost or investment	$82,000
2.	Income	64,000
3.	Operating cost	55,264
4.	Return	8,736
5.	Economic yield	10.65%
6.	Total operating cost plus expected return	$60,184
7.	Unit operating cost	17.27
8.	Unit operating cost plus expected return of 6 per cent	18.80
9.	Selling price per unit	20.00
10.	Break-even point for no gain—no loss	42.5%
11.	Break-even point for 6 per cent yield	64.0%

[1] For other capacities see break-even chart Fig. 11.3.

[2] For other capacities see operating-cost chart.

Automobile.—Data for 60 m.p.h., 40,000 miles per year:[1]

1. First cost	$850
2. Income	Not determinable
3. Operating cost	$948.36
4. Return	102 (estimated)
5. Economic yield	12% desired
6. Total operating cost plus expected return of 12 per cent	$1050.36
7. Unit operating cost	0.0237 per mile
8. Unit operating cost plus expected return of 12 per cent	0.02625 per mile
9. Selling price	Not specified

Insulation.

1. First cost: $4.20 for each $\frac{1}{2}$ in. thickness of insulation for 30 sq. ft.
2. Fixed cost: $0.38 for each $\frac{1}{2}$ in. thickness.
3. Variable cost (heat loss): Varies from $2.69 with no insulation to $0.47 for 6 in. Variation with thickness is curved.
4. Total operating cost: Varies from $5.37 with no insulation to $7.73 with 6 in. of insulation, going through a minimum value of $4.97 with 1 in. of insulation.

It should be observed that the use of several criteria thus brought together will depend upon what particular factors are up for discussion. It is obvious that to use one criterion is to lose the allround picture of the project. The several criteria together give a picture which should assist in making decisions.

In the housing and industrial enterprise problems, the main single criterion may well be unit operating cost plus expected return, since it gives a direct comparison with selling prices. It must not be forgotten that this criterion should always be tied in with some figure stating the rate of production or capacity use.

In the automobile and insulation problems the main single criterion will be the location of the minimum operating cost plus expected return.

The preceding illustrations have dealt with completely assembled projects. The method of analysis and use of criteria are just as applicable to a part of the project, such as the selection of materials out of which the part is to be made. However, it is not sufficient or correct to compare the purchased price or cost of the raw materials alone. Because various materials require different manufacturing processes and, therefore, diverse machinery and equipment, it is necessary to make the economic analysis on the process of converting the raw material into a finished product. The specific criterion that is applicable in this case is obviously the cost of production per unit of product.

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[1] For other capacities see break-even chart Fig. 11.5.

11.18. Capitalized Cost.—Capitalized cost is the application of the principle of valuation discussed in Sec. 4.6. It will be more fully developed and applied in Chap. 21.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following lettered projects, proposals, or situations has been brought to your attention. You are to determine income, draw a break-even chart, and determine the break-even point for no gain—no loss. Make a recapitulation of derived data, such as unit operating cost, etc. For additional data refer to the same lettered problems in Chaps. 1, 6, 8, 9, and 10.

A. The following data have been obtained for a commercial hotel:
First cost: $1,270,000
Working capital: One month's income at 100 per cent capacity
Fixed operating cost at 100 per cent capacity: $89,565
Variable operating cost at 100 per cent capacity: $62,500
Number of rooms: 250
Number of days operation per year: 365
Room rent, average charge per day: $4
Expected yield on investment: 8 per cent

B. A central mix concrete plant operates 150 days in the year and produces 4,500 cu. yd. of concrete mix. Data for its operation are as follows:
First cost of plant: $10,500
Working capital: Equal to sales for 30 days at $9 per cubic yard
Operating cost annually:

Fixed	$ 5,590
Variable	$35,176

Expected yield: 15 per cent

C. Use the data given in Chap. 10 for Prob. *C*, together with the average charge for electricity at $2\frac{1}{2}$ cents per kilowatt-hour.

D. Data for a plant manufacturing milk-bottle caps have been determined by a firm of consulting engineers to be as follows:
First cost: $54,920
Working capital: Cost of one month's production
Fixed operating costs at 80 per cent capacity: $72,100
Variable operating costs at 80 per cent capacity: $305,050
Expected return on investment: 15 per cent
Selling price: $2.70 per thousand
Discount: 2 per cent in 10 days (assume discount taken)
Production at 80 per cent capacity: 1,000,000 per day for 300 days

E. The following data have been obtained for a cold-storage locker plant:
First cost: $15,004, including working capital
Fixed operating cost at 100 per cent capacity: $5,081
Variable operating cost at 100 per cent capacity: $192
Total number of lockers: 320
Rental charge per year per locker: $30
Expected return on investment: 12 per cent

1. A pumping plant with a capacity of 1,000 g.p.m. delivers water against a head of 120 in. with an efficiency of 70 per cent at full load and 40 per cent at half load. The plant costs $850 to install; power costs 1 cent per horsepower-hour; fixed charges are 12 per cent of first cost. The plant operates at full load for 2,000 hr. per year and at half load for 1,000 hr. per year. Expected return on the investment is to be 10 per cent. Find the total annual cost of operation and the cost per gallon pumped at full load and at half load. (Modified from examination for Professional Engineer's license, New York State, Feb. 3, 1937.)

2. Snow removal from city streets is accomplished by machine loading into trucks. There will be required a machine whose first cost, including garage will equal $10,000. Labor to operate the machine will be 7 operators at $7 per day. Fuel, oil, and maintenance will amount to $28 per day. Garage maintenance will amount to $100 per year. Machine life estimated at 5 years. Disregarding return on the investment, draw the operating-cost curve for the method assuming 20 days operation.

3. A shoe manufacturer produces a pair of shoes at a labor cost of 90 cents per pair and a material costs of 80 cents per pair. The fixed charges on the business are $90,000 a month and the variable costs are 40 cents per pair. Draw an operating-cost curve assuming a maximum production of 150,000 pairs per month.

4. A power plant has a number of 1,200-hp. boilers having the following efficiency characteristics:

Per Cent of Capacity Rating	Per Cent of Over-all Efficiency
40	50
60	58
80	63
100	68
120	70
140	70
160	68

Draw the operating-cost curve, using the following additional data: coal having 12,000 B.t.u. per pound value costs $2.50 per ton (2,000 lb.), one boiler horsepower is 33,500 B.t.u. per hour. Maintenance and general operating costs are $5 per boiler horsepower. Fixed charges for depreciation, taxes, and insurance are equal to 9 per cent of first cost which is $40 per boiler horsepower. Expected return is 6 per cent.

5. An evaporator system is being set up in a chemical plant for the evaporation of a saturated salt liquor. In such a system, steam is used in the first evaporator of a series, called the "first effect"; the vapor from this effect is used in place of steam in the second effect; the vapor from this effect is used as steam in the third effect, and so on. There are a number of effects that will give the lowest unit operating cost. This number is to be determined.

As the number of effects is increased, it is obvious that the fixed operating costs increase. Also, since raw steam is used in only the first effect, the variable operating costs must decrease. At some number of effects, the sum of fixed and variable operating costs will be the least.

The desired rate is 200,000 lb. per day of 24 hr. to be evaporated for 300 days per year. The installation complete will cost at the rate of $15,000 per effect, including all accessories and the building to house them.

Evaporation will be at the rate of 0.80 lb. of water per pound of steam, times $1/N$ per setup, where N is the number of effects. Steam costs 40 cents per 1,000 lb. Labor required is one man at $1 per hour, regardless of the number of effects.

Depreciation, repairs, taxes, and insurance amount to 20 per cent of the *first cost.* Administration costs 20 *per cent* of the *labor cost* regardless of the number of effects. Expected return is 12 per cent. Cooling water costs 5 cents per 1,000 cu. ft. (62.5 lb. per cubic foot) and is used at the rate of 30 lb. per pound of steam (first effect).

Draw the operating-cost curve and the unit operating-cost curve. Determine the economical number of effects to use.

6. The size of a penstock pipe for supplying water to a hydroturbine is to be determined. The frictional loss of water flowing through the pipe is a function of the diameter of the pipe, the smaller the pipe the larger the frictional loss and vice versa. But the larger the pipe the more it costs. Hence the best size is the one that gives a minimum of operating cost. The following data are for an installation to provide flow of 100 cu. ft. per second through a vertical penstock 1,000 ft. long.

Diameter of pipe, inches	Friction loss head in feet per 500 ft.	Weight of pipe in pounds per foot
30	33.3	97.5
33	20.6	121.5
36	13.4	146.5
39	9.0	173.8
42	6.2	202.8
45	4.4	233.6
48	3.2	271.6
54	1.8	338.3
60	1.1	418.4
66	0.8	506.6

The cost of the steel pipe will be 7 cents per pound. The value of frictional loss will be $100 per horsepower. Taxes will be 1 per cent of first cost, life of pipe will be 50 years, depreciation to be straight-line method. Annual maintenance costs will be at the rate of $25 per inch of diameter of the pipe. Return on the investment is to be at the rate of 7 per cent annually. Draw the curves of total operating cost and unit operating cost. What size of pipe gives the lowest operating cost plus expected return?

7. In the design of a condenser for a power plant, that design is most economical which makes the sum of fixed charges plus operating costs a minimum. A surface condenser is to be designed which will condense 10,000 lb. of steam per hour under certain conditions of pressure and heat content. The engineers provide the following technical data for the operation of the condenser:

Circulating water, temperature rise, degrees Fahrenheit	Surface, square feet	Power for circulating pump, kilowatts
6	932	10.65
10	1,088	7.96
12	1,195	7.5
15	1,450	7.05

The base cost of the condenser shell is $3,000. The additional cost of the condenser is $3 per square foot of surface.

The cost of pumping water is 1 cent per kilowatt-hour. The condenser system operates 6,000 hr. per year. The fixed charges for depreciation, taxes, and insurance are 10 per cent of the first cost. Expected return to be 10 per cent. Draw the operating-cost curves and determine the best size of condenser.

8. An industrial concern manufactures a product for which there is a market for 2,000 units per year. It can be produced at the rate of 50,000 units per year of 300 days. The unit operating cost is $2.50 per piece. Storage cost is $3 per square foot per year. Storage space required is at the rate of 0.000625 sq. ft. per piece. If the setup cost per lot run is $200, draw the curve of operating costs. Interest on investment in stored goods is at 6 per cent. Determine the curve of operating costs and economical lot size.

9. Assume that all figures in Prob. 8, except the following, apply to another product and determine the curve of operating costs and the economical lot size. Storage space required is 0.125 sq. ft. per piece.

10. An industrial plant has the following data to be used in the determination of economical lot size:

Effective sales demand: 1,500 pieces per day
Rate of production: 2,000 pieces per day, 300 days per year
Setup cost: $415

Manufacturing cost: Labor at $0.02, material at $0.009, and overhead or fixed costs at $0.011 per piece

Storage space: 0.25 cu. ft. per 100 pieces

Cost of storage space: $3 per cubic foot per year

Interest on the investment: 12 per cent

Determine the operating-cost curve and the economical lot size.

11. An industrial concern finds that the following data pertain to a certain product:

Number parts used per day: 20

Production rate per day: 200

Number working days per year: 300

Total production cost including labor, material, and overhead: $0.25 per piece

Setup cost: $16

Insurance, and storage space charges: $0.01 per part per day. Interest on average inventory is 10 %

Determine the operating-cost curve and the economical lot size.

12. If in Prob. 11 the policy of the company is always to have at least one day's supply of parts on hand when the new lot is finished, what is the effect on the lot size?

CHAPTER 12

"IN-FAVOR-OF" ANALYSIS

12.1. Analysis of Alternative Projects.—When any project is under consideration, it is natural to compare its several factors with something else as a basis. If there is only one proposed solution, it is necessary to compare the factors of the project with some standard bases of comparison. These standard bases of comparison are measures that have been set up by the one who is to make the decision. The measures that he uses or rather the standards have been built up from his experiences on similar or related work.

For example, what rate of return or yield should an investment give? As was brought out in Sec. 10.7, there may be several bases depending upon the individual's own valuation of the intangible factors entering into the base.

When several solutions are proposed for the subject, a direct comparison may be made between the several factors entering into the project. The purpose of the analysis is to reveal the differences and segregate them so that they may be more easily evaluated.

Professor Fish, "Engineering Economics" suggests the "in-favor-of" analysis technique. It consists simply of a ruled form arranged somewhat as follows:

Item	Proposal or project		In favor of	
	A	*B*	*A*	*B*

The names of the several factors are listed in the first column. The values of these factors for the alternative proposals are listed in the second and third columns under their respective solutions.

The differences between the values of a factor for the proposals are listed in the fourth and fifth columns in accordance with the following simple rules:

1. The difference in first costs or investment is in favor of the one with the least first cost.
2. The difference in income is in favor of the one with the greatest income.
3. The difference in operating cost is in favor of the one with the least operating cost.
4. The difference in return is in favor of the one with the greatest return.
5. The difference in yield is in favor of the one with the greatest yield. It is obvious that the right combination here is lowest investment or first cost and greatest return.
6. The difference in valuation of an intangible factor is in favor of the one having the most desirable value.

12.2. Illustration of In-favor-of Analysis.—In several previous sections a housing development for a college community has been used for illustration. As an alternative proposal, an apartment development has been suggested. Assuming that all the values for the several factors have been determined, the in-favor-of analysis will be as follows:

TABLE 12.1.—IN-FAVOR-OF ANALYSIS: COMPARISON OF HOUSING DEVELOPMENT AND APARTMENT DEVELOPMENT
Economy Analysis

Item	Proposal or project		In favor of	
	Housing	Apartment	Housing	Apartment
First cost	$110,000	$110,000		
Income	15,600	16,800	......	$1,200
Operating cost	6,876	8,016	$1,140	
Return	8,724	8,784	......	$60
Economic yield	7.93%	7.98%	......	0.05%
Unit operating cost	$28.65	$33.40	$4.75	
Unit operating cost plus expected return	56.15	60.90	$4.75	
Rental price	65.00	70.00	$5.00	
Return	36.35	36.60	......	0.25
Extra return over expected return	8.85	9.10	......	0.25

Problems

K. Two apartment projects are under consideration, one having 920 rooms and the other 940 rooms. (See original statement of problem in Chap. 1.) Estimates have been made as follows:

	920 rooms	940 rooms
Total investment	$1,113,000	$1,203,000
Operating cost at 100% capacity:		
Fixed	42,640	47,057
Variable	39,100	43,240
Room rent per month	$11.80 per room From 920 rooms, 96 garages (2 garages = 1 room), and 5 stores (1 store = 14 rooms)	$11.80 per room From 940 rooms, 96 garages (2 garages = 1 room), and 5 stores (1 store = 14 rooms)
Expected yield on investment	$6\frac{1}{2}$%	$6\frac{1}{2}$%

Make an in-favor-of analysis.

M. (Refer to original statement of problem in Chap. 1.) The first costs for the two truck fleets, 1-ton and 2-ton, have been determined to be as follows:

	Ten 1-ton trucks	Five 2-ton trucks
Trucks	$10,000	$12,500
Garage	6,896	4,774
Repair equipment	5,000	5,000
	$21,696	$22,274

An analysis of yearly operating costs for the two fleets of trucks shows the following, on a per truck basis:

	1-ton truck	2-ton truck
Depreciation:		
Truck	Life, 3 years	Life, 4 years
Building	Life, 40 years	
Equipment	Life, 10 years	
Insurance	$29.69	$35.88
License	18.75	70.00
Driver	$26 per week, 52 wk.	
Gas consumption at 21 cents/gallon	1,691 gal.	2,901 gal.
Oil consumption at $1/gal	68 gal.	139 gal.
Tires	$128.00	$145.34
Miscellaneous:		
Upkeep and repairs	120.00	240.00
Repair parts	30.00	55.00
Mileage per year	12,704	11,901

Determine the operating costs for the two fleets assuming that normal operation is the indicated number of miles listed as mileage per year in the foregoing data. Make an in-favor-of analysis. Expected return to be 10 per cent.

1. An investor has $50,000. The choice is presented between buying tax-free bank stock at $250, which pays $6 per share annually, and buying a piece of rental property, the ground being worth $10,000 and the building $40,000. The property rents for $500 per month and the vacancy expectancy is 10 per cent. The building has a useful life of 20 years. Taxes are 3 per cent on an 80 per cent valuation. Insurance is $270 per year. The repairs will average $400 per year. The real-estate agency charges 3 per cent of the gross income. The owner pays 2 per cent income tax on net income. Make an in-favor-of analysis. (From examination for Professional Engineer's license, New York State, June 29, 1938.)

2. An untreated railroad tie has an average life of 7 years; the same tie treated with creosote has an average life of 18 years. The untreated tie will cost $1.42 in place. Assume depreciation on a 4 per cent compound interest equivalence basis. How much can a railroad afford to spend for treatment, if a 7 per cent return on the investment in treatment is required? (From examination for Professional Engineer's license, New York State, July 8, 1937.)

3. On the basis of the following data, state whether you would select a wood or a steel penstock for a hydroplant:

	Wood	Steel
First cost	$124,000	$169,000
Annual operation and maintenance	450	700
Life	25 years	50 years

Taxes, 1 per cent, depreciation 4 per cent sinking fund; expected return 6 per cent. (Modified from examination for Professional Engineer's license, New York State, Feb. 3, 1937.)

4. The replacement part of a wood-stave flow line 13 ft. in diameter costs $1,000,000. A 14-ft. steel flow line would be the equivalent hydraulically but would cost $1,400,000. Consider the return on the investment as 7½ per cent, taxes 1 per cent, maintenance ½ of 1 per cent, life of wood-stave pipe 25 years and of steel pipe 50 years. Depreciation is taken care of by a sinking fund at 5 per cent. Make an in-favor-of analysis. (Modified from examination for Professional Engineers' license, New York State, Jan. 27, 1936.)

5. A city requires 2,000,000 kw.-hr. of electric energy per year for various municipal purposes. It can purchase this energy from a power company for 1.3 cents per kilowatt-hour. The city may build its own power plant for $120,000. Annual operation and maintenance costs are estimated at $15,000. Depreciation is charged by straight-line method for a life of 20 years. Insurance is 1 per cent. If the city can borrow the money to build the plant at 4 per cent, will it be a good investment?

6. It is proposed to place a cable on an existing pole line along the shore of a lake in order to connect two points on opposite sides of it. Determine whether it would be more economical to place a submarine cable across the lake, assuming the following data:

	Land route	Submarine route
Length	10 miles	5 miles
First cost of cable per mile	$10,000	$18,000
Annual maintenance per mile	200	800
Expected return	7%	7%
Taxes	2%	2%
Life	15 years	15 years
Net salvage value per mile	$3,000	$3,000
Depreciation, sinking fund	7%	7%

Make an in-favor-of analysis. (Modified from examination for Professional Engineer's license, New York State, June 26, 1935.)

PART II

THE INTANGIBLE ANALYSIS

What factors in the project are intangible?
What factors are subjective?
What factors are irreducible?
How can the above factors be evaluated?

CHAPTER 13

THE INTANGIBLE ANALYSIS

13.1. The Intangible Analysis.—The economy analysis dealt with those factors pertaining to the project that could be described with monetary symbols. The intangible analysis has to do with a study of the project from a consideration of the factors that cannot be described with mathematical or monetary symbols.

The names "reducible," "objective," and "tangible" have been given to factors of the first class. The term "reducible" means that the factors are capable of being reduced or of being brought to a form in which money values can be assigned easily. "Objective" means that factors can be handled and seen; whatever variables they possess are easily seen and understood and can have money values assigned to them easily. "Tangible," likewise, means that the factors are capable of apprehension from many angles. The meaning is that abstract statistical values in some definite system of units can be assigned to this class.

To the second class of factors, the opposites of the words used to describe the first class are given, namely, "irreducible," "subjective," and "intangible." "Irreducible" means that the factors in question are of such a nature that their meanings, implications, or relationships elude the investigator. "Subjective" has the meaning, here, that the interpretation of the facts under consideration depends upon the faculty of judgment. It is said that a judgment is a decision or conclusion reached, as after consideration or deliberation; the belief or assertion that something is or is not. Very obviously the judgment that one makes on a certain set of factors depends largely upon the wealth of experience that he has had. The meaning of "intangible" is, of course, that the interpretation of the facts is difficult because the factors themselves are shadowy, hard to get hold of, and difficult to grasp.

When studies are made by the inexperienced person, the tangible set of factors is easily obtained. The intangible factors, being difficult to evaluate, are generally disregarded or at best only lightly considered.

13.2. The Project and Its Relationships.—As long as the project is in the stage of a proposal put forth and developed by some promoter who has not yet informed the public concerning it, its relationship is an individual one with the promoter. As soon as the promoter feels sure that suitable technical perfection of his project has been attained, he reaches the stage when he must bring the project to the attention of others than himself. Then other relationships come into the scene and with them numerous problems of relationship which are of an intangible nature.

The problem of making a decision does not depend entirely on the answer to the question, "Will it pay?" but also to other questions, such as "Should it be done this way?" and "Should it be done at all?"

Relationships other than those dealing with technical and functional design may be found among the following list:

1. *Consumer.*—The promoter of the project expects to sell the product or service to the consumer.

2. *Financier.*—The promoter expects to obtain funds with which to finance his project.

3. *Government.*—The promoter must design, operate, or sell his project in accordance with the safety regulations and business practices that have been set up by government as standard procedures in orderly society.

4. *Workmanship.*—The promoter must see to it that his project reflects good and proper handling and finishing of the materials and processes involved in it.

5. *Workmen.*—The promoter cannot operate his project alone. He must have assistance, technical, skilled, unskilled, clerical, and managerial, so that an efficient business will obtain.

6. *Community.*—The promoter has a responsibility to his community for the effect of his project on the many factors that go to make up community life.

7. *Morals and Ethics.*—The promoter must recognize higher laws than governmental laws in the conduct of his business and even in the selling of his project.

13.3. Intangibles in Consumer Relationship.—The promoter of the project expects to sell the product or service to the consumer. He must, therefore, determine what the consumer's reaction will be to the product or service.

The consumer is first of all interested in the product or service in terms of the specified service that will be given him. In other words, he wishes to know just what it is that is offered him. He wishes to know what functions the product is capable of performing. If the product is a new electric refrigerator, the consumer wants to know if it will keep the interior cold and if it will make ice cubes.

He is also interested in the quality of the service. For example, what degree of coldness can be maintained in the various parts of the refrigerator? How many ice cubes is it capable of making and in what time? Will it quick-freeze desserts? Is it so arranged that open dishes may be placed in the refrigerator without frosting up the freezing elements? Does the appearance of the case blend well with the consumer's kitchen? Is it dependable in operation; that is, do the control rheostats function properly?

The question of servicing and repair parts is a vital one. Are there many parts to get out of order and, if they do, are they expensive to replace? Can this servicing be done by the housewife or her husband or must it be done by a professional service man? If so, is he available? How serviceable are the parts? Will they wear out easily?

Price is very important to the consumer as well as to the promoter. The consumer wishes to buy at the lowest possible price, and the seller wishes to obtain the greatest possible price. The consumer judges the price in terms of what he is getting by way of service from the product. He measures the price against other things which he has bought and compares them on the basis of the relative services he will receive from them. From the promoter's point of view, price must be high enough so that it will cover the cost of production and in addition the cost of selling and distribution and still provide sufficient surplus or profit to make it worth while to invest his or other people's money in the business. If the price arrived at by the interchange of these ideas in the open market is insufficient to provide a reasonable profit, the promoter must either operate at a loss or take steps to reduce the production and distribution cost.

The consumer is also interested in what the cost of operation will be. The price or first cost may be quite reasonable, but the operating cost may be unusually high. In general, the higher the first cost, the better the product will be made and the lower the operating cost will be.

13.4. Intangibles in Financier Relationships.—The promoter must obtain funds with which to finance his project. What questions may arise in the minds of the persons to whom the promoter may go with requests for funds with which to finance his project?

What is the present demand for electric refrigerators? What are the extent and character of the potential consumer field for this product? Is the product one that will have a stable market, one that will endure and not just flourish for a short time and then die? In other words, how necessary to the modern housewife is a refrigerator, and especially an electric one?

Are there substitutes for an electric refrigerator? To be sure, there are the gas refrigerator and the ordinary icebox, but how satisfactory a substitute are they for an electric refrigerator? Turn the question around, how satisfactory a substitute is an electric refrigerator for an old-fashioned icebox or for the new type?

If a factory is started for the manufacture of this new refrigerator, what will be the extent of the competition to be met from factories already manufacturing? Are there other promoters interested in starting similar factories?

Before beginning a new enterprise, it is worth while to ask questions about these industries of a similar nature that are still in business. What is the history of their operations for the previous years? Much can be learned about the probable difficulties facing a new industry if the history of the old ones is studied.

The financier is interested in the relationship between the required capital investment and the operating cost resulting in a unit of production. This, together with an estimate as to probable price, will enable him to judge whether or not the venture is worth while.

The financier is also careful to examine existing and potential legislation which may affect the venture. Another question which the financier is anxious to know about is that of management. Are there available sufficient persons of the necessary training to manage the project?

The technical problems involved in the production of the new product are very important. Just where in the technical state are present projects? Many writers have shown that there is a curve of growth implicit in the development of a technology. It is illustrated in Fig. 13.1.

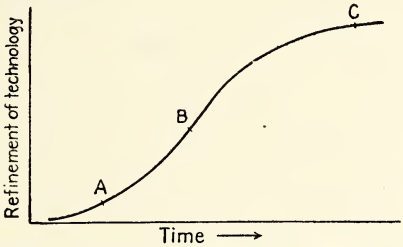


FIG. 13.1.—Curve of development or growth.

A discussion of three parts of the curve will be sufficient to show its meaning. The period of time during which state *A* exists may be called the developmental stage. It is that period in which the idea put forth by the inventor is gradually and sometimes very slowly worked out into practical accomplishments. It is the period during which the theory behind the project is tested and tried, modified, and refined. The net result of this stage is an experimental model which works.

Stage *B* is that period during which further refinement is made in the project looking toward more highly efficient operation. It is also that period during which the product is being manufactured in increasing numbers. It is for sale at comparatively high prices. It is the period of introduction to the public. The net result of this stage is a product whose theory has been highly refined and whose efficiency of performance has been nearly perfected.

The third stage, *C*, is that period during which manufacturing refinements have been achieved. It is the period of mass production during which production and distribution have reached such a high rate that the selling price has been reduced considerably.

The financier wants to know in which stage of development the venture is so that he may be able to judge the soundness of the venture.

13.5. Intangibles of Government Relationships.—The promoter must design, operate, or sell his project in accordance with safety regulations and business practices that have been set up by government as standard procedures in orderly society.

More and more, governments, Federal, state, and municipal, have entered the industrial scene in an attempt to protect the great majority of industrial enterprises and the consumers of their products from an unscrupulous minority. Witness the laws and regulations, food and drug acts, industrial safety codes, boiler and elevator inspection laws. These all have to do with the protection of the health and safety of the consumer. In this same category are the licensing procedures for persons engaged in certain occupations whose actions would affect the health and safety of consumers. Doctors, engineers, barbers, lawyers, and many others come under this heading. The promoter is aware of those conditions and must be sure that his project meets with these governmental restrictions.

Besides entering into the function of design of product, the government also enters into the manufacturing operations involved in the product. Through the National Labor Relations Board, it has an impact on the relations between the employer and the employee or labor. Through various regulations of state departments of labor and industry, it sets up standards of working conditions in terms of safety and health for the worker. Through the wages and hours administration, it attempts to set standards of hours of work and minimum wages.

The government also enters into the field of distribution. Through its agency of the Federal Trade Commission, it attempts to suggest and enforce good trade practices. In particular it is opposed to monopoly in any form. The Patman Act is an attempt to remove some of the abuses claimed to reside in the use of discounts to buyers. The Interstate Commerce Commission has to do with the control and regulation of commerce between the several states.

In still another field does the government enter. It is the field of financing the enterprise. Through the medium of the Securities Exchange Commission, it makes rules and regulations for exchange where shares of stock in enterprises are bought and sold. Not the least of its services is concerned with the manner in which prospectuses for issues of stocks and bonds for new enterprises are written. Its purpose in this respect is the protection of the consumer or buyer of interests in such enterprises.

In still another field the government affects enterprises. It is the field of taxation. The determination of the financial yield of an enterprise cannot be made without an understanding of the effect of taxation upon the operating costs and return of an enterprise. Certain taxes are a part of operating costs, being those on real estate and personal property. Other taxes are upon return, and depend upon certain relationships between return and capitalization or investment.

In all these fields and functions, the promoter relates his proposed project so that he will be able to go ahead with a minimum of interference in accordance with good and accepted legal practices.

13.6. Intangibles of Workmanship Relationships.—The promoter must see to it that his project reflects good and proper handling and finishing of the materials and processes involved in his project.

The good will of the consuming public is a most important asset for any industrial enterprise to have. The development of it depends largely on whether the public is pleased with the product or service. This in turn depends upon the quality inherent in the product or service. Generally speaking, high price and high quality go together. It should also be remembered that usually high quality and low operating cost go together. The need for servicing and the need for repairs are direct results of the kind of quality built into the product.

The promoter may skimp on quality in order to get a low selling price but may also get low volume of distribution because of the low quality. On the other hand, high quality will in general get good distribution of such an extent that quality can be improved, thus increasing sales. This increase in sales very often allows the use of mass production methods of manufacture resulting in lowered production costs. If these lowered production costs are reflected in lower prices, a still further increase in sales will result, so that it is possible to have high quality and low prices. In fact, the characteristics of present-day production are high rate of production, high quality, and low prices, and, strange as it may seem, high wages.

13.7. Intangibles of Workmen Relationships.—The promoter cannot operate his project alone. He must have assistance, technical, skilled, unskilled, clerical, and managerial, so that an efficient business will obtain.

It must never be forgotten that all technological developments have an effect on workmen. When changes are made in the design of products or when new methods of producing them are devised, they inevitably have an effect on labor.

Sam Lewisohn in "The New Industrial Democracy" suggests that labor is vitally interested in three things; namely, justice, status, and a chance to get ahead.

From the standpoint of justice, labor expects that it shall receive a just share of the returns from industry in its pay envelopes. One of the most difficult problems in modern management is the determination of what share of the returns from industry should go to labor. The distribution of this share to the various levels of labor that have contributed to it is vital to the continued operation of the enterprise.

Labor desires to be somebody. It takes as much interest in the development of the products of a company as do those who furnish the capital, if it is given the recognition of the importance of its contribution to such development. The psychological reaction of a man to doing a good job and such recognition is the satisfaction of mastery, of being somebody of consequence. Its extent is almost impossible to estimate in the smooth running of an organization.

The third thing that labor is interested in is a chance to get ahead. A worker wishes to improve his position in the organization or he wishes to improve his standard of living in the community.

In what manner will the proposed product or project affect these several interests? Will the proposed method affect his status? That is, will it result in unemployment or reduction in wages? Will he lose his position of importance in the organization in favor of someone else? The effect of changes on personnel must be studied so that its effects are readily seen. They must be weighed and studied.

13.8. Intangibles of Community Relationships.—The promoter has a responsibility to his community for the effect of his project on the many factors that go to make up community life.

Communities come into existence, in general, because of the location of an industry or of industries. What the industries do has considerable effect on the community. The appearance of the plant will influence the appearance of the homes of its workmen. The standards of workmanship required in the industry will affect the appearance of the community.

The standard of living in the community depends largely upon the success of the industry, especially in its effect upon employment and upon its wage scale.

Projects may have a municipal character in that they affect the lives of the whole community. A new sewage system, water works, municipal power plant, through highway, and many others have problems inherent in them which must be considered by their promoters.

13.9. Intangibles of Moral and Ethical Relationships.—The promoter must recognize higher laws than governmental laws in the conduct of his business.

Philosophers, from the beginning of time, have been setting forth their thoughts concerning the nature of this world and the various relationships therein. Morals, ethics, and religion have been presented to us. Morals and ethics represent man's attempt to form the best and highest relationships. Religion is the conception of these relationships as given to us by a supernatural creator. From experience with life many people have been convinced that there is one God who is the Creator of all things. They believe that He has revealed to us through the Bible and through His son Jesus the very highest of ideals or code of ethics and morals which we call the Christian religion.

Developing out of this worship of one God and His revelation has come the conception of Christian ways of living together. Generally, it is spoken of as the Brotherhood of Man. Implicit in it is the idea of brotherhood, of people working together for the common good, of honesty of purpose and performance, of unselfishness in the desire for wealth and position.

These are important concepts that the promoter must not forget. What will be the effect on such standards of living if his project is put into effect? Will it be a project that will degrade and lower man or will it be one that will elevate him?

13.10. The Engineer and the Intangibles.—The engineer and the engineering profession have been accused many times in recent years of being totally oblivious of their responsibilities toward the intangibles of life. The engineer has perfected his inventions and placed them on the market with an utter disregard for the effect they would have on the people of the world. Technology has been accused of being responsible for the great depressions in the past.

The engineer in the past has been engrossed in perfecting his fields of technology. He has been satisfied with passing on to financiers and businessmen the results of his work with technology. Therefore, the responsibility for the effects of the application of his technological achievements has been in the hands of those who have put them into commercial use. That those persons have always done it for the benefit of mankind is questionable. More and more, engineers are finding themselves in positions of high trust and responsibility. They cannot claim, then, that someone else was responsible for actions adverse to the welfare of society when they have to make the decisions themselves.

After all, no matter who makes the decisions, they must be made in terms of the moral, ethical, and religious concepts and convictions of the times.

13.11. Intangible Analysis of the Housing Development.—What are the applications of intangible analysis to the housing development which was studied in Chaps. 11 and 12?

1. *Consumer Relationship.*—The housing development will provide a service which is needed; namely, housing. The quality of the service is very high. The individual houses have six rooms: living room, dining room, kitchen, bathroom, and two bedrooms. The construction is of very good modern type. There are hardwood floors throughout; the bathroom is tiled, and a modern streamlined kitchen is provided.

The cost of this service is comparable with privately owned dwellings. The promoter engages to provide heat for the houses from a central plant. The rent is $60 per month, and includes heat and certain care of the properties such as repairs and supervision of shrubbery and plantings.

2. *Financier Relationship.*—This will be developed in Chap. 14, as far as the relationship between the promoter and financier is concerned. The financial relationship between promoter and consumer is in the form of a lease in the usual manner.

3. *Government Relationships.*—The development was built in full conformity with the zoning laws of the particular political sub-divisions in which it is located.

4. *Workmanship Relationship.*—The construction used was the best that could be obtained.

5. *Workmen.*—The promoter has had such relations with the workmen on the job that no difficulties have been experienced.

6. *Community Relationship.*—The building of the housing development has been in full conformity with the town planning commission in order to coordinate the town's utilities very satisfactorily. The contribution of the development to the growth of the town has been material.

7. *Ethical Relationship.*—As far as can be ascertained, an honest attempt was made to develop the project on the highest plan.

In-favor-of Analysis Comparison of Housing Development and Apartment Development

Intangible analysis

Item	Proposal		In favor of	
	Housing	Apartment	Housing	Apartment
Appreciation	Good	Good		
Safety	Good	Good		
Marketability	Good	Good		
Collateral value	Good	Good		
Type construction	Brick	Brick		
Layout of rooms	Two floors	One floor	?	?
Built-in equipment	Kitchen	Kitchen		
Method of heating	Warm air, coal-fired	Warm air, coal-fired	Complete responsibility	No bother or responsibility
Open ground around living place.	Considerable, private	Fair amount, common	Very good	Poor

In this analysis, it is very difficult for the promoter to evaluate certain of the intangible factors. The three factors, namely, layout of rooms, method of heating, and open ground, will be viewed differently by people preferring one method of housing to the other. The promoter will have to determine the number of possible renters. Since the differences between the housing development and the apartment are slight, the decision may turn around this factor.

It should also be remembered that the intangible analysis made by the promoter and the renter or prospective buyer may involve numerous different factors because of the viewpoints of the respective parties.

Problems

You are an engineer in the employ of an industrial enterprise, or a firm of consulting engineers, or a governmental agency, or operating your own business, as the case may be. One of the following projects, proposals, or situations has been brought to your attention. *See problems for Chap.* 1.

You are asked to prepare an outline of the data that need to be collected, the estimates that must be made, and any pertinent information that seems necessary to enable you or your employers to evaluate the project, proposal, or situation and that will assist them in arriving at a decision and a plan of action.

Prepare a report on the intangible factors for your assigned project.

PART III

THE FINANCIAL ANALYSIS

What funds are needed?
What funds are on hand?
What are the sources of funds?
What financial structure should be used?

CHAPTER 14

FINANCIAL RETURN AND YIELD

14.1. Financial Return.—Economic return was defined in Sec. 10.3 as the net income that was left after all operating costs had been subtracted from the total income. Economic return was the return to the project itself. It was not calculated with regard to the manner in which the project was financed. When this is taken into account, however, return becomes financial return.

The return may be a financier's return. In general, projects are financed by a combination of two types of persons, the financier and the organizer. The financier is one who supplies funds for financing at a stipulated rate of return. The organizer is one who supplies funds for financing with the hope of making an expected return. This expected return is usually somewhat larger than the return which the financier stipulates.

The organizer's share in the investment may be thought of as being that amount of funds contributed by the person or persons making up the organizing or promoting group. The financier's share is the amount of funds lent to the organizing group under certain conditions. These conditions are stipulated by the financier and are legally binding upon the organizer or promoter. The financier's share in a project is generally known as bonds or mortgages.

14.2. An Illustrative Example.—The data for the problem in Sec. 10.3 are here repeated.

Example: A project requires a total investment of $500,000. The income for the year is $350,000 and the operating cost is $300,000. The economic return is, therefore, $50,000.

Case 1.—Ownership:

Organizer's share	$500,000
Financier's share	0

Since the organizer has furnished all the finances required, he will receive the full return.

Distribution of net return:

Total amount of return	$50,000
Financier's share	0
Organizer's share	$50,000

Case 2.—Ownership:

Organizer's share	$300,000	
Financier's share	200,000	at 6 per cent per year
Total investment or net worth	$500,000	

Distribution of net return:

Total amount of return	$50,000
Financier's share (6 per cent of $200,000)	12,000
Organizer's share (balance)	$38,000

Case 3.—Ownership:

Organizer's share	0	(He still controls it.)
Financier's share	$500,000	
Total	$500,000	

Distribution of net return:

Total amount of return	$50,000
Financier's share (6 per cent of $500,000)	30,000
Organizer's share	$20,000

It should be noted in this case, which is an extreme one, that, even though the organizer did not contribute any of the finances for the project, yet he receives a return, because he controls the project.

14.3. Financial Yield.—The financial yield is the relation between the financial share in the project and the net return assigned to it. This may be illustrated by the example in Sec. 14.2.

Example:
Case 1.—Organizer's yield:

Net return	$ 50,000
Organizer's share	500,000

$$\text{Yield} = \frac{\$50,000}{\$500,000} \times 100 \text{ per cent} = 10 \text{ per cent}$$

It is to be noted by reference to Sec. 10.4 that the financial yield just calculated is the same as the economic yield for the project, namely 10 per cent. This will always be true when the entire financial share in the ownership is the organizer's. Since the financier has no share, he has no yield.

Case 2.—The financier's yield is set by the terms of the loan, namely, 6 per cent per year.

Organizer's yield:

Net return	$ 38,000
Investment	300,000

$$\text{Yield} = \frac{\$38{,}000}{\$300{,}000} \times 100 \text{ per cent} = 12.66 \text{ per cent}$$

Case 3.—The financier's yield is set by the terms of the loan, namely, 6 per cent per year.

Organizer's yield:

Net return	$20,000
Investment	0

$$\text{Yield} = \frac{\$20{,}000}{0} \times 100 \text{ per cent} = \text{infinity}$$

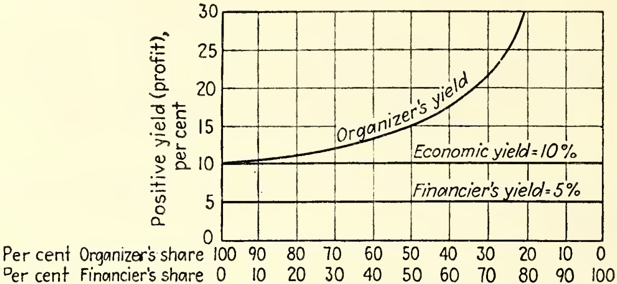
Thus it is seen that the financial yield to the organizer will vary in accordance with the proportion of funds that are loaned to the project. When the organizer provides all the funds, the financial yield is equal to the economic yield.

Figure 14.1 shows the variation in yield for the organizer under several conditions. In the upper part of the chart, the economic yield of the project is 10 per cent. The yield for the share that the financier supplies is 5 per cent. It is seen by consulting the scale that, as the organizer's share in the investment decreases from a 100 per cent share for a yield of 10 per cent to smaller proportions, the organizer's yield increases.

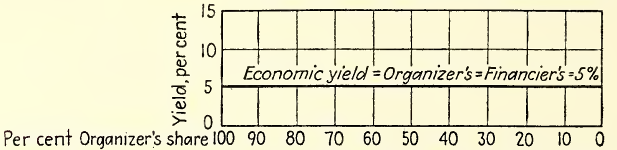
In the center of the chart, the conditions are that the economic yield is the same as the financier's yield. The organizer's yield is, therefore, the same as the financier's and the same as the economic yield no matter how much or how little the organizer's share is.

In the lower part of the chart, the economic yield is 2 per cent and the financier's yield is 5 per cent. It is seen by consulting the scale that, as the organizer's share in the investment decreases from 100 per cent for a yield of 2 per cent to smaller proportions, the organizer's yield decreases and becomes zero at 60 per cent share of the total investment and a loss below that point.

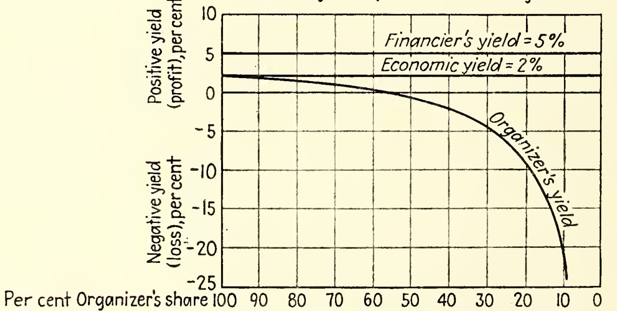
Thus it is seen that, as the organizer increases his borrowings, he must do so at an interest rate less than the economic yield if he would continue to receive a return. If he increases his borrowings at an interest rate greater than the economic yield, he can go only so far before he reaches a zero yield and after that a loss.



Economic yield higher than financier's yield



Economic yield equal to financier's yield



Economic yield less than financier's yield

FIG. 14.1.—Variation in yield to organizer in terms of share in total investment and in terms of relationship between economic yield and yield to financier.

Because of the principle just described, the financing of projects is generally made with a combination of funds from the organizer group supplemented by funds from the financier group. The sum of these funds constitutes the investment or net worth in the project. It is that investment upon which the economic yield is earned.

Problems

1. A project requires a total investment of $300,000. The income for a year is estimated at $250,000, and the operating cost per year is estimated at $200,000.

The promoter borrows one-half of the total investment and agrees to pay a return of 6 per cent per year on it. Determine the financial returns and financial yields to the promoter and financier.

2. Determine the financial returns and financial yields to the promoter and financier, using the data of Prob. 1, if the promoter borrows one third of the total investment.

3. Determine the financial returns and financial yields to the promoter and financier, using the data of Prob. 1, if the promoter borrows one-fourth of the total investment.

4. Determine the financial returns and financial yields to the promoter and financier, using the data of Prob. 1, if the promoter borrows two-thirds of the total investment.

5. Draw the graph that shows the variation of financial yield for the promoter and financier with amount of total investment provided by each, as determined from the data of Probs. 1 to 4. Indicate the economic yield rate.

6. The project in Prob. 1 during a certain year estimates that its income will be $225,000 and its operating cost will be $205,000.

The promoter borrows one-half of the total investment and agrees to pay a return of 6 per cent per year on it. Determine the financial returns and financial yields to the promoter and the financier?

7. Determine the financial returns and financial yields to the promoter and the financier, using the data of Prob. 6, if the promoter borrows one-third of the total investment.

8. Determine the financial returns and financial yields to the promoter and the financier, using the data of Prob. 6, if the promoter borrows one-fourth of the total investment.

9. Determine the financial returns and financial yields to the promoter and the financier, using the data of prob. 6, if the promoter borrows two-thirds of the total investment.

10. Draw the graph that shows the variation of financial yield for the promoter and the financier with amount of total investment provided by each or determined from the data of Probs. 6 to 9.

CHAPTER 15

FINANCIAL ANALYSIS

15.1. The Financial Analysis.—The financial analysis deals with the problem of money or funds with which to carry on the project, with the raising or finding of such funds, and with the financial returns to the participators in the enterprise.

15.2. Total Amount of Money Required.—The total amount of money required for the project may be determined from the analysis of investment as discussed in Chap. 2. In general, it will consist of the development, promotion, and construction costs and the circulating capital.

The time when these funds will be required depends upon the type and character of the project and the contractual relations between the promoter and the builder or operator. As construction proceeds, certain percentages of the cost of construction are required to be met by the terms of the contract. Thus all of the circulating capital is needed when the project is in such shape that materials and supplies must be bought and when the supervisory staff and other labor are being organized.

15.3. Sources of Funds.—There are three possible solutions to the problem of funds: namely, to borrow none of the requirements, to borrow part of them, and to borrow all of them. The extent to which any one of these methods is used depends upon the amount of available funds on hand and the ability to borrow or otherwise obtain funds.

15.4. Available Funds on Hand.—Available funds on hand means in accounting and business language the current assets, which are those assets of a person or enterprise that can readily be turned into cash. Such assets consist of cash on hand and in banks; notes receivable; accounts receivable; inventories of supplies, materials, and products; sinking funds; investments in securities of other companies or of the government.

Notes receivable are monies owed to a person or company, the legal record of which is a note. Accounts receivable are monies owed to a person or company for materials, products, or services, and which are to be paid within a short time, say 30 or 60 days. Inventories of supplies and materials and products are stores of such things as are on hand in storehouses and so can be sold readily. Sinking funds are funds kept outside the business, which, in general, are set aside for specific purposes. Securities are generally stock issues of corporations but also include issues of bonds.

A reserve for depreciation is an accounting term that represents the liability which the enterprise has toward its owners in that it is the manner in which capital is maintained for those assets that waste away or become obsolete. It is represented in the assets of the enterprise by cash and sinking funds and receivables. In a great many cases, especially for a company that is expanding rapidly, cash and funds that represent reserve for depreciation have been used for such expansion in buildings and equipment. In this case the reserve is represented by fixed assets of buildings and equipment.

15.5. The Balance Sheet Statement.—A balance sheet statement is a form of report used by businessmen to show the things that the business owns compared with the things that the business owes.

A typical balance sheet for a corporation is shown herewith:

Typical Balance Sheet Statement
Asset Side of Statement

Current assets:		
Cash in bank	$ x,xxx.xx	
Cash with cashiers, agents, etc	x,xxx.xx	
Notes receivable	xx,xxx.xx	
Accounts receivable—customers	x,xxx.xx	
Accounts receivable—others	xxx.xx	
Inventories	x,xxx.xx	$xxx,xxx.xx
Sinking funds		xx,xxx.xx
Investment securities		x,xxx.xx
Fixed assets:		
Manufacturing plant	$xxx,xxx.xx	
Less depreciation	x,xxx.xx	xxx,xxx.xx
Property other than manufacturing plant	$ x,xxx.xx	
Less depreciation	xxx.xx	x,xxx.xx
Other assets:		
Good will	$ xx,xxx.xx	
Patents, franchises, licenses	x,xxx.xx	
Unliquidated development	x,xxx.xx	
Other deferred charges	xxx.xx	xxx,xxx.xx
Total		$xxx,xxx.xx

The asset side shows those things that the business owns. Current assets are those things that are readily realized in cash when necessary. Most of these items were discussed in Sec. 15.4.

Fixed assets are those things that cannot be readily realized in cash, such as buildings, machinery, and equipment. Good will, patents, and unliquidated development charges are items that were discussed in Chap. 4 and represent, in the main, intangible values.

TYPICAL BALANCE SHEET STATEMENT

Liabilities Side of Statement

Current liabilities:		
Notes payable	$xx,xxx.xx	
Accounts payable	x,xxx.xx	
Accrued taxes	xx,xxx.xx	
Other accrued charges	x,xxx.xx	
Dividends declared and unpaid	xx,xxx.xx	
Interest accrued	x,xxx.xx	
Advance collection on orders	xxx.xx	$xxx,xxx.xx
Working reserves		xx,xxx.xx
Contingency reserves		x,xxx.xx
Funded debt:		
Mortgage bond issue (lender's share)		xx,xxx.xx
Capital stock (promoter's share)		xx,xxx.xx
Surplus (promoter's share)		xx,xxx.xx
Total		$xxx,xxx.xx

The liabilities side shows those things that the business owes. Notes payable are debts owed to lenders to the business. Accounts payable are debts owed to those who have sold the business supplies, machinery, or equipment. Accrued taxes are taxes which have accumulated at the time of the balance sheet report but which have not been paid. Dividends declared and unpaid are self-explanatory.

Working reserves and contingency reserves are amounts set aside for emergency or special projects.

Funded debt represents that part of the funds required for the project which are provided by someone other than the promoter. In the illustrative balance sheet it is listed as mortgage bond issue. Capital stock is that part of the required funds which is provided by the promoter. Surplus represents undistributed earnings that have been left in the business to be used for its expansion and to provide for increased working capital as expansion takes place.

The balance sheet may be used to illustrate the various ways in which funds are obtained for the project.

15.6. Having Sufficient Funds.—The promoter finds that he, personally, is in possession of the full amount of funds required for the project. For the illustrative problem used in Secs. 11.3, to 11.7, the housing problem statement of ownership is as follows:

BALANCE SHEET STATEMENT

Fixed assets, grounds and buildings	$110,000	Owner's equity	$110,000

It is to be noted that when complete ownership of all of the funds required by the project is in the hands of the promoter, the financial yield is the same as the economic yield (see Sec. 14.3).

15.7. Obtaining Funds by Borrowing.—Not having sufficient funds on hand with which to finance the project, the promoter must resort to other means in order to obtain the necessary funds. They may be obtained from banks, investment houses, friends, or other sources. Two types of borrowed funds are met with in business. The first may be called "credit funds" or "unsecured funds." They are lent on the good name of the enterprise. The other type may be called "secured funds." They are lent on certain security, such as property.

It is characteristic that, for the first type, the amounts lent are small percentages of the total investment in the project and are made for relatively short periods of time, after which they must be repaid or renewed. The amounts lent for the second type are for larger amounts and for longer periods of time depending upon the character of the project, length of life, and other significant factors.

15.8. Borrowing Unsecured Funds.—Assume that in the housing problem, the promoter has in hand $90,000. He is sucessful in borrowing from a friend the balance of $20,000 at an interest rate of 6 per cent yearly. The balance sheet statement will be

BALANCE SHEET STATEMENT

Fixed assets:		Owner's equity	$ 90,000
Grounds and buildings	$110,000	Loan from friend	20,000
	$110,000		$110,000

In Sec. 14.3, it was shown that the financial yield to the promoter depended upon how much of the funds was his and how much was the lender's, and the relationship between the return to the lender and the economic yield may be shown by an extension of the profit and loss statement discussed in Sec. 11.2.

PROFIT AND LOSS STATEMENT

Income from rental of houses at 100 per cent capacity	$15,600
Operating cost at 100 per cent capacity............	6,876
Net return (economic return)..................	$ 8,724
Interest on loan of $20,000 at 6 per cent (financial return and yield of lender)..................	1,200
Balance of return for promoter.................	$ 7,524

$$\text{Promoter's financial yield} = \frac{\$7,524}{\$90,000} \times 100 \text{ per cent} = 8.36 \text{ per cent}$$

It should be noted that, from Sec. 11.10, the economic yield was found to be 7.93 per cent. It should be remembered that so far no provision has been made for paying back the loan. If the promoter desires to repay the loan, it must be done out of his $7,524 return. In no case should the payment of the loan be considered a financial expense, since the effect is to increase the promoter's equity in the project. Some of the intangible problems or viewpoints associated with this method of financing will be discussed in a later section.

15.9. Borrowing Secured Funds.—Assume that in the housing problem, the promoter has on hand only $25,000 of the required $110,000. Being unable to obtain the additional funds from a lender of unsecured funds, he considers making the amount smaller, believing that by so doing he can get the required funds. This necessitates dividing the balance required, $85,000, into small amounts. He finds that he cannot even then obtain unsecured funds. Therefore, he must obtain them from sources of secured funds. The method of borrowing secured funds is to borrow on the security of a mortgage on the property in the project. For this purpose, he divides the large amount required into small shares called "bonds." There are certain intangible aspects of this method of borrowing that will be discussed in a later section.

After suitable investigation, the promoter finds that he can float a bond issue, with a mortgage on the property as security, in units of $500 per bond at a rate of 5 per cent interest. The balance sheet statement will be as follows:

BALANCE SHEET STATEMENT

Fixed assets:		Owner's equity..........	$ 25,000
Grounds and buildings.	$110,000	Funded debt:	
		First-mortgage bond issue...............	85,000
	$110,000		$110,000

The effect of this ownership is shown in a profit and loss statement.

PROFIT AND LOSS STATEMENT

Income from rental of houses at 100 per cent capacity	$15,600
Operating cost at 100 per cent capacity............	6,876
Net return (economic return)...................	$ 8,724
Interest on bond issue at $85,000 at 5 per cent.....	4,250
Balance of return for promoter.................	$ 4,474

$$\text{Promoter's financial yield} = \frac{\$4{,}474}{\$25{,}000} \times 100 \text{ per cent} = 17.9 \text{ per cent}$$

It should be remembered that, from Sec. 11.10, the economic yield was found to be 7.93 per cent.

15.10. Calculation of the Bond Issue.—Bonds may be redeemed at the end of the term. Very often one of the requirements of the bond agreement is that the person or persons issuing the bonds maintain a sinking fund for the accumulation of the face value of the bond issue so that its payment will be guaranteed at the end of its terms. The amount of the yearly payment to maintain this sinking fund is calculated by the methods of compound interest and in particular those applying to the series whose end total is S.[1]

For the illustrative housing problem, the term of the bond issue is 30 years, its face value is $85,000, and its interest rate is 5 per cent yearly. The promoter finds that he can accumulate this annual deposit in a bank which pays 3 per cent yearly on deposits at the end of the year.

The equation is

$$D = S\left[\frac{i}{(1+i)^n - 1}\right]$$

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[1] See Appendix A.

Substituting the proper values,

$$D = (\$85{,}000)\left[\frac{0.03}{(1.03)^{30} - 1}\right]$$
$$D = (\$85{,}000)(0.0210) = \$1{,}786$$

This amount is the yearly deposit to a sinking fund which will accumulate to the amount of the bond issue by the end of the life of the bond issue, or 30 years. The promoter may voluntarily set aside this amount or the bond contract may stipulate that he must do this.

The money with which to do this must come from the savings of the promoter. In this particular case, it was seen that the net return on his $25,000 cash in hand was equal to $4,474.

Since the sinking fund charge is $1,786, the promoter can easily pay it out of the return on his cash investment.

He may argue that to do this would mean a reduction in his return. It must not be forgotten, however, that the promoter in making his deposit to the sinking fund is in reality investing a part of his return in the project. For, at the end of 30 years he will exchange the amount in the sinking fund for the bonds which he issued. When he does that, he acquires an undivided ownership in the entire project, as shown in the following:

Cash in hand, beginning of project...............	$ 25,000
Redemption of bond issue, using sinking fund accumulation..............................	85,000
Total ownership...........................	$110,000

It is called to the promoter's attention, however, that he is paying out interest on his bond issue at the rate of 5 per cent and receiving from his deposits in the bank toward the sinking fund only 3 per cent. It would, therefore, reduce the interest payments if the promoter could pay off the bonds as soon as he has accumulated in the sinking fund sufficient money to do so. Since these bonds are in the amount of $500, as many bonds may be retired as there are units of $500 on hand in the sinking fund.

Very often the bond issue is so set up that no bonds may be retired during the first 5 years. Assume that such is the case for this problem. The promoter should draw up a schedule showing the accumulation of money in the sinking fund and also the retirement of bonds.

How much should the yearly payment be so that all the bonds will retire at the end of the life of the issue, which is 30 years? It was seen that, if the bond issue is not paid off until the end of the 30 years, the yearly payment to the bond issue would be \$1,786. Since interest money is saved if we retire bonds as we go, the yearly payment may be less. Since bonds may be retired only to their face amount and not in fractional amounts, it is not possible to devise an algebraic formula which will give the correct figure. It is necessary to get the figure by the "cut-and-try" method. As a suggested estimate to find this figure, let us determine the sinking fund deposit on the basis of the interest rate charged for the bonds.

By substituting in the same equation that was used before,

$$D = (\$85{,}000)\left[\frac{0.05}{(1.05)^{30} - 1}\right]$$
$$D = (\$85{,}000)(0.0151) = \$1{,}280$$

The charges against the bond issue are then as follows:

Interest on bonds	\$4,250 per year
Sinking fund	1,280 per year
Total charge	\$5,530 per year

The promoter has, therefore, the sum of \$5,530 with which to pay the interest on the bonds and the face value of the bonds when they become due.

He prepares a table showing how these funds are used and how fast the bonds are retired, based upon the following data:

Interest rate: 5 per cent on \$85,000 issue (\$4,250 annually)
Bank interest rate: 3 per cent annually
Yearly deposit: \$1,280
Total yearly payment: \$4,250 plus \$1,280 = \$5,530
Duration of issue: 30 years
No bonds redeemed during the first 5 years
Bonds in units of \$500

TABLE 15.1—BOND REDEMPTION TABLE

(1) Years	(2) Deposit in bank at end of year	(3) Amount of bonds outstanding	(4) Interest on bonds	(5) Balance of deposit used to retire bonds	(6) Interest on net balance in bank	(7) Total amount in bank	(8) Amount of bonds retired	(9) Net balance in bank
1	$5,530	$85,000	$4,250	$1,280		$1,280		$1,280
2	5,530	85,000	4,250	1,280	$ 38	2,598		2,598
3	5,530	85,000	4,250	1,280	77	3,955		3,955
4	5,530	85,000	4,250	1,280	118	5,353		5,353
5	5,530	85,000	4,250	1,280	160	6,793	$6,500	293
6	5,530	78,500	3,925	1,605	8	1,906	1,500	406
7	5,530	77,000	3,850	1,680	12	2,098	2,000	98
8	5,530	75,000	3,750	1,780	2	1,880	1,500	380
9	5,530	73,500	3,675	1,855	11	2,246	2,000	246
10	5,530	71,500	3,575	1,955	7	2,208	2,000	208
11	5,530	69,500	3,475	2,055	6	2,269	2,000	269
12	5,530	67,500	3,375	2,155	8	2,432	2,000	432
13	5,530	65,500	3,275	2,255	12	2,699	2,500	199
14	5,530	63,000	3,150	2,380	5	2,584	2,500	84
15	5,530	60,500	3,025	2,505	2	2,591	2,500	91
16	5,530	58,000	2,900	2,630	2	2,723	2,500	223
17	5,530	55,500	2,775	2,755	6	2,984	2,500	484
18	5,530	53,000	2,650	2,880	14	3,378	3,000	378
19	5,530	50,000	2,500	3,030	11	3,419	3,000	419
20	5,530	47,000	2,350	3,180	12	3,611	3,500	111
21	5,530	43,500	2,175	3,355	3	3,469	3,000	469
22	5,530	40,500	2,025	3,505	14	3,988	3,500	488
23	5,530	37,000	1,850	3,680	14	4,182	4,000	182
24	5,530	33,000	1,650	3,880	5	4,067	4,000	67
25	5,530	29,000	1,450	4,080	2	4,149	4,000	149
26	5,530	25,000	1,250	4,280	4	4,433	4,000	433
27	5,530	21,000	1,050	4,480	13	4,926	4,500	426
28	5,530	16,500	825	4,705	12	5,143	5,000	143
29	5,530	11,500	575	4,955	4	5,102	5,000	102
30	5,530	6,500	325	5,205	3	5,310	5,000	310

The explanation of the column headings is as follows:

Col. (2). This amount is the sum of calculated interest ($4,250) plus the estimated amount of retirement of $1,280, or $5,530.

Col. (3). This is the total amount of unredeemed bonds at the beginning of any particular year. It is found by subtracting from the previous year's amount the amount retired.

Col. (4). This is the interest on outstanding bonds at the rate of issue, 5 per cent.

Col. (5). This amount is the difference between the bank deposit and the interest on bonds.

Col. (6). At the end of each year after bonds have been retired, there will be left a small balance which varies with the year. This amount will draw interest at 3 per cent until the end of the year.

Col. (7). At the end of each year, this is equal to the sum of the net balance from the previous year, interest on the net balance, and the balance of deposit used to retire bonds.

Col. (8). This is determined from the total amount in bank. The amount retired is a multiple of $500, the unit size of the bond.

Col. (9). This amount is the balance left in the bank after withdrawing the necessary funds to retire the bonds.

Conclusions concerning Bond Table 15.1.—The estimate which was made was somewhat low. By correcting it, a new bond table (Table 15.2) may be drawn up which will show the payment of all the bonds in the thirtieth year. Each of the factors in the bond issue affects the estimated deposit.

Bond Interest Rate.—If the bond interest rate were reduced, the yearly deposit could be reduced and vice versa.

Number Of Years before Bonds Could Be Retired.—The smaller the number of years, the smaller the annual deposit can be. This is because full interest is paid on all bonds during the period when retirement is forbidden.

Bank Interest Rate.—When the bank interest rate equals the bond interest rate, it pays to retire as fast as possible. When the bank interest rate is higher than the bond interest rate, it pays to defer retirement until the last year. This condition is an extremely doubtful one.

Amount of bonds outstanding at the end of 30 years is $1,500. The amount of cash in the bank is $310. The cash deficiency is $1,500 − 310 = $1,190. An approximate increase of this amount ($1,190) divided by the number of years would probably take care of the whole amount. This amount would be $40. But, because some bonds would be paid off earlier, this amount might

TABLE 15.2.—REVISED BOND REDEMPTION TABLE

(1) Year	(2) Deposit in bank at end of year	(3) Amount of bonds outstanding	(4) Interest on bonds	(5) Balance of deposit used to retire bonds	(6) Interest on net balance in bank	(7) Total amount in bank	(8) Amount of bonds retired	(9) Net balance in bank
1	$5,550	$85,000	$4,250	$1,300	....	$1,300		$1,300
2	5,550	85,000	4,250	1,300	$ 39	2,639		2,639
3	5,550	85,000	4,250	1,300	79	4,018		4,018
4	5,550	85,000	4,250	1,300	120	5,438		5,438
5	5,550	85,000	4,250	1,300	163	6,901	$6,500	401
6	5,550	78,500	3,925	1,625	12	2,038	2,000	38
7	5,550	76,500	3,825	1,725	1	1,764	1,500	264
8	5,550	75,000	3,750	1,800	7	2,071	2,000	71
9	5,550	73,000	3,650	1,900	2	1,973	1,500	473
10	5,550	71,500	3,575	1,975	14	2,462	2,000	462
11	5,550	69,500	3,475	2,075	13	2,550	2,500	50
12	5,550	67,000	3,350	2,200	1	2,251	2,000	251
13	5,550	65,000	3,250	2,300	7	2,558	2,500	58
14	5,550	62,500	3,125	2,425	1	2,484	2,000	484
15	5,550	60,500	3,025	2,525	14	3,023	3,000	23
16	5,550	57,500	2,875	2,675	....	2,698	2,500	198
17	5,550	55,500	2,750	2,750	5	2,953	2,500	453
18	5,550	52,500	2,625	2,925	13	3,391	3,000	391
19	5,550	49,500	2,475	3,075	11	3,477	3,000	477
20	5,550	46,500	2,325	3,225	14	3,716	3,500	216
21	5,550	43,000	2,150	3,400	6	3,622	3,500	122
22	5,550	39,500	1,975	3,575	3	3,700	3,500	200
23	5,550	36,000	1,800	3,750	6	3,956	3,500	456
24	5,550	32,500	1,625	3,925	13	4,394	4,000	394
25	5,550	28,500	1,425	4,125	11	4,530	4,500	30
26	5,550	24,000	1,200	4,350	....	4,380	4,000	380
27	5,550	20,000	1,000	4,550	11	4,941	4,500	441
28	5,550	15,500	775	4,775	13	5,229	5,000	229
29	5,550	10,500	525	5,025	6	5,260	5,000	260
30	5,550	5,550	275	5,275	7	5,542	5,550	42

be too much. A conservative amount would be one-half of the $40, or $20.

The new annual deposit would be $5,530 plus $20, or $5,550. It will be observed from Table 15.2 that there is a balance of $42, at the time of maturity.

15.11. Obtaining Funds by Going into Partnership.—Another method of securing funds for a project is that of finding one or more persons who would be interested in joining with the promoter as partner. The partner provides such funds as are necessary and the return from the venture is divided according to the terms of the partnership. There are certain intangible aspects of partnership which will be discussed later.

For the illustrative housing problem used in previous sections the conditions are as set forth in Sec. 15.9. The total of required funds is $110,000, of which the promoter has $25,000. The balance sheet statement will be as follows:

BALANCE SHEET STATEMENT

Fixed assets:		Partnership shares:	
Grounds and buildings	$110,000	Promoter	$ 25,000
		Partner	85,000
	$110,000		$110,000

Since the partners will share the return in the proportion of their respective shares in the partnership, the financial yield to each partner is the same as the economic yield for the project, which is in this case (see Sec. 11.10) equal to 7.93 per cent. Other arrangements may be made by partners for the share of the return but the general and usual method is to share as indicated.

15.12. Obtaining Funds by Incorporating.—Another method of securing funds for a project is that of setting up a corporation. Such a device is indicated when it is found impossible to secure a partner with sufficient funds to make up the balance needed. Because the required amount is too large for an individual person to provide or because of intangible reasons, it is necessary to secure funds in smaller amounts from numerous persons. A corporation organization makes it possible for the promoter to issue shares of participation in the project, called "stock." These shares of stock may be sold at a certain value, such as $10, $25, $50, $100 or some other figure, per share. The number of shares issued would be sufficient to secure the required funds.

For the illustrative housing problem used in previous sections, the promoter has $25,000 which purchases 250 shares at $100 each. There is still required $85,000, or 850 shares. These are sold to several persons who take varying amounts. The balance sheet statement will be as follows:

BALANCE SHEET STATEMENT

Fixed assets:		Capital stock:	
Grounds and buildings.	$110,000	Common 1,100 shares at $100...............	$110,000
	$110,000		$110,000

Since all shares will receive a return proportionate to their respective shares in the ownership, the financial yield will be the same as for single ownership, or in this specific case, 7.93 per cent. There are intangible factors that affect the desirability of this method of securing funds which will be discussed later.

15.13. Incorporation with Common and Preferred Stock.—It is sometimes found advisable to introduce into the financial structure of a corporation what is known as preferred stock, which is similar to a bond because it has a stipulated rate of return or yield. It is unlike a bond in that it does not have to be paid back to the holder. It is like common stock in this latter respect. It is unlike common stock in that it receives a definite specified return which is a claim prior to the return on common stock. Usually preferred stock carries the stipulation that the return is cumulative. This means that, if in any year the return is not paid, then in a subsequent year the preferred stock must receive its back and unpaid return before the common stock receives anything.

The balance sheet of a corporation having both common and preferred stock will be as follows:

BALANCE SHEET STATEMENT

Fixed assets:		Stock:	
Grounds and buildings.	$110,000	Common, 700 shares at $100...............	$ 70,000
		Preferred (7 per cent), 400 shares at $100...	40,000
	$110,000		$110,000

The effect of this change in ownership on return is shown in the following profit and loss statement.

PROFIT AND LOSS STATEMENT

Income from rental of houses at 100 per cent capacity	$15,600
Operating cost at 100 per cent capacity	6,876
Net return (economic return)	$ 8,724
Return or dividend on preferred stock:	
$40,000 at 7 per cent	2,800
Balance of return for common stock	$ 5,924

$$\text{Promoter's or common stock financial yield} = \frac{\$5,924}{\$70,000} \times 100 \text{ per cent} = 8.46 \text{ per cent}$$

15.14. Incorporation with Common and Preferred Stocks and Bonds.—Still another modification is made in the financial structure of the corporation. It is the addition of bonds. The balance sheet of a corporation having common and preferred stocks and bonds will be as follows:

BALANCE SHEET STATEMENT

Fixed assets:		Stock:	
Grounds and buildings	$110,000	Common, 300 shares at $100	$ 30,000
		Preferred (7 per cent) 400 shares at $100	40,000
		First-mortgage bonds, 4 per cent	40,000
	$110,000		$110,000

The effect of this combination of financing on return is shown in the following profit and loss statement.

PROFIT AND LOSS STATEMENT

Income from rental of houses at 100 per cent capacity	$15,600
Operating cost at 100 per cent capacity	6,876
Net return (economic return)	$ 8,724
Interest on first-mortgage bonds:	
$40,000 at 4 per cent	1,600
Balance of return for common and preferred stocks	$ 7,124
Return or dividend on preferred stock:	
$40,000 at 7 per cent	2,800
Balance of return for common stock	$ 4,324

$$\text{Promoter's or common stock yield} = \frac{\$4,324}{\$30,000} \times 100 \text{ per cent} = 14.41 \text{ per cent}$$

It will be observed that, as other financing besides common stock enters into the financial structure, the financial yield for common stock increases considerably above the economic yield which was 7.93 per cent.

The total investment in the project is its net worth, in this case $110,000. The amount of the bond issue will, of course, have to be paid back some day. As was suggested in Sec. 15.10, the bond issue can be repaid by maintaining a sinking fund. If so, it must be done out of the return which the common stockholder receives. The final repayment will increase the equity of the common stockholder to $70,000 at that time. His financial yield will then be 8.46 per cent (see Sec. 15.13). If the common stockholders wish to receive a larger yield as above (14.41 per cent), then, instead of repaying the bond issue out of their return, they must either refinance the mortgage or find the funds in the assets of the corporation. For this purpose they might use the assets that represent the reserve for depreciation, if available. If they do, however, there will be no assets to provide for replacement of the buildings and equipment. In other words, the promotor cannot repay the bond issue and replace the equipment from the same funds.

15.15. The Break-even Chart and Financial Obligations.—In Chap. 11, the break-even chart was developed as an instrument for depicting the relationships between the several factors of cost, income, and rate of production.

It may also be used to show the financial obligations in so far as the distribution of the return from the project is concerned.

The housing project which was used as an illustration in this chapter is to be financed by the forming of a corporation having common and preferred stock, with part of its financial needs supplied from the proceeds of a bond issue in accordance with the following data:

BALANCE SHEET STATEMENT

Fixed assets:			
Grounds and buildings	$110,000	6 per cent first-mortgage bonds	$ 50,000
		8 per cent preferred stock	35,000
		Common stock	25,000
	$110,000		$110,000

With this financial setup, the distribution of the return would be

Interest on bonds at 6 per cent	$3,000
Preferred stock dividend	2,800
Common stock dividend	2,924
Total return	$8,724

Promoter's financial yield (common stock dividend) $= \frac{\$2{,}924}{\$25{,}000} \times 100$ per cent $= 11.69$ per cent

These various returns may be shown on the break-even chart by drawing in lines parallel to the total operating-cost line and at the several values apart. They should be drawn in the order in which they take preference: bond interest, preferred stock dividend, and common stock dividend (see Fig. 15.1).

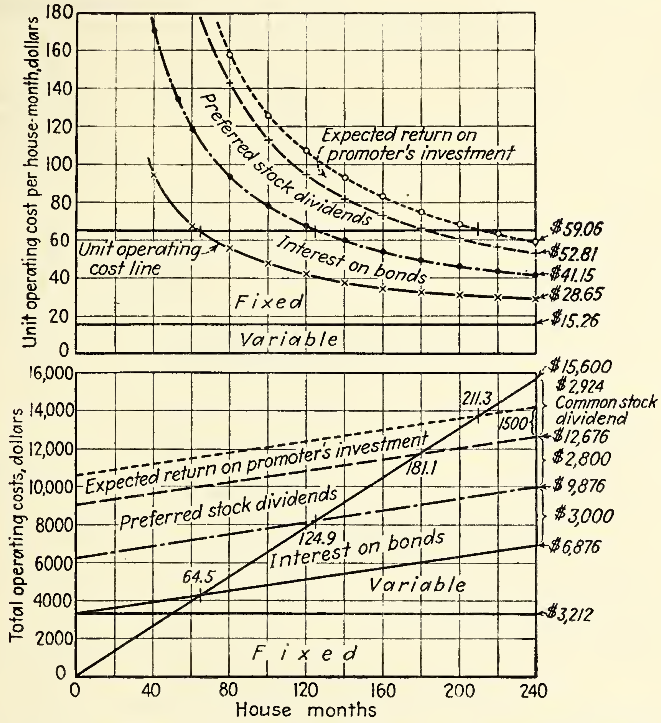

FIG. 15.1.—Break-even chart for housing development showing distribution of return in accordance with financial structure. See Figs. 11.1 and 11.3.

There are, thus, four break-even points:

1. No gain—no loss . . . . at 64.5 house-months
2. Interest on bonds . . . . at 124.9 house-months
3. Dividends on preferred stock . . . . . at 181.1 house-months
4. Expected return on promoter's investment . . . . at 211.3 house-months

The housing corporation must do the amount of business indicated at the break-even points in order to satisfy the several financial obligations.

TABLE 15.3.—IN-FAVOR-OF ANALYSIS

COMPARISON OF HOUSING DEVELOPMENT AND APARTMENT DEVELOPMENT

Financial Analysis

Item	Proposal		In favor of	
	Housing	Apartment	Housing	Apartment
1. Having Sufficient Funds				
Cash in hand	$110,000	$110,000		
Net return	8,724	8,784	..	$60
Financial yield	7.93 %	7.98 %	..	0.05 %
2. Borrowing Funds—Loan				
Cash in hand	$25,000	$25,000		
Amount to be borrowed	85,000	85,000		
Interest rate	6 %	6 %		
Interest charge	$5,100	$5,100		
Net return on cash in hand	3,624	3,684	..	$60
Yield on cash in hand	14.5 %	14.93 %	..	0.43 %
3. Borrowing Funds—Mortgage Bond (Bonds redeemed at end of term issue)				
Amount to be borrowed	$85,000	$85,000		
Interest rate	5 %	5 %		
Interest charge	$4,250	$4,250		
Net return on cash in hand	4,474	4,534	..	$60
Yield on cash in hand	17.9 %	18.13 %	..	0.23 %
(Bonds redeemed at end of term)				
Yield on cash in hand	17.9 %	18.13 %	..	0.23 %
Deposit to sinking fund to redeem bonds	$1,786	$1,786		
Balance of return after paying bond and sinking fund charge	2,688	2,748	..	$60
4. Borrowing Funds—Mortgage Bonds (Bonds redeemed before end of term of issue)				
Amount to be borrowed	$85,000	$85,000		
Interest rate	5 %	5 %		
Interest charge	$4,250	$4,250		
Net return on cash in hand	4,474	4,534	..	$60
Yield on cash in hand	17.9 %	18.13 %	..	0.23 %
Deposit to sinking fund to redeem bonds	$1,300	$1,300		
Balance of return after paying bond interest and sinking fund charges	3,174	3,234		
5. Partnership				
Yield on cash investment	7.93 %	7.98 %	..	0.05 %
For further details see recapitulation of method of financing under Housing Development.				
6. Incorporation				
Yield on cash investment	7.93 %	7.98 %	..	0.05 %

Problems

A. The first cost, operating cost, and income for a proposed hotel were given in Prob. *A* for Chap. 11. The proposed financial structure is as follows:

	Rate, per cent	Amount	Term, years
First mortgage	5	$700,000	30
Second mortgage	6	250,000	20
Cash by promoter	..	Balance	

a. Determine the shares of the return which go to each of the participants in the financial structure, assuming that the hotel is occupied 75 per cent of the time.

b. Prepare a bond redemption table for the first-mortgage bond issue. Bonds in units of $500 may be redeemed beginning at the end of the fifth year. The fund balances are deposited in bank drawing 2 per cent annually. The bonds are redeemed at end of the year.

c. What is the promoter's net cash return after having set aside a sum from his gross earnings to take care of the redemption of the bond issues?

d. A usual procedure would be for the hotel to be rented to a hotel operator who will manage it. Assume that the owner rents to an operator for $120,000 per year. Prepare a financial analysis for both owner and operator (see Prob. *A* in Chap. 9 for detailed operating costs). First cost of furnishings was $100,000. Owner pays taxes and takes care of depreciation on building and $5,000 of fixed operating expenses and all financial obligations concerning the bond issues. The operator takes care of furnishings and general operation. Assume 80 per cent capacity operation.

e. Indicate the distribution of return to first and second mortgage on a break-even chart.

B. The first cost, operating cost, and income for a proposed central concrete-mix plant were given in Prob. *B* of Chap. 11. The proposed financial structure is by cash in the hands of the promoter. What is his financial return at 150 days' operation?

C. The first cost and operating cost for a proposed power plant having either Diesel or uniflow engines for prime movers have been estimated in Prob. *C* in Chap. 10. Income is given in Prob. *C* in Chap. 11. A proposed financial structure is as follows:

Preferred stock	$150,000 at 8 per cent
First-mortgage bond issue	$300,000 at $4\frac{1}{2}$ per cent
Common stock	Balance

a. Determine the shares of the return to each of the classes of participants in the financial structure assuming that the plant is operated at 75 per cent capacity.

b. Prepare a bond redemption table for the first-mortgage bond issue. Bonds in units of $10,000 may be redeemed beginning at the end of the first year. Fund balances are deposited in the bank drawing 2½ per cent annually. The bonds are redeemed at the end of the year.

c. If it is required that redemption of the bond issue take place in accordance with the bond redemption table worked out in (*b*) above and that this payment for redemption of the principal be paid by the company before common stock dividends are paid, what will be the return to common stock and the yield on its share of the investment, year by year?

d. Indicate the distribution of return to first mortgage and preferred stock on a break-even chart.

D. The first cost, operating cost, and income for a plant to manufacture milk-bottle caps were given in Prob. *D* in Chap. 11. The ownership of the proposed plant is as follows: Mr. Jones, $\frac{1}{5}$; Mr. Smith, $\frac{1}{6}$; Mr. Green, $\frac{1}{4}$; the balance by small owners.

Determine the amount of ownership for each group and their shares of the return, assuming 85 per cent capacity operation.

E. The first cost, operating cost, and income for a proposed cold-storage locker plant was given in Prob. *E* in Chap. 11. The proposed financial structure of this plant is to be a cooperative one. Each owner takes a proportionate share of the total investment.

What is the share of each? What is his return on the investment? What is his financial yield?

1. A water district put out an issue of serial bonds with face value of $1,000,000, which sold at par. The interest rate was 3 per cent. Payment of interest and redemption of bonds were made in 20 equal installments beginning at the end of the first year. Prepare a bond redemption table for the issue. What was the yearly charge for amortization? Bank interest is at 3 per cent.

CHAPTER 16

INTANGIBLE ANALYSIS OF FINANCIAL STRUCTURE

16.1. Suggested Intangible Analysis of an Investment.—J. C. L. Fish in "Engineering Economics" suggests,

> The ideal of the capitalist is to put his funds in service in such place and manner and under such conditions as to
>
> 1. Give the maximum possible yield
> 2. Give the highest assurance of maximum appreciation (increase in value)
> 3. Incur minimum taxes
> 4. Leave no doubt as to the return of capital
> 5. Assure periodic return in uniform payments
> 6. Subject him to a minimum liability
> 7. Permit borrowing money advantageously at any time on his right in the investment as security
> 8. Permit his selling his rights in his investment quickly and advantageously at any time
> 9. Suit his wishes as to duration of investment
> 10. Permit him to control his investment to just the extent that he desired and no more
> 11. Permit him to manage the service to just the extent of his wishes
> 12. Have the units of investment of most convenient size and/or denominations

On analysis, it is seem that the first three fall in the class of facts that are easily reduced to monetary units; the others fall in the class of intangible facts.

Short-cut names for the above may be

1. Yield
2. Appreciation
3. Taxes
4. Safety of principal
5. Uniformity of yield
6. Liability

7. Collateral value
8. Marketability
9. Duration, or life
10. Control
11. Management
12. Units, or size

16.2. Intangible Analysis of Sole Ownership.—See Sec. 15.6. An application of the factors listed in Sec. 16.1 as they apply to the case of sole ownership of all the required funds shows

1. *Yield.*—The yield in this case is 7.93 per cent. Since he could obtain 6 per cent by investing in first mortgages or by lending his money out at interest, this may be considered satisfactory.

2. *Appreciation.*—The history of the college town is that it has continually grown. Other property has appreciated somewhat in value. The immediate prospect is that student enrollment will increase, thus making a larger faculty necessary.

3. *Taxes.*—The tax rate is not high considering the environment.

4. *Safety of Principal.*—As was cited under Appreciation, the college is growing. It is assured of continued support. There seems to be no reason why the project would not be a safe investment.

5. *Uniformity of Yield.*—This is assured because of the fact that, more and more, the college is in session for a greater length of time.

6. *Liability or Risk.*—The promoter's liability is his entire investment in the project.

7. *Collateral Value.*—Since it is good real estate, it will make good collateral against which to borrow. The fact that there are 20 separate houses makes it much better than if it were one individual project.

8. *Marketability.*—There is little question but that the project will be a success. Inasmuch as it fills a need and can be easily divided into individual houses, it will be readily marketable. This would also depend upon amounts of uninvested funds that are available in the community.

9. *Duration, or Life.*—Since the promotor has put all his funds into the project, its life will be as long as the promoter wishes to keep the investment.

10. *Control.*—Since the promoter owns the entire investment, having put his own money into it without borrowing, the control is completely his.

11. *Management.*—The promoter has complete say.

12. *Units, or Size.*—The project is for 20 houses. Although the project was planned as a whole, it would be easy to reorganize it on a single-house basis.

16.3. Intangible Analysis of Borrowed Funds.—See Sec. 15.8.

1. *Yield.*

PROMOTER.—8.3 per cent satisfactory.

LENDER.—6 per cent satisfactory.

2. *Appreciation.*

PROMOTER.—See Sec. 16.2.

3. *Taxes.*

PROMOTER.—See Sec. 16.2.

4. *Safety of Principal.*

PROMOTER.—See Sec. 16.2.

5. *Uniformity of Yield.*

PROMOTER.—See Sec. 16.2.

6. *Liability.*

PROMOTER.—$90,000 in cash plus any other personal assets, as well as the houses.

LENDER.—His liability is his investment protected by the cash put in by the promotor. In case anything goes wrong, it is not likely that the houses would depreciate $90,000.

7. *Collateral Value.*

PROMOTER.—See Sec. 16.2.

8. *Marketability.*

PROMOTER.—See Sec. 16.2.

9. *Duration, or Life.*

PROMOTER.—As long as he wishes to keep his investment.

LENDER.—Loans of this kind are usually for short periods of time. The lender asks the promoter when he can repay the loan. Since the promoter has no other funds in sight, and since the lender does not wish to lend for more than one year, the proposition is very unsatisfactory from the lender's viewpoint. The promoter could offer to pay back part of the amount each year, out of his return. Since his return is $7,524, it would take him

$$\frac{\$20{,}000}{\$7{,}524} = 2.65 \text{ years}$$

This the lender believes is too long a time and he will not make the loan. The lender suggests that, if he could be given greater security, he might be willing.

10. *Control.*

Promoter.—Has complete control.

Lender.—Has no control.

11. *Management.*

Promoter.—Has complete say.

Lender.—Has no say.

12. *Units, or Size.*

The loan is in one lump sum. It could be broken down into 20 notes, one on each house.

16.4. Intangible Analysis of Bond Issue.—See Sec. 15.9.

1. *Yield.*

Promoter.—17.9 per cent satisfactory.

Lender.—5 per cent satisfactory.

2. *Appreciation.*

Promoter.—See Sec. 13.13.

3. *Taxes.*

There may be additional taxes assessed by the state and Federal taxing authorities because of the nature of the bond.

4. *Safety of Principal.*

Promoter.—See Sec. 16.2.

5. *Uniformity of Yield.*

Promoter.—See Sec. 16.2.

6. *Liability.*

Promoter.—$25,000 in cash plus any other personal assets as well as the houses.

Lender.—His liability is his investment protected by the cash put in by the promoter. In case anything goes wrong, it is not likely that the houses would depreciate $25,000.

7. *Collateral Value.*

Promoter.—See Sec. 16.2.

8. *Marketability.*

Promoter.—See Sec. 16.2.

9. *Duration, or Life.*

Promoter.—As long as he wishes to keep his investment.

Lender.—Those who buy the bonds ask the promoter when he can pay the money back. Loans of this character when secured by real estate are usually for a long time. They may be issued for terms of 5, 10, 15, 20, 25, or 30 years. The promoter has a choice of three ways of handling this financing.

The first way is for him to set up the bond issue for, say, 30 years and agree to pay its face value back to the bondholders, at that time. In order to do this, he must be able to obtain that sum, in this case $85,000, at the time the bonds become due. This he may do by borrowing the indicated sum. Or he may deposit regularly in a fund drawing interest, such an amount as to be equal to the face of the bond issue at its date due.

10. *Control.*

PROMOTER.—Has complete control except as modified by bond conditions.

LENDER.—The lenders have no control except as provided in the bond itself. In case of default of interest, the bondholders usually take over control.

11. *Management.*

PROMOTER.—Has complete authority.

LENDER.—Has no authority unless bond interest is not paid promptly.

12. *Units, or Size.*

The size of bond units may be made to suit the ability to find small investors. Units of $100 are quite common.

16.5. Intangible Analysis of Partnership.—See Sec. 15.11.

1. *Yield.*

The yield is now the same for both partners, 7.93 per cent.

2. *Appreciation.*

PROMOTER.—See Sec. 16.2.

3. *Taxes.*

PROMOTER.—See Sec. 16.2.

4. *Safety of Principal.*

PROMOTER.—See Sec. 16.2.

5. *Uniformity of Yield.*

PROMOTER.—See Sec. 16.2.

6. *Liability.*

Here there is a change. Under partnerships, each partner is liable for all the debts of the firm. Creditors may bring action against the partners separately. The proper choice of a partner is, therefore, very essential. There are certain conditions under partnership law whereby the liability of one of the partners may be limited.

7. *Collateral Value.*

LENDER.—See Sec. 16.2.

8. *Marketability.*

LENDER.—See Sec. 16.2.

9. *Duration, or Life.*

In this situation, the duration will be for the lives of the partners. If the lender partner should die, the partnership would be dissolved and the promoter would have to find funds with which to pay the lender's share of the investment to the lender's estate. The partnership could also be dissolved by agreement. Should the lender partner desire to withdraw, the promoter would again have to find funds.

10. *Control.*

In the situations previously discussed, the promoter had all the control, modified only in the case of default of interest payments. In this situation, the control will be divided. Obviously, the new partner will desire to have his say in the control. He is also likely to believe that control should be in proportion to the share in the investment. This means that the promoter would have to give up control to the lender partner.

11. *Management.*

In the other situations, the promoter has been the manager of the project. In this situation, the management may be divided in terms of the talents and abilities of the partners.

12. *Units, or Size.*

Each partner has an amount equal to his share in the investment.

16.6. Intangible Analysis of Setting Up Corporation.—See Sec. 15.12.

1. *Yield.*—The yield for each stockholder will depend upon the structure of the corporation. If the corporation has common stock only, the yield is the same for all stockholders. It is to be noted that the economic yield is 7.93 per cent. The officers of the corporation may wish to set aside some of the return to form a surplus. This procedure would reduce the stockholder's yield from the calculated 7.93 per cent. It would be 7.93 per cent only if no surplus was set aside. Note, however, that the effect of creating a surplus is to enhance the value of a share of stock. The corporation may have both common and preferred stock. Usually the owners of the preferred stock will be given a

stated yield, say 7 per cent, and priority as to the return. The balance is then distributed among the holders of common stock.

Preferred stock	$ 40,000	
Common stock	70,000	
Total	$110,000	
Total return (economic analysis)		$8,724
Dividends to preferred stock at 7 per cent		2,800
Balance, dividends to common stock		$5,924

$$\text{Yield for common stock} = \frac{\$5{,}924}{\$70{,}000} \times 100 = 8.4 \text{ per cent}$$

This yield may again be reduced because of the creation of a surplus as indicated in the discussion of common stock.

2. *Appreciation.*—See Sec. 16.2.

3. *Taxes.*—There may be additional taxes assessed by the state and Federal taxing authorities because of the nature of the corporation and its financial policies with regard to the creation of surpluses.

4. *Safety of Principal.*—See Sec. 16.2.

5. *Uniformity of Yield.*—See Sec. 16.2.

6. *Liability.*—The liability of each stockholder is solely to the extent of his stock ownership.

7. *Collateral Value.*—The value of the shares of stock will depend upon how the project is run. The market price of the stock will determine its borrowing value.

8. *Marketability.*—Since the size of the stock will be much smaller than the single amount of $85,000, the shares of the stock will be more readily marketable. Even the promoter would not find it difficult to dispose of his shares of the stock if the project is as successful as it seems to be.

9. *Duration, or Life.*—The period of life for the corporation should be infinite. The death of a stockholder will not affect the operation of the corporation.

10. *Control.*—The control which each stockholder may exert depends upon the amount of stock he owns. Obviously, anyone having over 50 per cent of the stock can control the corporation and the project. Our promoter has only 25 per cent of the stock. He can control the corporation as long as no other person or group of persons possesses more than he does. The promoter will see to it that sufficient shares of stock are owned by persons who trust him and are willing to have him control the project.

It is to be noted that when preferred stock is a part of the financial structure of the corporation, certain modifications are made in control. Usually holders of preferred stock, in exchange for their fixed and assured dividend positions, give up any control. The exception is that when the corporation defaults on its dividend to preferred stockholders, they step in and assume control.

11. *Management.*—The management of the project is delegated to a group of officers or a manager. Obviously, that stockholder or group of stockholders which controls over 50 per cent of the stock can select the manager and dictate the policies by which the project is to be operated.

Obviously, if preferred stockholders are in control, they can select the management.

12. *Units of Size.*—The size of stock units may be made to suit the ability to find small investors. Units of $100 are quite common.

Problems

Discuss the intangible aspects of the financial structures given in the problems for Chap. 15.

PART IV

SPECIAL METHODS AND APPLICATIONS

The break-even chart
The increment cost principle
The problem of replacement
The endowment principle
Some aspects of public projects
Studies and reports

CHAPTER 17

BREAK-EVEN CHART

17.1. The Break-even Chart.—In Sec. 11.12, the break-even chart was described as a pictorial profit and loss statement. It is formed by adding an income line to the chart of operating costs.

In Sec. 4, the third type of problem met in engineering economics was "Rate of production of the equipment." Many pieces of equipment and machinery are operated at varying capacities. Likewise, industrial enterprises are operated at varying rates of production. In order to make correct economic analyses of the relationships between first cost and investment, operating cost, income, return, and yield, the effect of varying any one of the several factors upon the others must be studied.

The effect of varying any of the factors that enter into the profit and loss statement or the break-even chart may be stated as follows:

1. Any increase in fixed costs will make the break-even point move to the right, or increase in value.

2. Any increase in variable costs will make the break-even point move to the right, or increase in value.

3. Consequently, any decrease in fixed or variable costs will make the break-even point move to the left, or decrease in value.

4. Any increase in the income will make the break-even point move to the left or decrease in value. Obviously any decrease will make the break-even point increase in value.

5. Any decrease in the normal work day, work week, or work year, will increase the proportionate position of the break-even point. Obviously, any increase will decrease the proportionate position of the break-even point.

17.2. An Illustrative Problem.—In Sec. 11.12, an industrial enterprise was used as an illustrative problem and a break-even chart developed. The effect of varying the factors of cost, etc., will be studied, using this problem and keeping in mind the principles of Sec. 17.1 (see Fig. 17.1).

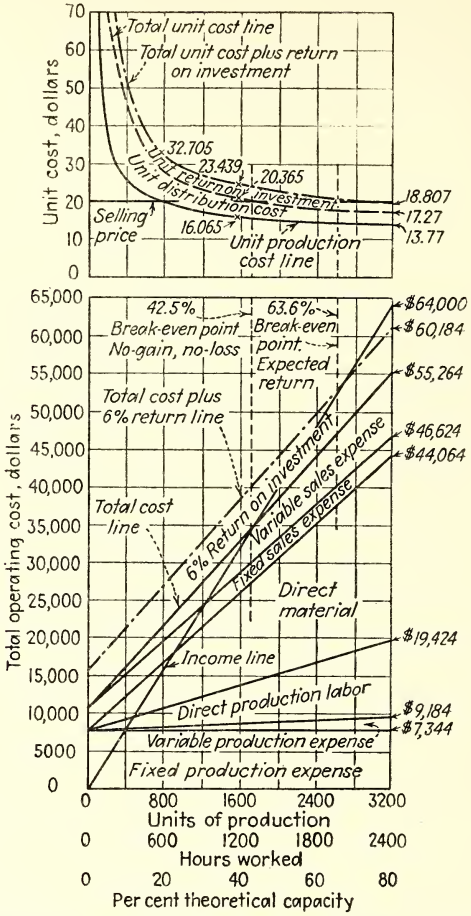

FIG. 17.1.—Break-even chart for an industrial enterprise. See Figs. 11.2 and 11.4.

17.3. Materials Prices Increase.—See Fig. 17.2. Because of conditions in the materials market, the cost of materials has been increased by $3,200.

Previous materials cost	$24,640
Price increases	3,200
New materials cost	$27,840

The effect of this change is to raise the point where the direct-materials line intersects the 80 per cent-capacity line to $47,264; the fixed-sales-cost line to $49,824; the variable-sales-cost line to $58,464; and the 6 per cent-yield line to $63,385.

The upward shift of these lines causes the intersection of the total-cost line and the income line to shift toward the right. The same effect is seen with regard to the 6 per cent-yield line and the income line.

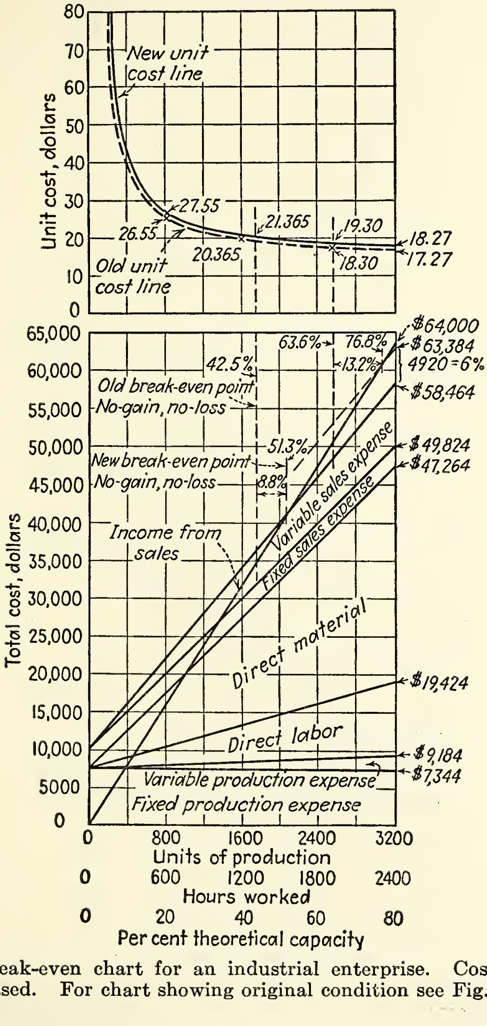

FIG. 17.2.—Break-even chart for an industrial enterprise. Cost of materials increased. For chart showing original condition see Fig. 17.1.

The break-even point for no loss—no gain now becomes about 51.3 per cent and the break-even point for 6 per cent yield becomes about 76.8 per cent.

The change in the cost of materials will also affect the unit-cost curve. Since the change is $3,200 for a total production of 3,200 units, the net change per unit of production is, therefore,

$$\frac{\$3{,}200}{3{,}200} = \$1$$

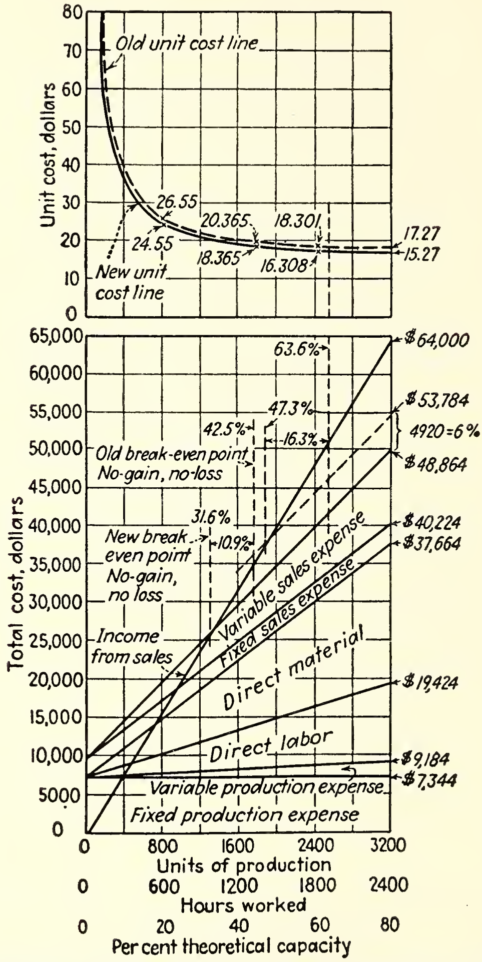

Fig. 17.3.—Break-even chart for an industrial enterprise. Cost of materials has decreased. For chart showing original condition see Fig. 17.1.

The curve for unit costs, taking into account the price rise, may be drawn $1 higher than that shown on Fig. 17.1. The old unit-cost curve before the price rise is shown dotted; the new curve is shown in solid lines.

17.4. Materials Prices Decrease.—See Fig. 17.3. Because of conditions in the materials market, the cost of materials has been decreased by $6,400.

Previous market cost	$24,640
Price decreases	6,400
New materials cost	$18,240

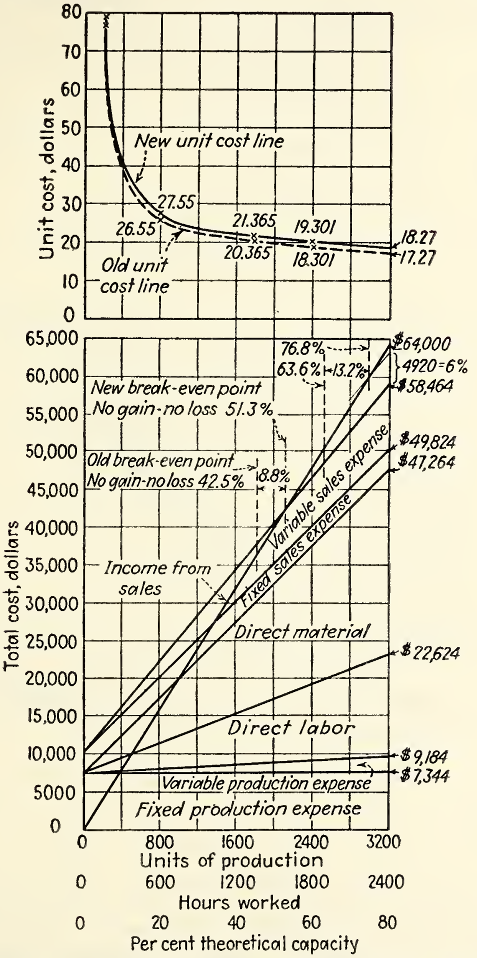

FIG. 17.4.—Break-even chart for an industrial enterprise. Cost of labor has increased. For chart showing original condition see Fig. 17.1.

The effect of this change is to lower the point where the direct materials line intersects the 80 per cent-capacity line to $37,644; the fixed-sales line to $48,864; and the 6 per cent-yield line to $53,784.

The downward shift of these lines causes the intersection of the total-cost line and the income line to shift toward the left. The same effect is seen with regard to the 6 per cent-yield line and the income line.

The break-even point for no loss–no gain now becomes about 31.6 per cent and the break-even point for 6 per cent yield becomes about 47.3 per cent.

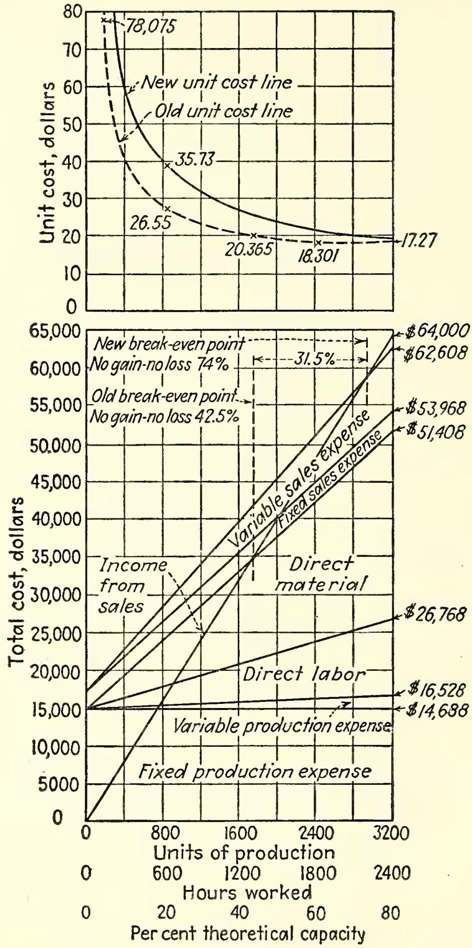

FIG. 17.5.—Break-even chart for an industrial enterprise. Fixed expense has increased. For chart showing original condition see Fig. 17.1.

The change in the cost of materials will also affect the unit-cost curve. Since the change is \$6,400 for a total production of 3,200 units, the net change per unit of production is, therefore,

$$\frac{\$6{,}400}{3{,}200} = \$2$$

The curve for unit costs, taking into account the price decrease, may be drawn $2 lower than that shown in Fig. 17.1. The old unit-cost curve is shown as a dotted line. The new curve is solid.

17.5. Direct Labor Increase.—See Fig. 17.4. Because of conditions in the labor market, an increase in wages has been demanded by the workmen and has been granted. It amounts to $3,200.

Previous labor cost	$10,240
Rate increase	3,200
New labor cost	$13,440

The effect of this change is to raise the point where the direct-materials line intersects the 80 per cent-capacity line to $22,624; the direct-materials line to $47,624; the fixed-sales-cost line to $49,824; the variable-sales line to $58,464; and the 6 per cent-yield line to $63,384.

The upward shift of these lines causes the intersection of the total-cost line and the income line to shift toward the right. The same effect is seen with regard to the 6 per cent-yield line and the income line.

The break-even point for no gain—no loss now becomes 51.3 per cent and the break-even point for 6 per cent yield becomes 76.8 per cent. The change in the cost of labor will also affect the unit-cost curves. Since the change is $3,200 for a total production of 3,200 units, the net change per unit of production is, therefore,

$$\frac{\$3{,}200}{3{,}200} = \$1$$

The curve for unit costs, taking into account the rate increase, may be drawn $1 higher than that shown in Fig. 17.1. The old unit-cost curve is shown as a dotted line; the new curve is solid.

17.6. Fixed Expense Increase.—See Fig. 17.5. Certain conditions have caused the fixed expense to increase to the amount of $7,344.

Previous fixed expense	$ 7,344
Increase	7,344
New fixed expense	$14,688

The effect of this change is to raise the point where the fixed expense line intersects the 80 per cent-yield line to $14,688; the variable-expense line to $16,528; the direct-labor line to $26,768; the direct-materials line to $51,408; the fixed-sales-expense line to $53,968; the total-cost line to $62,608.

The upward shift of those lines causes the intersection of the total-cost line and the income line to shift toward the right. The 6 per cent-yield line does not intersect the income line within the 80 per cent-capacity point. If the income line could be extended outside the 80 per cent-capacity point to the right, the 6 per cent-yield line would probably intersect at about 100 per cent capacity.

The break-even point for no loss–no gain now becomes about 74 per cent.

The change in fixed expense will also affect the unit-cost curve. Since the change is uniform for all productive capacity points, the change per unit of production will vary with the productive capacity. In other words, the change will be nonuniform in nature.

At 80 per cent capacity the change per unit will be

$$\frac{\$7,344}{3,200} = \$2.295$$

Previous unit cost	$17.27
New unit cost	19.565

At 60 per cent capacity the change per unit will be

$$\frac{\$7,344}{2,400} = \$3.06$$

Previous unit cost	$18.301
New unit cost	21.361

At 40 per cent capacity the change per unit will be

$$\frac{\$7,344}{1,600} = \$4.58$$

Previous unit cost	$20.365
New unit cost	24.945

At 20 per cent capacity the change per unit will be

$$\frac{\$7,344}{800} = \$9.18$$

Previous unit cost	$26.55
New unit cost	35.73

At 4 per cent capacity the change per unit will be

$$\frac{\$7,344}{160} = \$45.80$$

Previous unit cost	$ 78.075
New unit cost	124.875

The new curve for unit cost is drawn as a solid line. The old curve is dotted. It will be noticed that the vertical distance at any capacity point increases as the rate of production decreases. This is typical of all fixed expenses. Since they are the same for all rates of production and do not vary with rate of production, the change is nonuniform.

It will be noticed that the unit-cost curve crosses the selling price line close to the 80 per cent-capacity point. Had the increase in fixed expense been just a little more, the unit-cost curve would have crossed at the 80 per cent line, which would have been the break-even point for no loss–no gain.

17.7. Capacity Decrease.—See Fig. 17.6. The company has decided to operate on a 36-hr. week for 50 weeks, or 1,800 hr. per year instead of the present 2,400 hr. There is to be no increase in the rate of production. Therefore, all the relationships that existed as drawn in Fig. 17.1 will obtain with the exception that the new chart will be drawn only up to the 60 per cent-capacity line.

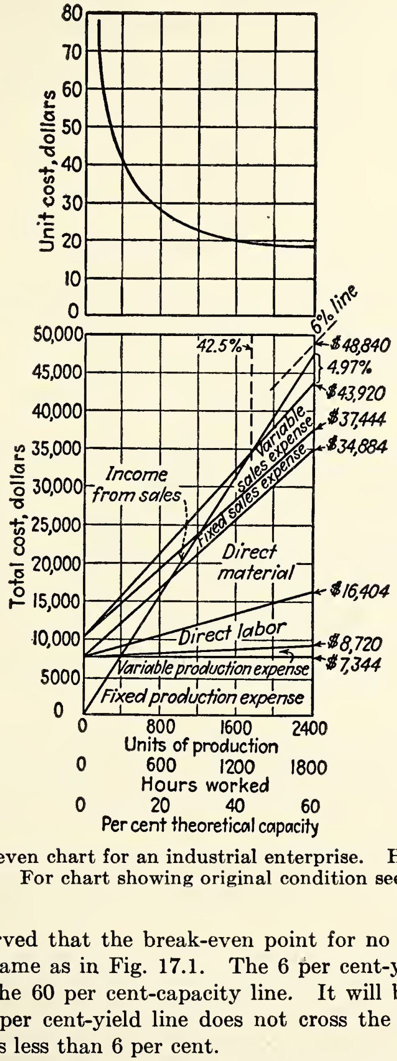

FIG. 17.6.—Break-even chart for an industrial enterprise. Hours per year have decreased. For chart showing original condition see Fig. 17.1.

It will be observed that the break-even point for no loss–no gain is at 44 per cent, the same as in Fig. 17.1. The 6 per cent-yield line is drawn where it crosses the 60 per cent-capacity line. It will be noticed that at that point the 6 per cent-yield line does not cross the total-income line. Hence, the yield is less than 6 per cent.

If this condition of operating at 1,800 hr. is to be the standard for some time, it may be advisable to change the values for capacity and the break-even point. If this is done, the capacity values for the abscissas will become

Old Value, Per Cent	New Value, Per Cent
80	Not on chart
60	80
40	53.3
20	26.5

A new scale may then be placed on the abscissas. It will look like the following:

0	600	1,200	1,800	2,400
		Units of Product		
0	450	900	1,350	1,800
		Hours Worked		
0	20%	40%	60%	80%
		Per Cent Theoretical Capacity		

This scale is now in terms of the new method of operation.

It will also be necessary to change the value of the break-even point. The new value will be

$$\frac{42.5}{60} \times 80 \text{ per cent} = 56.6 \text{ per cent}$$

The unit-cost curve will be the same as in Fig. 17.1, since the values of the elements of cost of production and sales have not changed.

17.8. Modifications of Figure 17.6.—See Fig. 17.7. It will be readily observed that the condition shown in Fig. 17.6 is not good. What changes must be made in order to duplicate the conditions shown in Fig. 17.1?

In Fig. 17.1 the break-even point for no loss–no gain was 42.5 per cent for the old scale. This means that the break-even point will have to be shifted from 56.6 to 42.5 per cent. It will be noticed that the total-cost line and the total-income line at this 42.5 per cent capacity are quite a distance apart. There are three possibilities available for bringing these two lines together.

The first method is to increase the angle which the total income from the sales line makes with the base line so that it crosses the break-even point at the total-cost line. The net result of this change is to increase the unit selling price. This may be observed from Fig. 17.7 by projecting the 42.5 per cent capacity line upward to cross the unit-cost curve. This line crosses the unit-cost curve at about $2 more than the previous selling price. Unless the selling price can be raised $2, this method of changing the break-even point will be useless.

Since the chart may not be accurate, the exact selling price necessary to satisfy a breakdown point of 42.5 per cent may be calculated.

From the original data the cost of production and of selling at the 42.5 per cent-capacity point for the new setup may be determined as follows:

The 42.5 per cent-capacity point represents a capacity in units of

$$\frac{42.5}{80} \times 2{,}400 = 1{,}275 \text{ units}$$

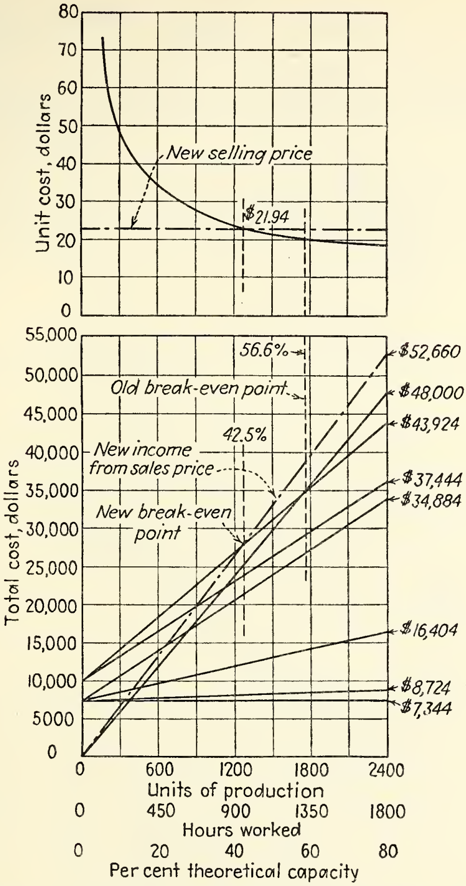

FIG. 17.7.—Break-even chart for an industrial enterprise. Selling price increased. For chart showing original condition see Fig. 17.1.

Item	Cost
Fixed production expense	$ 7,344.00
Variable expense $\left(\frac{1,275}{3,200} \times \$1,840\right)$	733.13
Direct labor $\left(\frac{1,275}{3,200} \times \$10,240\right)$	4,080.00
Direct material $\left(\frac{1,275}{3,200} \times \$24,640\right)$	9,817.50
Fixed sales expense	2,560.00
Variable sales expense $\left(\frac{1,275}{3,200} \times 8,640\right)$	3,442.50
Total cost of production and sales	$27,977.13

$$\text{Unit cost} = \frac{\$27,977.13}{1,275} = \$21.94$$

The second method is to decrease the angle which the total cost line makes with the base line so that it crosses the break-even point at the income line.

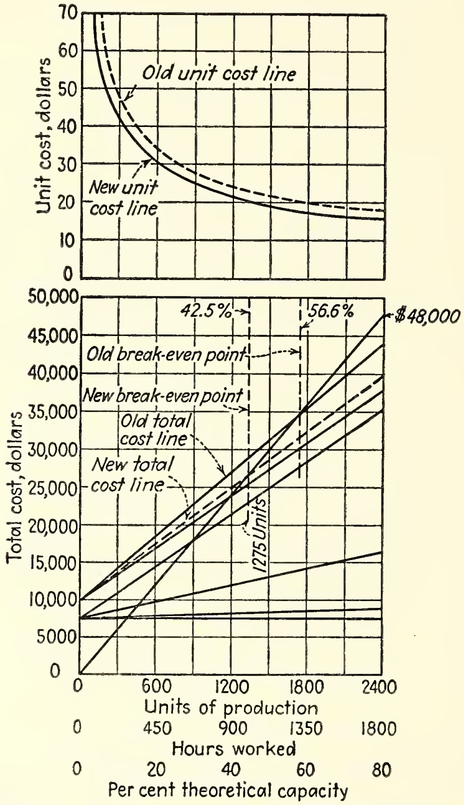

FIG. 17.8.—Break-even chart for an industrial enterprise. Figure 17.6 with 60 per cent capacity changed to 80 per cent capacity. Total operating cost line decreased to get original break-even point.

The net result of this change is to decrease the unit cost of production and sales cost.

This may be observed from Fig. 17.8. The unit-cost curve will have to be lowered in order to cross the selling-price line at the break-even point. This line crosses at about $2 less than the previous unit cost. Unless some one or more of the factors that make up unit cost can be reduced, this method of changing the break-even point will be useless.

It will be observed that changes in one or more of the following will be necessary:

Fixed production expense
Variable expense
Direct labor
Direct material
Fixed sales expense
Variable sales expense

Since the chart may not be accurate, the exact unit cost necessary to satisfy a break-even point of 42.5 per cent may be calculated.

From the original data the unit cost may be obtained as follows:

The 42.5 per cent-capacity point represents a capacity in units of

$$\frac{42.5}{80} \times 2{,}400 = 1{,}275 \text{ units}$$

The total income for a capacity point of 42.5 per cent will be equal to

$$1{,}275 \times \$20 = \$25{,}500$$

From the previous calculation for Fig. 17.8, it was shown that the cost of production and sales at the 42.5 per cent-capacity point (without any changes in the several costs) was $27,977.13. It is apparent, therefore, that this cost must be reduced to $25,500.

Present cost	$27,977.13
New cost	25,500.00
Required reduction	$ 2,477.13

As previously indicated, this reduction of $2,477.13 may be accomplished in any of the several elements of production or sales cost, or in several at one time. This calls for a review of manufacturing and selling methods in order to determine how the changes may be made. Since this method involves no change in the rate of production, the reduction in costs must occur in reduced rates for materials, labor, and expense. This calls for better purchasing and utilization of materials if the reduction is to be made in materials cost. A reduction in labor rate per hour would permit the change to take place in this element of cost. For the change to take place in fixed or variable expenses, a reduction in some element of these costs is called for.

A third method is to increase the rate of production within the present total cost of production and selling. This means spreading the present charges for materials, labor, and expense over a large number of units produced and sold. Assuming that the material charges will not change per unit no matter what the rate of production, the required changes must occur in labor cost and variable and fixed expense. These are possible only when the rate of production is increased without the labor or expense rates per hour of work increasing.

From the calculation previously made for the second method it is seen that the cost of production and sales is equal to $27,977.13. Since the selling price is $20 per unit and at the break-even point the cost of production and sales is the same, it follows that the required production must be equal to

$$\frac{\$27{,}977.13}{\$20} = 1{,}399 \text{ units}$$

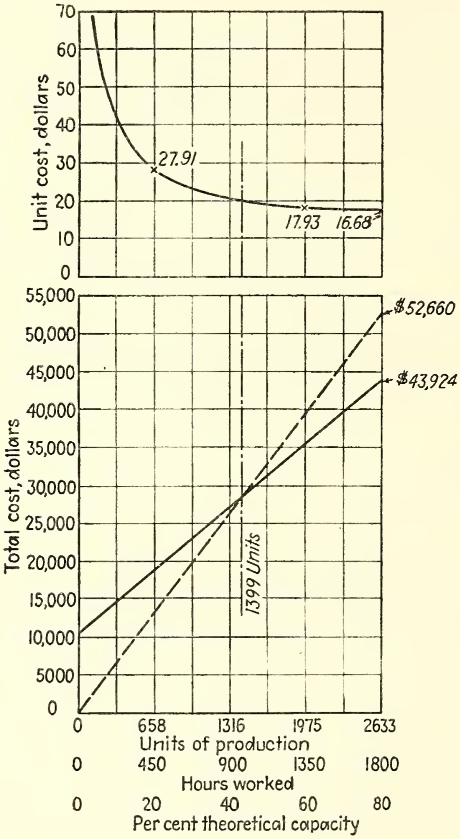

FIG. 17.9.—Break-even chart for an industrial enterprise. Figure 17.6 with 60 per cent capacity changed to 80 per cent capacity. Rate of production changed to get original break-even point.

This increased rate of production requires a change in the scale for the abscissas marked "units of product."

If the rate of production at 42.5 per cent capacity point is to be 1,399 units, then the rate of production at the 80 per cent capacity point will be

$$\frac{80}{42.5} \times 1{,}399 = 2{,}633 \text{ units}$$

A revised unit-cost curve will have to be constructed using the new rate of production.

Capacity point, per cent	Units produced	Unit cost
80	2,633	$16.68
60	1,975	17.93
42.5	1,399	20.00
40	1,316	20.45
20	658	27.97
4	131	88.58

A word of caution is needed at this point. It is rarely possible to increase the rate of production without increasing the fixed and variable expenses. This is true because some kind of new equipment or machinery is needed to accomplish the increased rate of production, thereby giving rise to decreased charges for direct labor and expense. Figure 17.9 is drawn assuming no changes in fixed expenses. If fixed expenses are increased, there must be a still greater decrease in direct labor and variable expense, and perhaps in materials costs.

17.9. Standard Cost, Normal Capacity, and Fixed Expense.—The term "standard cost" is applied to the use of cost figures for the control of an industrial plant. It is the modern method of cost accounting. Control of costs is secured by comparing actual costs with this standard which has been previously set up.

In the illustrative problem that preceded this section, Fig. 17.1 has a unit-cost curve. This is a nonuniform curve in which the unit cost decreases as the production increases. When production is at the 80 per cent-capacity point, the unit cost is $13.77 for production and $3.50 for sales, making a total unit cost of $17.27. At other capacity points this total cost will be different.

In this figure the right-hand limit of the abscissas was fixed at 80 per cent of the theoretical capacity. This rate of production or capacity point may be called the "normal capacity." It is that capacity to be expected by the industrial plant when it is operated with a reasonably efficient management. For many plants, this capacity has been assumed at 80 per cent.

Hence the total unit cost, in this case $17.27, at this normal capacity point of 80 per cent has become known as the standard cost.

Normal capacity has another significance. It is that capacity point or rate of production whose standard cost absorbs all of the fixed expense. If the plant operates at less than 80 per cent but does not change the unit cost to correspond to the capacity point, then a portion of the fixed expense is not absorbed in the price charged the customer.

For example, from the problem of Fig. 17 1.

Capacity Point, Per Cent	Unit Cost
80	$17.27
60	18.30
40	20.36
20	26.65
4	78.07

It is easy to see that, if an industrial plant manufactures at any point less than the point at which standard cost is figured, there will be a decrease in profit because all of the fixed expense is not charged against the selling price. This reduces the profit until the rate of production reaches the break-even point of 42.5 per cent. Below this there is a loss.

If the plant operates above the standard-cost-capacity point, the unit cost will be less because the fixed cost is spread over more units of production and there will be an increase in profits because of this action.

The normal-capacity point is assumed at that rate of production which represents the average of operations over the business cycle of the industry. The purpose is to pick a point so that the volume of production below the standard cost is about equal to the volume of production above the standard-cost point.

When an industry is overcrowded with operating plants, so that a particular plant is able to get only enough orders to operate its plant at 60 per cent capacity on the average, then that plant should set its standard-cost point at about the 60 per cent-capacity point. This obviously means that there is a considerable amount of idle machinery around the plant. The fixed-expense proportion of the unit cost must be much higher than before. For the problem illustrated in Fig. 17.1 the standard cost must be increased from $17.27 at 80 per cent capacity to $18.30 at the 60 per cent-capacity point. This increase will, in turn, affect the selling price. In fact, if applied to Fig. 17.1, there will be a decrease of $18.30 minus $17.27, or $1.03, in the unit profit. The effect on the yield of the business is shown where the 60 per cent-capacity line crosses the income-from-sales line and the 6 per cent-yield line. The yield has dropped from 10.65 per cent at 80 per cent capacity to about 5 per cent at 60 per cent capacity.

17.10. Application to a Single Machine.—The graphical method of showing the relationships between the various factors of cost for an industrial plant, as presented, is also applicable to a single machine. Such a chart would be similar to Fig. 11.5, which shows the cost of production. There may be, for example, in the same plant two or more machines on which similar work can be done, each machine with certain characteristics as to its operation and with certain cost characteristics.

In a printing plant there are two printing presses, one more modern than the other. The printer wishes to run a job which consists of 6,000 sheets. Which press shall he use? The characteristics of the two presses are as shown at top of page 233.

This problem may be solved by making a chart of the cost of production, similar to Fig. 11.5, for each press and comparing the two.

Item	Press No. 1	Press No. 2
Direct labor cost	$0.50 per hour	$0.50 per hour
Direct material cost	$1 per 1,000 sheets	$1 per 1,000 sheets
Machine rate	$0.25 per hour	$0.30 per hour
Time required to set up and make ready the press	72 min.	90 min.
Time to print one sheet	0.001 hr.	0.0008 hr.
Investment in press	$5,000	$6,000

Press No. 1.—The scale for the abscissas can be hours of work per day, namely eight. This scale may be shown as production in sheets printed.

$$\text{Number of sheets printed} = \frac{\text{hours per day}}{\text{hours per sheet}}$$

$$\frac{8}{0.001} = 8{,}000 \text{ sheets per day}$$

Fixed Expense.—In this problem, the fixed expense is that expense which is incurred for the job and which applies no matter how many sheets are printed. In this problem it is the time required to set up and make ready the press. The charge for this time will consist of labor, the machine-hour charge, and interest on the investment. Return on the investment is added because there are no data given concerning income or return.

Labor for setup = 72 min. at $0.50 per hour = $0.60
Machine-hour rate = 72 min. at $0.25 per hour = $0.30

Return on Investment.—The investment in this press is $5,000. The company expects to earn 12 per cent on its investment, or $600 per year. Assuming a work year of 300 days of 8 hr. per day, the return per hour would be equal to

$$\frac{\$600}{(300)(8)} = \$0.25 \text{ per hour}$$

For 72 min. the return charge would be 30 cents. Thus the total fixed expense for the job would be

Setup labor	$0.30
Machine-hour rate	0.30
Return	0.30
Total	$1.20

Direct Labor for Printing.—This is a variable charge. For 8 hr. it would be $4.

Machine-hour Rate.—This is a variable charge. For 8 hr. it would be 8 × $0.25 = $2.

Return on Investment.—This is a variable charge. For 8 hr. it would be 8 × $0.25 = $2.

The total cost, including return on the investment but not including material, would then be

Fixed expense for the job	$1.20
Direct labor	4.00
Machine-hour rate	2.00
Return	2.00
Total	$9.20

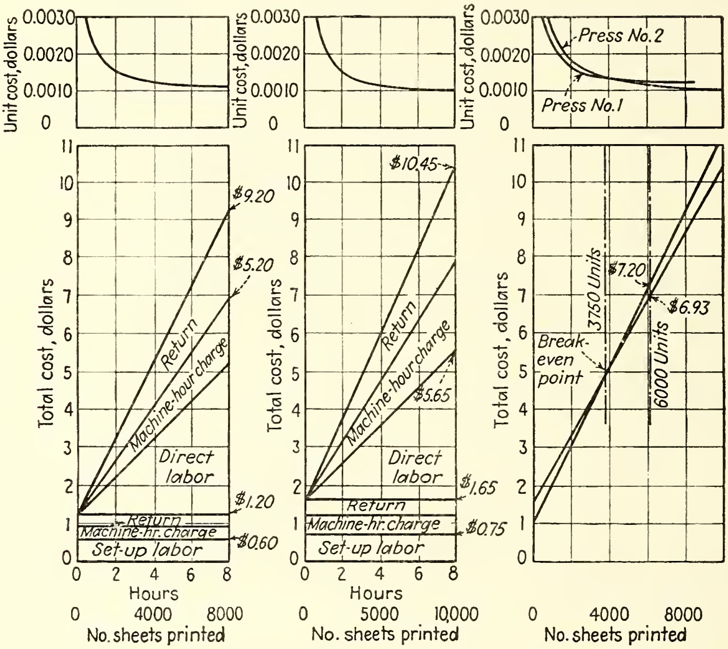

FIG. 17.10.—Operating cost chart for press No. 1.

FIG. 17.11.—Operating cost chart for press No. 2.

FIG. 17.12.—Break even point for presses No. 1 and No. 2.

Comparison of doing a printing job on two different presses to determine the most economical one for different rates of production.

Figure 17.10 is drawn for those costs. A unit-cost curve is also drawn at the top of the chart.

The unit-cost figures at several quantities of production are

Quantity	Unit Cost
500	$0.0034
1,000	0.0022
2,000	0.0016
4,000	0.0013
6,000	0.0012
8,000	0.00115

Material cost is not included in the figure because it is the same for both presses.

The total cost of 8,000 sheets printed on Press No. 1, including direct material, would then be

Cost, including interest, setup labor, etc.	$ 9.20
Cost of direct material	8.00
Total cost	$17.20

This figure represents the total cost of all of the factors of production and includes the interest on the investment. The job could therefore be sold for this figure if there was no additional cost for selling and administration. The yield would be the rate of return allowed on the investment as calculated, or 12 per cent.

The total cost not including return on the investment would be

Complete total cost	$17.80
Return	4.00
Total cost—return	$13.80

Press No. 2.—When the same calculations are performed for this press the following are the results. The total cost including return but not including material will be:

Fixed expense for the job:		
Setup labor	$0.75	
Machine-hour rate	0.45	
Return	0.45	$ 1.65
Direct labor		4.00
Machine-hour rate		2.40
Return		2.40
Total		$10.45

Figure 17.11 is drawn for these costs. A unit-cost curve is also drawn at the top.

The unit-cost figures at several quantities of production are

Quantity	Unit cost
500	$0.00418
1,000	0.00253
2,000	0.00170
4,000	0.00129
6,000	0.00115
8,000	0.00108
10,000	0.00104

Material cost is not included in the chart because it is the same for both presses. The total cost of 10,000 sheets printed on Press No. 2, including direct material, would then be

Cost including interest, setup labor, etc.	$10.45
Cost for direct material	10.00
Total	$20.45

This figure represents the total cost of all of the factors of production and includes the return on the investment. The job could therefore be sold for this figure if there was no additional cost for selling and administration. The yield would be the return rate allowed on the investment as calculated, or 12 per cent.

The total cost, not including return on the investment, would be

Complete total cost	$20.45
Interest	2.40
	$18.05

17.11. Comparison of the Two Presses.—There are three methods by which the operation of the two presses may be compared.

First Method. A Break-even Chart (Fig. 17.12).—The charts prepared previously for Press No. 1 and Press No. 2 may be superimposed, one on the other. Before this is done, the chart for Press No. 2 must be redrawn to correspond with the scale of the chart for Press No. 1. It will be remembered that both charts are drawn with the abscissa as a time scale. Because of the decreased time for printing a sheet on Press No. 2, the production will be greater per 8-hr. day. Therefore, the new scale should be one of production quantities. When this is done, the third chart is obtained. It will be observed that the total-cost-including-return lines cross each other at 3,750 sheets. This is the break-even point. It will also be observed that the unit-cost curves cross at the same point, as they should.

The significance of this chart is that for all jobs requiring less than 3,750 sheets to be printed, Press No. 1 gives the smaller cost per sheet. For all jobs above 3,750 sheets, Press No. 2 gives the smaller cost.

Second Method. Comparison of Unit Costs.—These units costs have been previously calculated. They are here repeated.

Quantity	Unit cost, press No. 1	Unit cost, press No. 2
500	$0.0034	$0.00418
1,000	0.0022	0.00235
2,000	0.0016	0.00170
4,000	0.0013	0.00129
6,000	0.0012	0.00115
8,000	0.00115	0.00108

It will be observed that for a job of 2,000 sheets, Press No. 1 will cost less; for a job of 4,000 sheets, Press No. 2 will cost less. The crossed lines in the accompanying table show that the break-even point lies between 2,000 and 4,000 sheets. This same solution may be noted on the chart at the intersection of the unit-cost curves for both presses.

Third Method. Comparison of Job Cost.—The original order was for 6,000 sheets. The characteristics of the presses have already been charted. It is seen that the total cost, including return on the investment but not materials, consists of two parts, a fixed cost and a variable cost.

Time for processing 6,000 sheets on the presses is

Press No. 1:	Press No. 2:
0.001 × 6,000 = 6 hr.	0.0008 × 6,000 = 4.8 hr.

The following is the total cost:

Item	Press No. 1	Press No. 2
Fixed cost	$1.20	$1.65
Direct labor	3.00 (6 hr.)	2.40 (4.8 hr.)
Return	1.50 (6 hr.)	1.44 (4.8 hr.)
Machine-hour rate	1.50 (6 hr.)	1.44 (4.8 hr.)
Totals	$7.20	$6.93

It will be observed that the smaller cost is incurred if the job is done on Press No. 2. This same fact could be observed from the break-even chart at the 6,000-sheet production ordinate.

Problems

A. Data for a commercial hotel have been given in Prob. *A* in Chaps. 8, 9, 10, and 11.

a. Make a statistical analysis.
b. Draw a break-even chart.
c. Determine the break-even point for no gain–no loss.
d. Determine the break-even point for expected return.
e. What should the selling price be to give the break-even point for no gain–no loss at 50 per cent capacity?
f. What should the selling price be to give expected return at 75 per cent capacity?
g. If the selling price is fixed at $4.50 per room per night, how much could variable operating costs be increased to give a break-even point of expected return at 60 per cent capacity?
h. If the selling price is fixed at $3.50 per room per night, how much must fixed operating costs be reduced to give a break-even point of expected return at 60 per cent capacity?
i. If variable operating costs increase by $3,000 at 50 per cent capacity, what is the effect upon the break-even points for no gain–no loss and expected return.

B. Data for a central mix concrete plant have been given in Prob. *B* for Chaps. 8, 9, 10, and 11.

a. Make a statistical analysis. Extend the operations data from 150 to 200 days.
b. Draw a break-even chart.

c. Determine the break-even point for no gain–no loss.

d. Determine the break-even point for expected return.

e. What should the selling price be to give expected return at 150 days?

f. What should the selling price be to give break-even point for no gain–no loss at 100 days?

g. If the break-even point is at 100 days for no gain–no loss, where is the break-even point for earning expected return?

h. If the selling price is fixed at $9 per cubic yard, how much must variable operating costs, per cubic yard, be reduced to give a break-even point of no gain–no loss at 100 days?

i. If the selling price is fixed at $9 per cubic yard, how much must the fixed operating costs be reduced per cubic yard to give a break-even point of no gain–no loss at 100 days?

j. If variable operating costs increase by $2,000 at 150 days, what is the effect upon the break-even point for no gain–no loss and for the expected return point?

C. Data for a power plant having uniflow engines as a prime mover have been given in Prob. *C* for Chaps. 8, 9, 10, and 11.

a. Make a statistical analysis.

b. Draw a break-even chart.

c. Determine the break-even point for no gain–no loss.

d. Determine the break-even point for expected return.

e. What should the selling price be to give the break-even point for expected return at 60 per cent capacity?

f. If the price of coal should increase to $5 per ton, what would be the effect on the break-even points for no gain–no loss and expected return?

g. If the fixed operating costs decrease $5,000, what would be the effect on the break-even points for no gain–no loss and expected return?

D. Data for a plant to manufacture milk-bottle caps have been given in Prob. *D* for Chaps. 8, 9, 10, and 11.

a. Make a statistical analysis.

b. Draw a break-even chart.

c. Determine the break-even point for no gain–no loss.

d. Determine the break-even point for expected return.

e. What should the selling price be to give expected return at 40 per cent capacity?

f. If the selling price should drop to $1.70 per 1,000 caps, what would be the effect on the break-even points of no gain–no loss and expected return?

g. If material costs increase $45,000 per year at 60 per cent capacity, what effect would this have on the break-even points for no gain–no loss and expected return?

E. Data for a cold-storage locker plant have been given in Prob. *E* for Chaps. 8, 9, 10, and 11.

a. Make a statistical analysis.

b. Draw a break-even chart.

c. Determine the break-even point for no gain–no loss.
d. Determine the break-even point for expected return.
e. If the selling price per year were changed to $25, what would be the effect on the break-even points for no gain–no loss and expected return?
f. If the fixed operating costs increased to $6,000, what would be the effect?
g. What would be the allowable fixed labor charge in order to operate the plant at 75 per cent capacity and make expected return?

1. In developing an ore deposit, two methods of mining are available. One will recover 80 tons for every 100 tons in the deposit and will cost $2.10 per ton recovered. The other will recover 88 tons for every 100 tons and will cost $2.30 per ton recovered. What is the break-even point in value of the recovered ore below which the low recovery method should be used and above which the high recovery method should be used? (From Examination for Professional Engineer's license, New York State, June 26, 1935.)

2. A shoe manufacturer produces a pair of shoes at a labor cost of 90 cents a pair and a material cost of 80 cents a pair. The fixed charges on the business are $90,000 a month and the variable costs are 40 cents per pair. If the shoes sell for $3 a pair, how many pairs must be produced each month for the manufacturer to break even? (From Examination for Professional Engineer's license, New York State, June 28, 1933.)

3. A manufacturer is selling chairs for $5 each, with an unlimited demand. The first year he turns out 100,000 at an average cost of $4 each. The second year he speeds up his factory and makes 150,000 chairs, but finds that his average cost has jumped to $4.20 each. The third year with 200,000 chairs, the average cost has risen to $4.50 each and the fourth year, with a production of 200,000 chairs, the average cost reaches the selling price of $5. When did the manufacturer begin to lose money by his speeding-up process, and why? (From examination for Professional Engineer's license, New York State, Feb. 3, 1937.)

4. Snow removal from city streets is accomplished either by hand loading into trucks or by machine loading into trucks. For hand loading a certain depth of snowfall will requie 100 men at $5 per day. Hand loading will also require shovel cost of $5 per year per man and $50 per year for shovel storage. Maintenance of shovel storage will amount to $25 yer year. For machine loading there will be required a machine whose first cost, including garage storage, will equal $10,000. Labor to operate the machine will be 7 operators at $7 per day. Fuel, oil, and maintenance will amount to $28 per day. Garage maintenance will amount to $100 per year. Disregarding return on the investment, draw the operating-cost curves for the two methods. Machine life estimated at 5 years. Assume 20 days operation.

CHAPTER 18

ECONOMIC CHARACTERISTICS OF POWER-GENERATING AND POWER-USING EQUIPMENT

18.1. Operating-cost Curve.—It was shown in Chap. 9 that there was a type of operating-cost curve in which unit operating costs decreased to a certain point and then increased. It was also described as a curve of diminishing returns. The curve of unit operating costs for power-generating and power-using equipment follows this diminishing-returns curve.

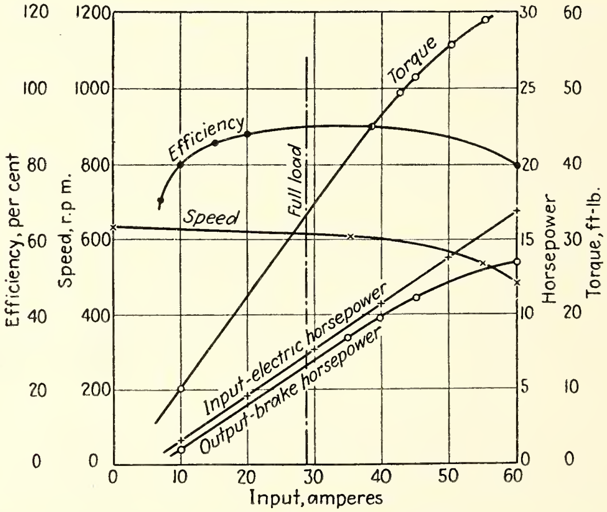

FIG. 18.1.—Performance characteristics of a 220-volt direct-current shunt motor.

The curve diagram in Fig. 18.1 is that of a direct-current shunt motor.

The data for constructing a cost curve (Fig. 18.2) are as follows:

First cost: $140
Estimated life: 10 years
Maintenance costs: $5 per year

Return on investment: 10 per cent
Electric power: 5 cents per kilowatt-hour
Motor to be operated 300 days per year, 8 hr. per day

Fixed operating costs:

Depreciation	$14
Return on investment	14
Maintenance, taxes	5
Total	$33

$$\text{Operating cost per day} = \frac{\$33}{300} = \$0.11$$

$$\text{Operating cost per hour} = \frac{\$0.11}{8} = \$0.013$$

The cost of electric power per hour is found from the equation;

$$\frac{\text{(Hours)(amperes)(voltage)(\$.05 per kilowatt-hour)}}{1{,}000} = \frac{(1)(7)(220)(\$0.05)}{1{,}000} = \$0.077 \text{ per hour}$$

Amperes	Watts	Horsepower-hour input	Efficiency, per cent	Horsepower-hour output	Power cost per hour	Fixed cost per hour	Total cost per hour	Total cost per horsepower-hour, output
7	1,540	2.06	70	1.44	$0.077	$0.013	$0.090	$0.0625
10	2,200	2.95	80	2.32	0.110	0.013	0.123	0.0530
20	4,400	5.90	89	5.25	0.220	0.013	0.233	0.0444
30	6,600	8.85	90	7.96	0.330	0.013	0.343	0.0431
40	8,800	11.80	89	10.50	0.440	0.013	0.453	0.0439
50	11,000	14.75	87	12.83	0.550	0.013	0.563	0.0439
60	13,200	17.70	81	14.31	0.660	0.013	0.673	0.0470

Again, as in the case of the automobile, the best economy is had when the apparatus is operated at or near full load.

It is to be observed in this case that the range of best economy lies between about 6 and 10 hp. of output. Full load of 8-hp. output shows a minimum operating cost of $0.0431 per horsepower of output. Power equipment is usually so designed that the best economy will be had at full load and to a slightly lesser degree for moderate variations above and below it. It will be observed from the curve of unit operating costs plus expected return on investment that as the motor is operated at outputs considerably more or less than full or rated load, the unit operating costs will increase. This is an illustration of diminishing returns. Power apparatus should, therefore, be operated at as near full load as is possible. If smaller loads are to be provided for over long periods of time, the motors should be selected with these principles in mind.

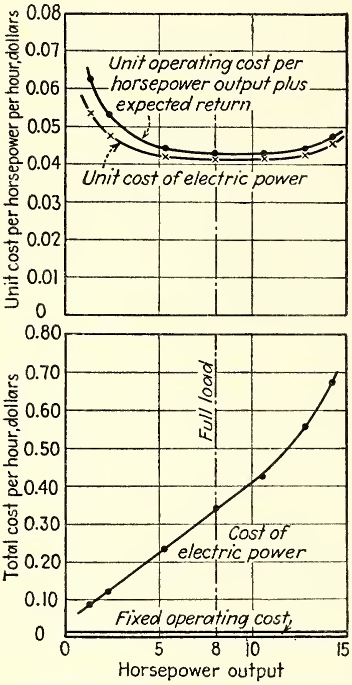

Fig. 18.2.—Operating-cost chart for a 220-volt direct-current shunt motor.

These principles are especially true for motors of small horsepower. As the motor becomes large, the extent of the flat or minimum part of the curve increases the range somewhat, so that under- or overloads may be carried with good economy.

18.2. Capacity Factor or Plant Factor.—Capacity factor or plant factor may be defined as the ratio between average load and the total rated capacity of the apparatus, which is the optimum load.

$$\text{Plant factor} = \frac{\text{average load}}{\text{total rated capacity}}$$

When the motor or engine is operated at rates on either side of the rated capacity, the unit operating cost will increase. This is because of the two inherent characteristics of power apparatus which were shown in the analysis of the motor in Sec. 18.1. One is the characteristic that when the apparatus is operated at less than rated capacity, the fixed operating cost is distributed over fewer units of output thus increasing the unit cost of the output. The other characteristic is that when power apparatus is operated either above or below the rated capacity, a greater quantity of power input is required for unit of output than at rated capacity.

This factor may also be applied to the operation of industrial plants and even to individual machines The development of the operating-cost patterns utilized this idea when the per cent of capacity was used as a measure of production or use.

18.3. Load Factor.—Load factor may be defined as the ratio between the average load and the maximum demand of a power system.

$$\text{Load factor} = \frac{\text{average load}}{\text{maximum demand}}$$

Maximum demand is defined as the maximum load that occurs in an interval of time during a given period. The interval of time may be instantaneous, or for 15, 30, or 60 min. The period may be the day, week, month, or year.

To have the best economic results this factor should, like that of plant factor, be around 100 per cent or at least on the flat portion of the unit operating-cost curve.

In the case of the automobile, the average load may be assumed to be that required for normal driving conditions. The maximum demand is called upon when the driver of the car wishes to pass someone or when accelerating very rapidly in order to escape some impending danger; it is the sudden calling on the machine to produce power at a considerably higher rate than ordinary.

The electric motor is used under somewhat similar conditions. If it is driving a machine tool, the average load is that load necessary to perform the usual cutting requirements. Occasionally a harder place in the metal to be cut is encountered. This calls for a sudden increase in the power required, corresponding to a maximum demand.

A similar situation exists with regard to a powerhouse. Assume that under ordinary conditions, at any period of the day, the demand for power may be considered the average load. Some customer is apt suddenly to apply additional loads to which the powerhouse must respond.

In each of the above cases, the apparatus in question is being operated under conditions that call for an average load during most of the time and for exceptionally higher loads at other times. If the apparatus has been selected to operate in accordance with the principle discussed in Sec. 18.2, it is being operated at or near its rated load. Any additional demand will overload the apparatus. If this sudden application is not too great and is not applied for a great length of time, the machine may have enough overload capacity to take care of the increased demand. If, however, the maximum load is too great and if it is to be applied for too long a time, a machine must be selected that will permit that usage. This may make it necessary to install a system with a low load factor.

It is evident, therefore, that as the ratio decreases from a value of one to a lesser value, a correspondingly larger sized machine must be provided. This means a larger investment and leads to a larger fixed operating cost per unit of output. When coupled with the fact that the machine is being operated for most of the time at considerably below rated capacity, except when the demand is on, a considerable increase in unit cost will be expected.

This condition may be remedied if the condition expressed by the next factor can be made to exist.

18.4. Diversity Factor.—This factor may be defined as the ratio between the sum of the maximum demands of the separate parts of the system and the maximum demand of the whole system.

$$\text{Diversity factor} = \frac{\text{sum of maximum demands of parts of system}}{\text{maximum demand of whole system}}$$

In effect it means that diversity exists when the respective demands for power of the several parts of the system do not come upon the single source at the same time.

In the case of the automobile there can be no application of the diversity factor principle. The maximum demand is made by one machine on one source of power and is therefore not divided.

In the case of a motor that is connected to several machines, however, there can be diversity factor. The motor is driving several machines at one time. It may happen that the maximum demands of the several machines do not occur at the same instant.

With the maximum demands from the several machines occurring at different times, the load is maintained for longer periods on the motor and the load factor thereby is higher. A powerhouse is similar. Maximum demands are made by several areas on the one source. But because of diversity, a powerhouse of smaller capacity can be used than if the maximum demands all came at the same time. Thus the load factor is made high and the unit cost of current becomes lower.

18.5. Demand Factor.—This factor may be defined as the ratio between the maximum power demand and the total connected load of the system.

$$\text{Demand factor} = \frac{\text{maximum power demand}}{\text{total connected load}}$$

It is a measure of the requirements that a power system might be called upon to supply if the entire connected load were to be used at one time. Obviously, the nearer the maximum demand is to the total connected load, the smaller will be the power plant necessary to supply the power. Low demand factor, therefore, necessitates that the power company charge the customer for the extra size of the power plant necessary to meet the maximum demand, in order to pay the fixed charges.

An overmotored factory is an example of poor demand factor. The motor driving a group of machines may have a horsepower rating equal to the sum of the horsepowers of the individual machines at maximum capacity. This is the total connected load. The work being done on the machines, however, may be such that the maximum power demands from the individual machines is low. The net result is a low demand factor. Too much motor is being supplied for the actual demand. A smaller sized motor would suffice and would produce a saving on first cost and unit operating costs.

18.6. Power Factor.—Power factor is the ratio between the true power and the apparent power. It is defined as the ratio between kilowatts per phase as measured by a wattmeter and kilovolt-amperes per phase as measured by an ammeter and voltmeter separately with the readings multiplied together. It is applicable only to electrical apparatus using alternating current, since the power factor for direct current is always 100 per cent, or 1.

$$\text{Power factor} = \frac{\text{kilowatts}}{\text{kilovolt-amperes}}$$

It is characteristic of alternating-current motors that the product of the values of voltage and current at any one instant is less than the product of the voltage and current in the electrical circuit. The first product is kilowatts and the second is kilovolt-amperes. The first product is true power; the second product is apparent power. But the generating equipment must be capable of producing power having the required characteristics of voltage and current. The higher the power factor, the closer does the true power approach the apparent power. This may be accomplished without increasing the size of the generating apparatus. Thus, increasing the power factor makes it possible to provide more power with the same apparatus. On the other hand, if the power factor decreases, then the generating apparatus is taking care of less power. If an induction motor is rated at 800 kw. and its kilovolt-ampere rating is 1,000, then its power factor is

$$\text{Power factor} = \frac{\text{kilowatt rating}}{\text{kilovolt-ampere rating}} = \frac{800}{1{,}000} = 0.80$$

This means that for every 800 kw. used by the motor, the powerhouse must have 1,000 kva. of power-generating equipment. If the power factor can be raised to approach 1, then the powerhouse will not have to provide so much equipment.

The effect of the power factor is, therefore, to increase the first cost of the powerhouse installation. Consequently, power plants must charge the customer for the added fixed expense brought about by this extra investment.

Power-factor correction may be accomplished by the installation of the proper equipment. Means for correcting power factor are synchronous and unity power-factor motors, capacitors, static condensers, and synchronous condensers. The net result of such installation is a reduction in the bill for service to the extent of the fixed operating cost for the extra equipment that is not necessary. Very often the total of the operating costs plus expected return on the investment in corrective apparatus is less than the cost of poor power factor. If so, it is worth while to install this equipment.

Problems

1. The XYZ Power Company has a power rate of $1 per month per installed horsepower plus an energy charge of 2.35 cents per kilowatt-hour. Assuming a 30-day month, find the cost per kilowatt-hour for the power users with a 5, 10, 25, 50 and 100 per cent capacity factor respectively. (From examination for Professional Engineer's license, New York State, July 8, 1937.)

2. A manufacturing plant has a motor installation as follows:

10 motors	5 hp. each
5 motors	10 hp. each
5 motors	20 hp. each

These motors are operated continuously at 75 per cent load over a period of 8 hr. daily for 26 days during the month. The efficiency at this load is 85 per cent. The power bill for this month is $410.75 with no demand charge included. What is the cost of electrical energy per kilowatt-hour? (From examination for Professional Engineer's license, New York State, June 24, 1936.)

3. A $7\frac{1}{2}$-hp., 110-volt, 1,400 r.p.m. motor operates at 75 per cent load over a period of 8 hr. daily, 6 days per week. The efficiency of the motor at this load is 85 per cent. At 4 cents per kilowatt-hour, what is the weekly cost of the energy consumed by the motor? (From examination for Professional Engineer's license, New York State, June 28, 1933.)

4. An electric household refrigerator is fully equipped with a $\frac{1}{6}$-hp. motor which operates 30 per cent of the time. The efficiency of the motor is 50 per cent. What is the energy cost to operate the refrigerator 30 days at 3 cents per kilowatt-hour? (From examination for Professional Engineer's license, New York State, June 26, 1935.)

5. An industrial plant has an installation of electrical motors which have an electrical load of 611-kva. demand corresponding to 455-kw. load at 74.5 per cent power factor. The load is on for 16 hr. per day for 300 days per year.

The monthly charges for electrical service by the power company are
Demand charge:
  First 100 kva. at $3 per kilovolt-ampere
  All over 100 kva. at $1.50 per kilovolt-ampere
Energy charge:
  First 200 kw.-hr. at $0.007
  All over 200 kw.-hr. at $0.005

It is proposed to reduce the power bill by installing a 210-kva. capacitor. The installation of this machine will result in a reduction of the demand load to 489 kva. The machine, however, will have a power loss of 0.76 kw., for 16 hr. per day for 300 days per year. The cost of this power is at $0.005 per kilowatt-hour.

The total installed cost of the capacitor is $2,313. Fixed charges consisting of depreciation, taxes, and insurance amount to 9 per cent of first cost. What will be the comparative cost of electric power if the new equipment should earn an expected return of 12 per cent?

6. An industrial plant has an installation of alternating-current motors which are being operated at a billing demand of 364 kw. at a power factor of 67.7 per cent. The current used has been a total of 83,040 kw.-hr. per month. The first cost of present installation is $3,500 having an estimated life of 15 years.

The monthly charge for power is in two parts:

Demand charge:
  First 50 kw. at $2 per kilowatt
  All over 50 kw. at $1 per kilowatt
Energy charge:
  First 40,000 kw.-hr. at $0.013 per kilowatt-hour
  All over 40,000 kw.-hr. at $0.009 per kilowatt-hour

It is proposed to reduce the power bill by installing a 150-hp. 220-volt synchronous motor to take the place of a 150-hp. squirrel-cage motor. This would cost a total of $3,970, installed, life estimated at 12 years.

The effect would be that the power factor would be corrected to an average of 94.8 per cent and a reduction in billing demand to 288 kw.

Operating costs for the synchronous motor will be the same as for the squirrel-cage motor which it replaces.

What is the difference in monthly bills before and after the installation of the synchronous motor? Expected return to be 10 per cent.

7. An industrial plant uses 30,000 kw.-hr. of power monthly and has a power factor of 50 per cent.

The billing for this power is done as follows: the energy charge at 100 per cent power factor is 30,000 kw.-hr. at $0.02 per kilowatt-hour and equals $600. Corrected for 50 per cent power factor, the billing would be

$$(\$600)\left(\frac{80 \text{ per cent power factor}}{50 \text{ per cent power factor}}\right) = \$960$$

It is proposed to install a 148-kva. capacitor at a total cost of $1,957. Depreciation, taxes, and insurance will be at 10 per cent of first cost. Power loss will amount to 0.25 kw. The apparatus is operated 200 hr. per month. This installation will reduce the power factor to 80 per cent, at which rate the power cost will be

$$(\$600)\left(\frac{80 \text{ per cent power factor}}{80 \text{ per cent power factor}}\right) = \$600$$

What will be the cost of power before and after the application of the power factor correction apparatus? Expected return 10 per cent.

CHAPTER 19

THE INCREMENT COST PRINCIPLE

19.1. Increment Cost.—Increment cost is the cost difference, either first cost or operating cost, that is incurred in the extension of the scope of a project beyond its original conception. The economy of this extension should be judged on the basis of the parts of the first cost, operating cost, and income which are directly attributable to the extension, and not on these factors for the entire extended project. This is logical because the extension is the thing that causes the increases in first cost and operating cost and from which additional income may be derived.

Another way to look upon increment cost is to say that it is the "out-of-pocket" cost brought about by the thing which extends the scope of the project.

An economy study for an increment cost situation is no different from the usual economy study except that the analysis is made on the several factors as they are influenced by the change in scope.

An addition to the scope of a project will generally entail an increased first cost and investment. Operating cost will be increased. Income from the new project is expected to increase. Obviously the return will be the difference between the increment operating cost and the increment income brought about by the change. This return is a return on the increment investment and is often spoken of as "return on extra investment."

19.2. A Home-owner's Problem.—Mr. Johnson owns a home in a certain town where the rates for electricity are as follows:

1. First 25 kw.-hr. at $0.07 per kilowatt-hour
2. Next 125 kw.-hr. at $0.03 per kilowatt-hour
3. Next 100 kw.-hr. at $0.0175 per kilowatt-hour
4. All above 250 kw.-hr. at $0.01 per kilowatt-hour

At the present time he uses electricity for the following pieces of equipment in his house:

1. Lights

2. Radio
3. Vacuum sweeper
4. Clothes washer
5. Refrigerator

His consumption of electricity during an average month for all of the above equipment is 110 kw.-hr. This requires a payment in accordance with the above rates as follows:

1. First 25 kw.-hr. at $0.07	$1.75
2. Next 85 kw.-hr. at $0.03	2.55
Total, 110 kw.-hr	$4.30

This is an average cost per kilowatt-hour of $4.30 ÷ 110 = $0.039.

A salesman of electrical apparatus wishes to sell him an electric stove. The salesman tells him that it will use on the average about 120 kw.-hr. per month for a family of his size. Mr. Johnson talks the matter over with a friend who shows him a method of figuring just what the cost of operating the stove would be.

Mr. Johnson reports that his bill was on the average $4.30 for 110 kw.-hr., or an average of $0.039 per kilowatt-hour. His friend reasons that, if the stove uses 120 kw.-hr. at the rate of $0.039 it will cost 120 × $0.039, or $4.68.

Mr. Johnson disagrees with his friend's method and suggests this method:

Total kilowatt-hours consumed equals 110 for the present equipment plus 120 for the stove, or a total of 230 kw.-hr.

1. First 25 kw.-hr. at $0.07	$1.75
2. Next 125 kw.-hr. at $0.03	3.75
3. Next 80 kw.-hr. at $0.0175	1.40
Total, 230 kw.-hr	$6.90

Average cost per kilowatt-hour = $6.90 ÷ 230 = $0.03

Therefore the cost for operating the stove is (120) ($0.03) or $3.60 per month.

The salesman, using some of Mr. Johnson's figures, makes the following calculations:

Total cost of 230 kw.-hr. for present equipment and stove	$6.90
Total cost of 110 kw.-hr. for the present equipment	4.30
Increment cost (cost of stove)	$2.60

Average cost per kilowatt-hour = $2.60 ÷ 120 = $0.02.

Thus there are three solutions to Mr. Johnson's problem. An examination of each shows the following:

His Friend's Solution.—If this solution is correct, Mr. Johnson's total bill should be

Amount for present equipment	$4.30
Amount for stove as calculated	4.68
Total	$8.98

This is in error because, when the total bill is properly calculated according to the rate schedule, the bill is $6.90.

Mr. Johnson's Solution.—If this solution is correct, Mr. Johnson's total bill should be

Amount for present equipment	$4.30
Amount for stove as calculated	3.60
Total	$7.90

This is likewise in error, since the bill is actually $6.90.

The Salesman's Solution.—This solution must be correct because

Amount for present equipment	$4.30
Amount for stove as calculated	2.60
Total	$6.90

The total bill, $6.90, is therefore correct. Notice that in this solution Mr. Johnson, pays only the additional cost of electricity for the operation of the stove. If he decided not to buy and install the stove, he will still pay $4.30 for the electricity to operate the present equipment. This is known as the principle of increment costs.

Several months later, the salesman visits Mr. Johnson to try to sell him an electric hot-water heater. He tells Mr. Johnson that the heater will use on the average 150 kw.-hr. per month and in view of his present average bill, this piece of equipment would cost $1.65 to operate. Mr. Johnson makes the following calculation in order to verify the salesman's estimates:

1. First 25 kw.-hr. at $0.07 per kilowatt-hour	$1.75
2. Next 125 kw.-hr. at $0.03 per kilowatt-hour	3.75
3. Next 100 kw.-hr. at $0.0175 per kilowatt-hour	1.75
4. Next 130 kw.-hr. at $0.01 per kilowatt-hour	1.30
	$8.55
Amount for present equipment including 120 kw.-hr. for stove, 230 kw.-hr	6.90
Increment	$1.65

Having applied the method which the salesman showed was the correct one, he calculated the total bill including the hot-water heater, and then subtracted from that total the previous cost of the other equipment up to and including the stove.

19.3. Other Applications.—A typical problem where this principle applies is in the isolated power plant. An industrial company is debating whether to install a power plant.

The company can build a complete plant for heat and power or it can build one for heat alone and buy the power from a public utility company. It is not sure that it will pay to build the complete power plant because the cost of electricity when purchased from the public utility is very favorable. The answer to this question may be determined by the application of the increment cost principle. What is wanted is the additional first cost required for generating power and the attendant operating cost.

There are two ways in which the estimate may be made. One is to estimate the cost of conversion of the power plant for steam load only to one for steam load and power load. The other method is to estimate the cost of separate power plants, one designed for steam load only and the other for steam load and power load. The difference between the estimated costs of the two plants will be the increment cost. The following estimate has been received:

1. Power plant for heating load:	
First cost	$150,000
Operating cost	60,000 per year
2. Power plant for heating and power load:	
First cost	$270,000
Operating cost	90,000 per year

Increment First Cost:

Power plant for heating and power	$270,000
Power plant for heating alone	150,000
Increment first cost for power equipment	$120,000

In other words, at an additional cost of $120,000 the power plant originally designed for the heating alone can be built to provide not only the heating load but also the power.

Increment Operating Cost:

Power plant for heating and power loads	$90,000
Power plant for heating alone	60,000
Increment operating cost for power equipment	$30,000

In other words, at an additional cost of $30,000 per year the power plant, originally designed for heating load, can be operated to provide not only the heating load but also the power load.

Increment Income.—Since, in this case, the company makes its own power and does not sell it, an income figure cannot be determined.

Increment Return.—In this case return, because of inability to determine income, would be equal to the percentage yield expected for this type of installation. Assuming a yield of 8 per cent, the return on increment first cost of $120,000 would be $9,600.

Operating Cost Plus Expected Return.—The return as a percentage yield on the investment having been determined, income will be equal to operating cost plus return.

Operating cost	$30,000
Return	9,600
Estimated income	$39,600

Unit Cost per Kilowatt-hour.—The power plant will provide 5,000,000 kw.-hr. per year. The unit cost of power will therefore be equal to

$$\frac{\text{Operating cost plus expected return}}{\text{Output}} = \frac{\$39{,}600}{5{,}000{,}000}$$
$$= \$0.00792, \text{ or } 0.79 \text{ cents per kilowatt-hour}$$

If the cost of power to be purchased outside is greater than this, it will pay the industrial plant to manufacture its own power.

Intangible Factors.—There are also judgment factors to be considered before a final decision is made. The company will be independent of a public utility power company. However, if a breakdown occurs, the plant may have to be shut down unless there is provided a connection with the power company. There will be a charge for such service which must be added to the operating cost of the private plant and may reduce the margin of gain between making and buying power. It will be necessary to maintain a much larger power organization than before.

Financial Analysis.—The principal problem is to find the money necessary to finance the changes in the project. The same principles will be used as those developed in Chap. 15.

19.4. Capacity and Increment Cost.—The Atlas Manufacturing Company is operating in accordance with the data given in Secs. 17.1 to 17.11, and illustrated in graphical form in Fig. 17.1.

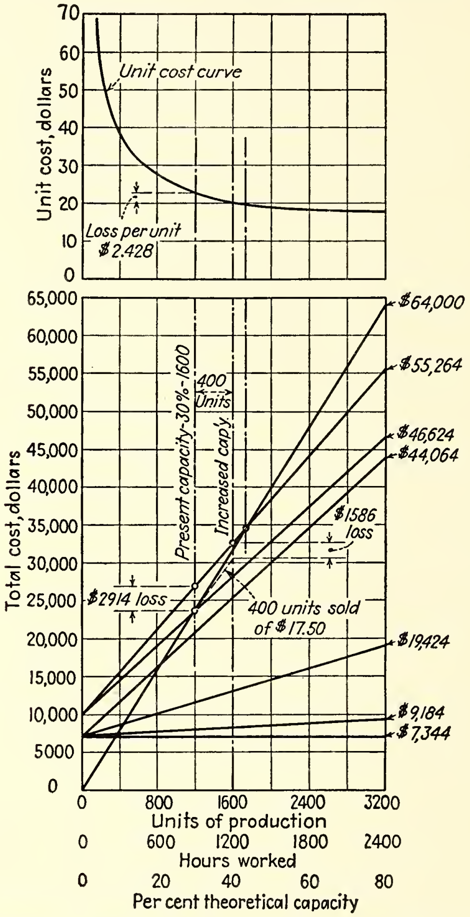

FIG. 19.1.—Break-even chart for an industrial enterprise. Increment-cost principle to reduce a loss.

So far, during the present year, the company has orders for 1,200 units of the product. This is at the 30 per cent capacity point, where the total cost line in Fig. 17.1, reproduced in Fig. 19.1, is above the total income from sales line. This shows a loss. The amount may be calculated from the original data as follows:

Item	30 Per Cent Capacity Point
Fixed production expense	$ 7,344
Variable expense	690
Direct labor	3,840
Direct material	9,240
Fixed sales expense	2,560
Variable expense	3,240
Total cost	$26,914

Unit cost = $26,914 ÷ 1,200 = $22.428

Since the article sells for $20, there is a loss of $22.428 − $20.00, or $2.428 per unit. For the total production, at this rate, the loss would be 1,200 × $2.428, or $2,913.60.

This loss may also be calculated by subtracting the cost of sales and production from the income.

Cost of production and sales	$26,914
Income (1,200 × $20)	24,000
Loss	$ 2,914

The sales manager does his best to get additional sales. He succeeds in getting a bid for 400 units more per year, provided he is allowed to sell them for $17.50 per unit.

This is a problem in "dumping." Here the company is considering the selling of additional units to a customer at a lower selling price than it is now receiving from a prior customer. Under what conditions should the company take the order?

Obviously, the company will not wish to take the order if it is going to increase the loss now being sustained. Consequently, it is asked, how much will it cost to produce the additional 400 units and will the $17.50 offered for them be sufficient to cover the cost or will the loss be increased?

How much will it cost to produce and sell 400 additional units? The factors that enter into this cost are

Fixed production expense
Direct material
Direct labor
Variable expense
Fixed sales expense
Variable sales expense

Since in the calculation for the present production of 1,200 units there was included all the above elements, the entire charge for fixed production expense and fixed sales expense will have been paid for by that order.

Therefore, the only items of cost to be considered now are the out-of-pocket costs of producing and selling the additional 400 units. These costs are

Direct material
Direct labor
Variable production expense
Variable sales expense

Since these are proportional to production, the costs for 400 units may be calculated as follows:

Variable expense $\left(\frac{400}{3,200}\right) \times \$1,840$	$ 230
Direct labor $\left(\frac{400}{3,200}\right) \times \$10,240$	1,280
Direct material $\left(\frac{400}{3,200}\right) \times \$24,640$	3,080
Variable sales expense $\left(\frac{400}{3,200}\right) \times \$8,640$	1,080
Total cost	$5,670

This cost figure is called the "increment cost" because it is the additional out-of-pocket cost necessary to provide and sell the additional 400 units. The unit cost would be $5,670 ÷ 400, or $14.18. Since the units were to be sold for $17.50 and the cost was $14.18, then the unit return will be 17.50 − $14.18, or $3.32. The total return or profit from this deal will be 400 × $3.32, or $1,328. The net effect of taking this order would be to reduce the loss of running the business from $2,914 by $1,328 to $1,586.

Before making a final decision, the company would survey the possible effect that this reduction in selling price would have on the market for their goods.

19.5. Return on Extra Investment.—The application of the increment cost idea may also be shown by another situation. In Chap. 12, the case of building an apartment house as an alternative to the housing development of Chap. 11 was discussed. The promoter having drawn his plans for a two-story 20-apartment building wonders what the return would be if he were to erect a three-story building with 27 apartments.

He estimates that the following are the additional costs that would affect his project if a third floor with seven apartments were erected:

First cost	$25,000.00	
Income		$5,880.00
Operating cost (including depreciation)		1,905.00

What is the effect on his investment of $25,000 additional in the project?

Return is the difference between income and operating cost, or $5,880 − $1,905.00 = $3,975.00.

The economic yield on his investment would be

$$\frac{\$3.975}{\$25,000} \times 100 \text{ per cent} = 15.9 \text{ per cent}$$

Since the promoter's yield on the original apartment house was 7.98 per cent, a yield for the additional floor of 15.89 per cent is very attractive to him.

Data for the enlarged apartment building would be equal to the sum of the original plus the addition, as follows:

Item	Original	Addition	Total
First cost	$110,000	$25,000	$135,000
Income	16,800	5,880	22,680
Operating cost	8,016	1,905	9,921
Return	8,784	3,975	12,759

Had the yield been figured on the entire enlarged building, it would have been

$$\frac{\$12,759}{\$135,000} \times 100 \text{ per cent} = 9.4 \text{ per cent}$$

Thus it is seen that the use of the principle of increment costs gives a better and truer picture of the effect of the investment in an additional floor to the apartment building. Obviously a new financial analysis must be made to cover the financing of the additional floor.

Problems

A. A commercial hotel, having the economic data given in Prob. *A* for Chaps. 8, 9, 10, and 11, estimates its yearly expenses as follows for various percentages of its capacity:

	0%	25%	50%	75%	100%
Taxes	$22,815	$ 22,815	$ 22,815	$ 22,815	$ 22,815
Depreciation	41,750	41,750	41,750	41,750	41,750
Fixed operating cost	25,000	25,000	25,000	25,000	25,000
Variable operating cost	....	15,625	31,250	46,875	62,500
Total	$89,565	$105,190	$120,815	$136,440	$152,065

If 100 per cent capacity represents 250 rooms occupied for 365 nights, what is the unit cost per room-night at each capacity figure?

What is the increment cost per room-night for the first 25 per cent capacity?

What is the increment cost for each additional 25 per cent capacity?

C. The yearly operating costs for a 1,500-kw. power plant having uniflow engines as a prime mover for the data given for Prob. *C* in Chaps. 8 to 11 have been estimated as follows:

	Per cent of capacity				
	25	50	75	100	125
Fuel	$ 29,105	$ 44,807	$ 57,159	$ 73,584	$100,849
Labor	11,580	12,660	12,660	12,660	13,200
Lubricants	500	550	600	650	730
Supplies	3,300	3,500	3,700	3,900	4,150
Variable maintenance	8,000	10,800	12,000	12,800	13,750
Depreciation	30,291	30,291	30,291	30,291	30,291
Taxes and insurance	15,345	15,345	15,345	15,345	15,345
Fixed maintenance	12,276	12,276	12,276	12,276	12,276
Totals	$110,397	$130,229	$144,031	$161,506	$190,591

If 100 per cent capacity represents 13,140,000 kw.-hr., what is the unit cost per kilowatt-hour at each capacity figure? What is the increment cost per kilowatt-hour at the first 25 per cent capacity point? What is the increment cost at each additional 25 per cent capacity point?

1. The residential rate for electric energy is as follows: first 25 kw.-hr. per month at $0.08 per kilowatt-hour; next 25 kw.-hr. at $0.04 per kilowatt-hour; next 100 kw.-hr. per month at $0.03 per kilowatt-hour; all additional electric energy at $0.02 per kilowatt-hour. Assume that your present monthly use of electric energy for lights and miscellaneous appliances is about 60 kw.-hr. You are contemplating the purchase of an electric range which will use about 100 kw.-hr. per month. For purposes of making a comparison of the economy of the electric range with that of some other type of stove, how much should you consider as its monthly cost for electric energy? (From Examination for Professional Engineer's license, New York State, June 27, 1934.)

2. If in the preceding question, after installing the electric range, it was proposed to install an electric hot-water heater, what would the monthly cost of energy be if the heater used 150 kw.-hr. per month?

3. A college has a heating load on the average of 20,950,000 lb. of steam per month. Its corresponding electrical power load is 160,000 kw.-hr.

The college is considering whether to build a power plant for heating only or a combined heating and power plant. The following estimates have been made for the operation of the two proposed plants:

	Heating plant	Heating and power plant
First cost:		
Land	$ 15,000	$ 18,000
Boiler plant	188,000	228,000
Power generating plant	....	100,000
Total first cost	$203,000	$346,000
Monthly operating cost:		
Fuel	$ 3,984.00	$ 4,704.00
Labor	550.40	1,037.40
Maintenance	477.25	783.50
Supplies	103.75	197.50

Fixed charges, depreciation, insurance, and taxes are 8 per cent of first cost.

If electric power can be purchased for $0.0216 per kilowatt-hour, should the college install the combined plant or install the heating plant and buy power?

NOTE: The college does not anticipate the necessity of earning a return on investment.

If the college were a private one and expected to earn a return of 5 per cent on its investment, what should it do, assuming that working capital of $8,000 is necessary?

4. The following data have been estimated for the erection and operation of a tall office building:

Height: 12 stories
Total investment: $15,900,000
Total income: $1,670,000
Total operating cost: $766,000

What are the return and yield on the investment?

It is proposed to enlarge the building by increasing the number of stories to 15, 20, 25, or 30. The following data have been estimated for these increases. The amounts are in thousands of dollars.

Height, stories	15	20	25	30
Total investment	$17,150	$19,100	$21,200	$23,250
Total income	2,010	2,450	2,900	3,151
Total operating cost	868	1,025	1,192	1,351

Determine the increment investment, income, operating cost, and return as well as the increment yield for each change in story height? Also determine the return and yield for each suggested story height.

CHAPTER 20

PROBLEM OF REPLACEMENT

20.1. The Problem.—After an industrial enterprise or project has been in operation for some time, a recurring problem for the executive is that of the replacement of apparatus and equipment and construction work. The problems of replacement may be classified as follows:

1. The replacement of a piece of equipment at the expiration of the estimated useful life of the equipment
2. The replacement of a piece of equipment before the expiration of the estimated useful life of the equipment
3. The replacement of manual labor by machinery

20.2. The Replacement Principle.—The replacement principle is that, when a new method, operation, or piece of equipment is being considered, its economy should be compared with the economy of the then existing actual value of the equipment rather than with its original cost. It looks upon the original transaction as a "sunk" proposition. The reasons that warranted the investment then have long since changed more or less. Therefore, when replacement is contemplated, all analyses should be of present actualities. This is logical because the new setup is in competition with the old setup as it might be installed in a competitor's factory, he having purchased the old setup in the secondhand market just at the time the change is being considered.

The competitor who purchases a piece of secondhand equipment that is just the same as the present installation has for his first cost the following:

Purchase price	$xxx
Transportation	xxx
Installation	xx
Total	$xxx

This is equivalent to the in-place value which was presented and discussed in Sec. 4.2. The purchase price is the same as the salvage value if the present equipment were sold at this time. A competitor buying it would still have to pay for transportation and installation before he could put it into operation. But his total investment is as indicated above. Hence, when compared with a new setup, the first-cost value to be used in the economy study for the old setup should be the in-place value.

An additional part of the replacement principle is that the unrecovered depreciation is not to be considered as a part of the first cost of the new equipment. This is logical since there is no connection between the procurement and operating costs of the new equipment and the old. The unrecovered depreciation results because of a poor estimate as to the life of the equipment. This principle is known as the "sunk" cost principle. It means, in effect, that those decisions which were made in the past should not affect present decisions. There still remains the difficulty arising from the unrecovered depreciation. This will be treated as a financial problem from the accountant's viewpoint.

20.3. Replacement at Expiration of Useful Estimated Life.—Two questions are raised when the age of a piece of equipment reaches the end of its estimated life, which is the one that is used in the calculation of the depreciation charge.

The first question asks if the equipment can still be operated satisfactorily. The second question asks if the equipment can still be operated satisfactorily after repairs have been made. Each of these situations is in competition with the proposal to replace the old equipment with new.

The solution of the problem lies in making the several analyses of the situation; namely, economy, financial, and intangible.

The situation in the Monarch Furniture Company may be used as an illustration.

Case I. *No Repairs Needed.*—There has been in the shop of the Monarch Furniture Company a double arbor saw bench. This machine consists of a table or bench under which is arranged an arbor fitted with a rip and a cutoff circular saw. By a suitable controller handle either saw may be rotated up through the table or bench so that either crosscutting or ripping may be done.

The saw was purchased 15 years ago. At that time, an economy study was made. Its installed cost was $550. The life of the machine was estimated to be 15 years. Its salvage value at the end of that time was estimated to be $70. The net depreciation in value would be $550 − $70 or $480. The depreciation charge was calculated by the straight-line method. The yearly charge was, therefore, $480 ÷ 15, or $32.

It is proposed to replace the machine with a new one. The estimated cost of purchasing, installing, and preparing the machine for operation is $700. The life of this machine is estimated to be 15 years and the salvage value at the end of that time is to be $100. The new machine is quite similar to the old one in general design. The principal difference is in the method of driving the new arbors. Instead of the belt drive as in the old one, the new machine has an electric motor attached directly to the arbor of the saw. There is to be expected, therefore, much better operation of the new machine. It is fitted like the old one with the necessary fences and guides. Since the general design of the new machine is a great improvement over that of the old one, it is to be expected that much more accurate work may be done on it. In addition, because of the better balancing of the rotating parts, brought about by the direct drive of the arbor, it is expected that better work will be done with greater speed and less waste.

A study of the operating costs charged against the old machine shows an average charge for repairs and upkeep of $100 per year. It is estimated that for the new machine similar charges will be about $95 per year on the average over its life. The space and power requirements for the new machine will be approximately equal to that used by the old one. There is also the information that the old machine will sell for $70, if disposed of now, which is the same as the estimated salvage value. It was also shown that the old machine could be used for 3 years more but must be replaced at the end of that period. After 3 years the salvage value is estimated at zero.

There are, therefore, two situations with regard to the replacement of this equipment:

1. The present machine may be kept for 3 years and then be replaced by a new one.
2. The present machine may be replaced immediately.

Accordingly, economy studies and financial studies must be made for both situations.

FIRST SITUATION: KEEP PRESENT MACHINE FOR THREE YEARS, THEN REPLACE.

OPERATING-COST ANALYSIS

	Old machine up to the present		Old machine for next 3 years	
Upkeep and repairs		$100		$100.00
Depreciation	($480 ÷ 15)	32	($70 ÷ 3)	23.33
Total		$132		$123.33

Thus it is shown that keeping the machine for 3 more years will reduce the yearly operating costs from $132 to $123.33, a saving of $8.67 per year.

SECOND SITUATION: REPLACE IMMEDIATELY.—Applying the replacement principle introduced in Sec. 20.2, the in-place value must be found for the old equipment. The present salvage value is $70. It is estimated that if this machine were bought secondhand, there would be charges of $15 for transportation and $25 for installation. The in-place value would, therefore, be

Salvage value	$ 70
Transportation charges	15
Installation charges	25
Total value	$110

ECONOMY ANALYSIS

Item	Old machine for next 3 years		New machine	
First cost			$700	
In-place value	$110			
Upkeep and repairs		$100.00		$ 95.00
Depreciation	($110 ÷ 3)	36.66	($600 ÷ 15)	40.00
Total		$136.66		$135.00
Expected 10 per cent return on investment		11.00		70.00
Total operating cost plus expected return on investment		$147.66		$205.00

Thus it is shown that keeping the machine for the next 3 years will entail less yearly operating cost plus expected return on the investment than replacing it immediately.

Case II. *Machine Needs Repairs.*—If the old machine, however, requires rebuilding or repairing to the extent of $175, what is the value, assuming that its rebuilt life will be 3 years?

The in-place value will now be

Salvage value	$ 70
Rebuilding value	175
Transportation charges	15
Installation charges	25
Total value	$285

ECONOMY ANALYSIS

	Old machine repaired		New machine	
First cost			$700	
In-place value	$285			
Operating cost:				
Upkeep and repairs		$100.00		$ 95.00
Depreciation	($285 ÷ 3)	95.00	($600 ÷ 15)	40.00
Total		$195.00		$135.00
Expected 10 per cent return on investment		28.50		70.00
Total operating cost plus expected return		$223.50		$205.00

Thus it is shown that under these conditions the new machine will entail less operating cost plus expected return than the old machine; hence, the old machine should be replaced immediately.

FINANCIAL ANALYSIS

Case I. First situation. Keep present machine for 3 years
There will be no cash outlay involved.

Case I. Second situation. Replace immediately

Cash on hand:

Balance[1] in reserve for depreciation account (15 × $32)	$480
Salvage value of old machine when sold	70
	$550

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[1] It is assumed that the depreciation charges over the life of the machine have been accumulated and are available. Very often in industrial practice this accumulation may have been used for other financial purposes and is not available as cash (see Sec. 3.7).

Total cash required	$700
Cash on hand	550
Cash required	$150

Thus it is seen that cash to the extent of $150 is required.

Case II. First situation. Repairs are needed.

There will be a cash outlay for rebuilding to the extent of $175 needed immediately.

These problems of financial matters are for the accountant and the treasurer to solve.

20.4. The Accountant's Problem.—His problem concerns the relationship between the cost of the machine and the charges for depreciation which he credits to the reserve for depreciation account. Each year a charge, in this case $32, is made against operating expense and applied to the reserve for depreciation account. If continued for the estimated life of the machine, which was 15 years, there would have been collected in this account $32 × 15 or $480. If to this sum is added the salvage value as originally estimated, when the machine is sold, namely, $70, the original cost of the machine, $550, will be met.

It is to be noted that the salvage value now is the same as was estimated, namely $70. Therefore, the accountant is satisfied with the proposition in Case I, in which the machine is kept or replaced. He notes, however, that if replaced, an additional $150 will have to be found. Since he is not responsible for matters of this sort, he passes the problem on to the treasurer.

He also notes that in Case II, when repairs are needed, an additional amount of $175 is needed. Since this is for rebuilding, he passes this on to the treasurer.

20.5. The Treasurer's Problem.—The treasurer must find out where to get the additional funds. In the case of buying a new machine it is $150, the increase in purchase price for new machines. In the other case, it is $175, the cost of rebuilding the old machine. He will examine the balance sheet of the enterprise in order to see if there are sufficient funds available.

Assume that the asset side of the balance sheet shows that there is sufficient cash and other quick or current assets over and above the required working capital. These assets may have been increased by keeping some of the profits or return in the business instead of paying them out in the form of dividends. On the credit side of the balance sheet these unpaid dividends are represented by the Surplus account. It, like the Reserve for Depreciation account, may be represented on the debit side of the balance sheet by either current assets such as cash, accounts receivable, notes receivable, materials inventory, or fixed assets such as buildings and equipment or machinery. This source is the first important one and the one to which the treasurer will turn first. A second possibility, especially when the amount required is large, is to sell more stock and thus acquire new capital or to borrow the money by means of a bond issue. Still another way, if the amount is small, is to make short loan arrangements with financiers or to buy on the installment plan.

20.6. The Intangible Analysis.—The intangible analysis will be concerned with the respective merits of the machines:

Item	Old machine for next 3 years	New machine
Construction of machine	Good	Excellent
Accuracy of work	Fair	Excellent
Speed of doing work	Average	Fast
Waste from width of saw cut	Much	Little

The intangible factors seem to favor the new machine over the old. It might not be difficult to show that they have a monetary value equal to or exceeding the difference in operating cost in Case I, second situation, of \$205.00 − \$147.66 or \$57.34 per year.

20.7. Replacement Prior to Expiration of Useful Estimated Life.—The replacement of equipment before the expiration of its useful estimated life may be because of either physical depreciation or functional depreciation. Physical depreciation may have been brought about by the wearing out of the elements of the machine which are subject to wear and tear. Lack of proper maintenance may be the cause of the physical breakdown. Functional depreciation may have been brought about by a change in design, change in materials, change in method of operation, or improvements in the equipment. In other words, obsolescence has occurred.

The question to be answered in the first case is whether equipment should be replaced by a like piece, or whether a new type of equipment should be used. New equipment may be compared with a new machine of the same type as the original, in order to determine whether to continue using the old method or to devise a new one.

The financial analysis will be concerned with the problem of taking care of the unrecovered depreciation and price increase, if any.

Example: Assume a machine, whose installed cost is $3,000, with an estimated life of 10 years and no salvage value at the end of that time. The machine is now 5 years old and its present salvage value is $500.

Using straight-line method for calculating depreciation,

Annual depreciation charge: ($3,000 − 0) ÷ 10 = $300

Accumulated depreciation reserve at 5 years: 5 × $300 = $1,500

Depreciated book value:	
Original cost	$3,000
Accumulated reserve	1,500
Depreciated book value	$1,500
Salvage value	500
Unrecovered depreciation	$1,000

It is this $1,000 which must be settled by some financial means.

As in the previous problem, the question is put up to the treasurer and the accountant. The treasurer says that the problem is not his because it is not the finding of new capital. Somebody made a poor guess about the life of the equipment, hence, it is a problem for the accountant.

The accountant admits that it is his problem and suggests the following solutions:

1. Charge the unrecovered amount to current profit and loss. It can be argued that this is a legitimate charge against profit and loss because it is in reality an additional charge against operating cost for obsolescence which could not be forecast. If this charge is more or less uniform, the results may be quite satisfactory. It does, however, reduce the net return for the period. It can be contended that in effect this charge should have been made in the past had somebody estimated life more accurately. From this viewpoint operating costs have been understated.

2. It would seem logical, then, to charge this amount to surplus. It can be contended that surplus is, in effect, undis-

tributed return or profit and arises at least in part because such charges as depreciation are understated and make return unduly great. Since surplus is a credit balance sheet item and is represented on the debit side of the balance sheet by other forms of assets, this method can be used quite logically.

20.8. Replacement of Equipment because of Obsolescence.—The question to be answered in the second case is whether or not the present equipment should be replaced by the proposed equipment. The solution of this problem lies in the application of the several basic analyses of the situation; namely, economic, financial, and intangible. The replacement principle must be applied in order to obtain the valuation of the present equipment.

Illustrative Problem: Certain equipment has been in operation in a manufacturing plant. The industrial engineers have been studying the product which this company produces in order to find better and cheaper ways of producing it.

One of the products is a special bolt made of machinery steel. In the past it has been made on the engine lathes. It is now proposed to make it on turret lathes. The industrial engineers have prepared the following analysis:

Present Practice.
Machines used: 2 engine lathes
Direct labor: 1 man per machine per 50-wk. year; $0.70 per hour for 2,000 hr.
Direct material: $0.10 per piece
Rate of production: 10 per hour, 40,000 pieces per year
Power cost: $0.01 per hour
Tools: $50 per machine per year
Indirect expenses consisting of taxes, heat, light, fire insurance: $0.50 per square foot
Repairs and upkeep: $100 per year per machine
Workmen's compensation, indirect labor, supervision: $1 per hour
First cost of equipment: $2,500 per machine
Estimated life made when purchased: 10 years
Present age: 6 years
Estimated salvage at end of 10 years: $300
Floor area required: 75 sq. ft.
Proposed Practice.
Machine used: 1 turret lathe
Direct labor: 1 man per machine, $0.60 per hour
Job setter and check-up man: (part time) $10 per 8-hr. week
Direct material: $0.09 per piece (less waste)

Rate of production: 20 per hour, 40,000 pieces per year
Power cost: \$0.01 per hour
Tools: \$150 per year
Indirect expenses consisting of taxes, heat, light, fire insurance: \$0.50 per square foot
Workmen's compensation, indirect labor, supervision: \$1.00 per hour
Repairs and upkeep: \$300 per year
First cost of equipment: \$6,000 per machine
Estimate of life made when purchased: 6 years
Estimated salvage value at end of estimated life: \$600
Floor area required: 100 sq. ft.

The usual economy analysis and financial analysis are to be made.

20.9. Economy Analysis.—As was explained in Sec. 20.2, the replacement principle requires that the economy analysis be made between the valuation of the present machine at its immediate value and the valuation of the new and proposed machine. The industrialist, however, has on hand the original economy analysis and will refer to it as a means of comparison even though he recognizes that it is a "sunk" analysis, made some time previously and not valid now. Hence the original analysis will be made in the succeeding paragraphs solely for reference purposes.

20.10. First Cost. *Present Practice. Original First Cost.*—The number of machines depends upon the rate of production. This will be equal to

$$\frac{\text{Production per year}}{(\text{Pieces per hour})(\text{hours per year})} = \frac{40{,}000}{(10)(2{,}000)} = 2$$

Since the first cost of one machine was \$2,500, the cost of two will be \$5,000.

Present Practice. In-place Valuation.—The valuation of the old machines is made in accordance with the replacement principle. Therefore, the in-place value must be determined. This is equal to salvage value at present age plus costs for transportation and installation.

Salvage value	\$600
Transportation	25
Installation	275
Total	\$900, or \$1,800 for two

Proposed Practice.—The number of machines will be

$$\frac{40{,}000}{(20)(2{,}000)} = 1$$

The first cost of 1 turret lathe is $6,000.

20.11. Operating Cost. *Present Practice.*

Direct labor: (No. machines)(no. hours per year)(labor rate per hour) = (2)(2,000)($0.70) = $2.800

Direct material: (No. pieces per year)(cost per piece) = (40,000)($0.10) = $4,000

Expenses:

Power. (No. machines)(no. hours per year)(cost per hour) = (2)(2,000) ($0.01) = $40

Tools: (No. machines)(cost per machine) = (2)($50) = $100

Indirect expenses: (No. machines)(square foot per machine)(cost per square foot) = (2)(75)($0.50) = $75

Workmen's compensation, etc.: (No. machines)(hours direct labor per year)(cost per hour) = (2)(2,000)($1) = $4,000

Repairs and upkeep: (No. machines)(cost per machine) = (2)($100) = $200

Depreciation (original first cost): $\frac{(2)(\$2{,}500) - (2)(\$300)}{10} = \$440$

Depreciation (replacement value): It is estimated that at present age of 6 years the machines will be good for 5 years more, and that there will be no salvage value at that item.

$$\text{Charge} = \frac{(2)(\$900) - (2)(0)}{5} = \$360$$

RECAPITULATION OF OPERATING COST

	Original first cost	In-place value
Direct labor	$ 2,800	$ 2,800
Direct material	4,000	4,000
Expense, power	40	40
Tools	100	100
Indirect expenses	75	75
Workmen's compensation, etc	4,000	4,000
Repairs and upkeep	200	200
Depreciation	440	360
Totals	$11,655	$11,575

Proposed Practice.

Direct labor = (1)(200)($0.60)	$1,200	
Job setter = (No. weeks)(cost per week) = (50)($10)	500	$1,700
Direct material = (40,000)($0.09)		3,600
Expenses:		
Power = (10)(2,000)($0.01)		20
Tools = (1)($150)		150
Indirect expenses = (1)(100)($0.50)		50
Workmen's compensation, etc. = (1)(2,000)($1)	$2,000	
Job setter = (1)(8)(50)($1)	400	2,400
Repairs and upkeep = (1)($300)		300
Depreciation = ($6,000 − $600) ÷ 6		900
Total operating cost		$9,120

20.12. Return Analysis.—Since there were no data by which income could be determined, no figure can be given for return in the usual way. Those who manage this particular business believe that the proper return for their investment should be on the order of 12 per cent.

On that basis, the return to be expected from the investment in the two methods would be

A. Present practice (first cost) = ($5,000)(0.12) = $600
B. Present practice (replacement) = ($1,800)(0.12) = 216
C. Proposed practice = ($6,000)(0.12) = 720

20.13. Income Analysis.—The expected income may now be determined. It is the sum of the operating cost and the expected return.

A. Present practice (first cost):	
Operating cost	$11,655
Expected return	600
Expected income	$12,255
B. Present practice (in-place):	
Operating cost	$11,575
Expected return	216
Expected income	$11,791
C. Proposed practice:	
Operating cost	$ 9,120
Expected return	720
Expected income	$ 9,840

The income in this case would be derived from the charges made against the sales department of the business as a cost per piece. Therefore, operating cost plus expected return per piece would be total cost plus expected return divided by yearly production.

A. Present practice (original cost):

$$\frac{\text{Total cost}}{\text{No. pieces per year}} = \frac{\$12{,}255}{40{,}000} = \$0.3064 \text{ per piece}$$

B. Present practice (in-place value):

$$\frac{\$11{,}791}{40{,}000} = \$0.2947 \text{ per piece}$$

C. Proposed practice:

$$\frac{\$9{,}840}{40{,}000} = \$0.2460 \text{ per piece}$$

A recapitulation of the three analyses will be useful for purposes of comparison.

Item	Present, original	Present, in-place	Proposed replacement
First cost	$ 5,000	$ 1,800	$6,000
Income	Not determinable		
Operating cost:			
Direct labor	$ 2,800	$ 2,800	$1,700
Direct material	4,000	4,000	3,600
Power	40	40	20
Tools	100	100	150
Indirect expense	75	75	50
Workmen's compensation, supervision, etc	4,000	4,000	2,400
Repairs	200	200	300
Depreciation	440	360	900
Total operating cost	$11,655	$11,575	$9,120
Expected yield	12%	12%	12%
Expected return	$ 600	$ 216	$ 720
Total operating cost plus expected return	12,255	11,791	9,840
Unit operating cost including expected return	0.3064	0.2947	0.2460

Thus it is seen that the operating cost for the proposed method is lower than the present method valued at original first cost and the present method valued at in-place value.

20.14. Intangible Analysis.—The following factors of an intangible nature must be considered before a final conclusion is reached:

Purpose of the Machines.

Engine Lathes.—These machines are what are called "general-purpose" machines. This means that the work done on them is general in scope and not narrowed down to one special type of work. In general, operations which may be performed on these machines are turning, boring, thread cutting, chucking, and facing. The size of the work is limited by the length of the bed and the diameter that will swing across the ways.

Turret Lathes.—These machines are what are called "special-purpose" machines. This means that the work done on them is special in scope and narrowed down to special types of work. In general, operations that may be done by the machine under consideration are turning, boring, thread cutting, and facing. The size of work is limited to the size of bar that may be fed into the machine.

Comment.—Thus it is seen that the new machine has a restricted use. If there is not sufficient work of the bar type, such as bolts, bars, spindles, etc., then the machine may lie idle for long periods of time. It will limit the kind of work that the company may accept. On the other hand, because of its specialization and because of the possibility of its being able to reduce cost of manufacture, new sales opportunities may be opened.

Type and Quality of Work.

Engine Lathes.—The type of work has been discussed under Purpose of the Machines. The quality of the work depends largely upon the workman or mechanic who operates the machine.

Turret Lathe.—The type of work has already been discussed. The quality of the work depends upon the skill with which the job setter sets up the machine. The operator has little control over the quality after the machine has once been set up. His principal duty is to feed the material, check the dimensions, and keep the machine in operation. There is a greater probability that repetitive quantities of the product will produce more consistent quality than when made on the other lathe.

Comment.—It is seen that the turret lathe produces better quality, when large amounts are produced.

Labor Required.

Engine Lathes.—Two mechanics are employed under the present method.

Turret Lathe.—This method requires the services of one operator and the part-time services of a job setter. The net result for the present, at least, is a reduction in the amount of labor employed.

Comment.—It is to be noted that there is an accompanying reduction in the cost of manufacture. Assuming that this saving in cost is passed on to the consumer, the worker, the management, and the person who furnished the capital, it is not difficult to see that increased demand will result in increased production and increased employment.

Plant Expansion.

Engine Lathes.—These machines require 150 sq. ft.

Turret Lathes.—These machines require 100 sq. ft.

Comment.—A reduction of 50 sq. ft., when added to other changes, provides needed room for expansion without any additional capital outlay.

Effect on Organization.

Turret Lathes.—The acquisition of these machines changes the complex of the personnel of the organization. The job of lathe hand is abolished and instead there are two jobs: turret-lathe operator and job setter. No doubt the better of the lathe hands will become the job setter and the poorer will become the turret-lathe operator.

The most profound change is that the new position of job setter is created. This calls for the development of highly trained all-round mechanics who can be trusted to set up the special-purpose machines with extreme accuracy. A new department of tool engineering would seem to have been established.

20.15. Financial Study.—The economy study and the intangible study have shown that the proposed method is worth while. A financial study must be made in order to determine what money is required and if it is available.

The following data are at hand:

Cost of new machine: \$6,000
Balance in reserve for depreciation account: (No. machines)(yearly depreciation charge) (No. years) = (2)(\$220)(6) = \$2,640
Present salvage value: 2 (\$600) = \$1,200

The data may be summarized as follows:

Cash required for the new machine		\$6,000
Balance in reserve for depreciation account	\$2,640	
Salvage value	1,200	
Net cash on hand from old machine		3,840
New cash required		\$2,160

In Sec. 3.7, there was presented a discussion of the manner in which the charges for depreciation are handled in the usual going business. The assumption is made in the study of a project by the principles of engineering economics that the reserve for depreciation has been so managed that liquid funds are available when needed. The young engineer who is working in the organization is not expected to know just what happens in an actual way to this reserve for depreciation. He does know that a charge is made for depreciation against the operating costs of machines and equipment and that it is made for the purpose of recovering the investment in the equipment. He assumes rightly, therefore, that it will be available when wanted.

Since the economic analyses are made for some person in authority above the young engineer, he assumes that other information will be brought to bear upon the analyses which he submits to this person. So, if top management has allowed the money set aside in the reserve for depreciation to become inextricably mixed with other general moneys of the business, it is possible that the money may not be available for this replacement purpose. It does not destroy the validity of the financial analysis, however, to assume that the money is available.

20.16. The Accountant's Problem.—The accountant's problem concerns the relationship between the original cost of the machine and the charges for depreciation to the Reserve for Depreciation account. Each year a charge, in this case \$440, is made against operating expense and applied to the Reserve account. If continued for the estimated life of the machines, which was 10 years, there would have been collected in this account (\$440)(10) or \$4,400. If to this sum is added the salvage value, at that time \$600, the original cost of the machines, \$5,000, would have been accumulated.

But all that can be collected, if the machines are disposed of now, is \$2,640 from the Reserve account and \$1,200 from salvage, a total of \$3,840. This leaves a total of \$1,160 to be accounted for. The accountant may claim that this should be added to the first cost of the new machine and reflected in the operating expenses. This is not fair since the proposed machine had nothing to do with the poor guess that was made on the depreciation rate on the old machine. In other words, little allowance was made for the factor of obsolescence by this company.

Although the question of what to do about this unabsorbed depreciation may affect the financial problem, it should have no bearing on the economy of the new machine. The operating cost of the new machine should not be affected by the financial measures that may be made to absorb this deficiency.

There are several ways of solving the accountant's problem:

1. Charge the unabsorbed depreciation of \$1,160 to Profit and Loss (see Sec. 20.7).

2. Charge the unabsorbed depreciation of \$1,160 to Surplus (see Sec. 20.7).

3. Assume that the present operating cost is used until the unabsorbed depreciation is made up by the difference between operating cost of the replaced equipment and the operating cost of the proposed equipment. Since the saving in operating cost, including expected return on the investment, per piece, is

$$\frac{\$12{,}255 - \$9840}{40{,}000} = \frac{\$2{,}415}{40{,}000} = \$0.0604$$

it will require a production of \$1,160 ÷ \$0.0604 = 19,205 pieces or 0.480 year. Thus a delay of 19,205/40,000 or nearly 6 months, will be required before the savings by the new method may be reflected in the selling price.

20.17. The Treasurer's Problem.—The treasurer may be quite satisfied with the accountant's solution of the unabsorbed depreciation. He wants to know where to get the additional capital which is the difference between the first cost of the lathes and the turret lathe, which is \$6,000 − \$5,000, or \$1,000.

In Sec. 20.5, several ways of finding the necessary additional funds were discussed. Still another way would be to make use of the third suggestion in Sec. 20.16, namely, to apply the saving in operating cost, including expected return, per piece, to making up the difference or \$1,000. It would require the production of

$$\frac{\$1,000}{\$0.0604} = 16,556 \text{ pieces}$$

Thus a further delay of 16,556/40,000 or 0.414 year, or nearly 5 months, will be required before the savings by the new method can be reflected in the selling price.

This delay of 0.414 year added to that caused by unrecovered depreciation of 0.480 year equals 0.894 year or 10.72 months. From this time on, the new low cost of manufacture may reflect itself in the determination of new price policies.

It is doubtful if this is a good solution. A good bit depends upon the competitive position of the company and the trend of prices for the specific product. If the company has a patent monopoly, it may be able to keep its selling price up for this required period without difficulty. If the field is highly competitive, it may not be able to do so. This method is, in its essence, nothing more than maintaining the price level with a lower operating cost, the increased return or profit being charged against the purchase price of the new equipment instead of being placed in the surplus account.

Problems

1. In Chap. 4, Prob. 1, a manufacturing company bought an engine lathe. The estimated life of the lathe, 15 years, has passed. An investigation reveals that lathes have been very much improved during that time. However, this lathe is still in good condition. It can be sold for its estimated salvage value of \$300. It can be kept and operated for 2 years more. Another lathe may be purchased to replace it at a cost completely installed for \$4,500. Should it be replaced immediately or kept for 2 years and then replaced? Outline the reactions of the shop foreman, cost accountant, and treasurer. Assume that operating costs do not change but continue at \$300 per year as before. The yield on the investment is 10 per cent.

2. A certain part has been made on an engine lathe. It is proposed to make it on a turret lathe. The following facts apply:

	Old method	New method
Process time per piece	56.1 min.	40.1 min.
Labor cost	60 cents/hr.	65 cents/hr.
Overhead (not including depreciation)	$1.50/hr.	$1.50/hr.
Machine life	20 years	8 years
Original purchase price	$2,800	$3,915
Installation cost	250	500
Estimated salvage value, end of life	200	500
Present age, old machine	5 years	
Present salvage value, old machine	$1,850	

Assume 300 working days per year at 8 hr. per day. Also assume that full time of the machine is to be used for this particular piece. The expected return on the investment is to be 12 per cent. Make a complete replacement study, economy, intangible, and financial.

3. The present practice of a machine company is to grind a taper hole in a lock bolt bushing on an *A* grinder. It is proposed to replace the *A* grinder by a *B* grinder. The facts are as follows:

	A grinder	*B* grinder
Setup time	45 min.	60 min.
Production time	6 min.	1.6 min.
Lot size	300 pieces	300 pieces
Direct labor	60 cents/hr.	60 cents/hr.
Direct material	10 cents/pc.	10 cents/pc.
Overhead (not including depreciation)	90 cents/hr.	90 cents/hr.
Salvage value, old machine	$500	
Original cost	$4,000	$6,315
Life	10 years	10 years

Assume that a depreciation reserve has been maintained for the entire life of the old machine. The expected return on the investment is to be 8 per cent. Assume 280 days per year at 8 hr. per day. Make a replacement study.

4. A newspaper company uses a foundry type in making up advertisements. When the advertisements are broken up, the type must be sorted by hand. This operation may be eliminated by the installation of three machines using cast type that can be melted and used again. The three machines will cost $25,000. Their use requires two operators at $55 a week each. Power and repairs will average $70 a month over the life of the machines. Salvage value of the foundry type will equal the cost of type metal for the new machines. Renewal cost for worn-out foundry type averages $1,100 per year. The new machines will replace six printers at

$45 per week each. The expected rate of return is 6 per cent and depreciation on new machines 25 per cent annually. Make a replacement study. (Modified from examination for Professional Engineer's license, New York State, Jan. 30, 1935.)

5. An old light-capacity highway bridge may be strengthened at a cost of $9,000 or it may be replaced by a modern bridge of sufficient capacity at a cost of $55,000. The present net salvage value of the old bridge is $13,000. It is estimated that the reinforced bridge will last for 5 years after which additional reinforcement or replacement will be necessary. The net salvage value of the reinforced bridge will be $10,000 at the end of 5 years and that of the new bridge will be $15,000 after 30 years. The additional cost of maintenance and inspection on the old structure is $300 per year. Assuming return at 6 per cent and depreciation on a straight-line basis, state whether it is more economical to reinforce the bridge or replace it. (Examination for Professional Engineer's license, New York State, Jan. 30, 1935.)

CHAPTER 21

ENDOWMENT PRINCIPLE

21.1. The Endowment Principle.—An endowment is understood to be a sum of money which is set aside at compound interest and from which a periodic payment is made. This payment may be used to pay for the periodic operating cost of the project under consideration.

There are two types of application of this method. The first is applied to a project to be operated for a restricted or limited period of time. The second is applied to a project to be operated for an indefinite period of time or perpetually.

21.2. A Restricted Endowment.—The application of the endowment principle will be illustrated by means of an automobile. The following data are available:

A man plans to purchase an automobile at a first cost of $800. He plans to purchase a new car every year. He finds that the salvage value of his old car at the end of each year will be $570. The operating costs not including depreciation per year will total $300.

What sum of money would be sufficient to provide for the foregoing program over a period of 10 years?

Operating Cost per Year:	
Operation	$300
Depreciation ($800 − $570)	230
Total annual cost	$530

The required sum of money must be sufficient to provide at the end of each year the sum of $530, which is the yearly operating cost. This amount is needed at the end of each year for 10 years. The sum required is the sum of the present worths of the yearly operating costs referred back to the beginning of the first year.

The present worth of the annual cost ($530) is the amount of an R series of payments for which the yearly payment is $530 and the term is 10 years.

$$R = (\$530)\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \quad \text{(See Appendix } A\text{)}$$

where n is 10 years, and i is the rate of 3 per cent.

$$R = (\$530)(8.530), \text{ or } \$4,520$$

This sum would provide $530 at the end of each year for 10 years. Of this sum $300 would be used for ordinary operating expenses of the car. The balance ($230) would be used every year to provide, with the salvage value of $570, the required $800 necessary to replace the old car. This depreciation has been determined by the straight-line method on the assumption that this amount is kept in a fund at no interest until needed to replace the old car. If the depreciation charge is kept in the fund, then it should be calculated by the sinking-fund method.

If to this amount is added the original cost of the car,

Present worth of first cost	$ 800
Present worth of operating cost	4,520
	$5,320

then this amount will be sufficient to make the original purchase and, as previously explained, provide sufficient funds for the ordinary operating costs and the replacement of the car every year. If the fund earns more than 3 per cent, the amount needed will be less. Should the fund earn less, the amount must be increased.

21.3. Present Worth of First Cost and Operating Cost.—This is another name for the restricted endowment, explained in Sec. 21.2. The restricted endowment assumed that an investment was made at the beginning of the life of the project only. This method assumes that an investment is made at the beginning of the life of a project and also that additional investments will have to be made at different periods after the initial investment. It is applicable, as a method of analysis to assist judgment, to those projects which are not complete with the first installation but which must be added to from time to time because of increased demand in capacity.

There are several ways in which the project may be carried out. Enough capacity may be provided at the start to take care of requirements for a considerable period of time. The cost will usually be great. Or enough capacity may be provided for a short period, to be followed by additions at more or less regular intervals.

In many cases, the latter method is quite feasible, but there are cases when this method will cost considerably more than the first one.

A good method of studying the relative economy of projects which are of this nature is to compare the costs of making the additions at various times, as referred to a base year, namely, the year when the project is started. Present worths are calculated for each investment back to that year. The sums of the present worths of the various schemes are then compared so as to determine the best one. See Equivalence, Sec. A.9 of Appendix.

21.4. A Typical Case.—A certain town has a sewer problem. It plans to lay a sewer main in a certain street of a rapidly developing section. Two alternatives are presented. One is to lay an 8-in. main immediately and at the end of 15 years lay an additional 12-in. main. The other is to lay a 15-in. main immediately.

The cost of the 8-in. main will be \$6,000, of the 12-in. main will be \$11,250 (includes cost of tearing up street and repairing), and of the 15-in. main will be \$10,500. The operating cost will include depreciation, maintenance, and interest and sinking fund on a bond issue. Depreciation is to be calculated by the straight-line method on a life of 50 years with no salvage value. The maintenance will cost as follows:

Size of pipe	8 in.	12 in.	15 in.
Maintenance cost per year	\$300	\$750	\$1,000

The sewer will be financed by a bond issue at 3 per cent over a life of 30 years.

21.5. First Cost Comparison.—It is usual to set up a period of time called a "study period." In this case, 30 years will be used, since it is the life of the bond issue.

Plan A.—Lay 8-in. main immediately, then a 12-in. main in 15 years.

Time Scale 0 5 10 15 20 25 30

8-in. main

$\$6{,}000p^{0}$

\$ 6,000 = (\$6,000)(1.00)

12-in. main

$\$11{,}250\ p^{15}$

7,221 = ... (\$11,250)(0.6419)

\$13,221 = present worth of first cost of 8- and 12-in. mains

Plan B.—Lay 15-in. main immediately.

Time Scale 0 5 10 15 20 25 30
\$10,500 p^0
$10,500 = ($10,500)(1.00)

Thus from the standpoint of first cost Plan *B* is the better.

21.6. Operating-cost Comparison.—This comparison is made on the same study period basis as for first cost. Operating costs for the two plans are as follows:

	Plan *A*		Plan *B*
	8-in. main	12-in. main	15-in. main
Maintenance cost	$300	$ 750.00	$1,000.00
Depreciation	120	225.00	210.00
Bond interest	180	337.50	315.00
Sinking fund, bond	126	236.25	220.50
Totals	$726	$1,548.75	$1,745.50

Plan A.—Lay 8-in. main immediately, then a 12-in. main in 15 years.

Time Scale 0 5 10 15 20 25 30
8-in. main
← $726 yearly →
Present worth of yearly payments or *R* value
$14,229.60 = ($726)(19.60)
(19.60 from Table B-5 of Appendix B.)
12-in. main
← $1,548.75 yearly →
Present worth of yearly payments or *R* value at fifteenth year.
$18,492 = ($1,548.75)(11.94)
Present worth of *R* value at fifteenth year carried to beginning year equals:
$11,870.06 = ... ($18,492)(0.6419)
$26,099.66 = Total

Plan B.—Lay 15-in. main immediately.

Time Scale 0 5 10 15 20 25 30
|←——$1,745.50 yearly——→|
Present worth of yearly payments or *R* value
$34,202 = ($1,745.50)(19.60)

Thus from the standpoint of operating cost Plan *A* is better.

21.7. Combined First Cost and Operating-cost Comparison.—This comparison is made by adding the present worths of first cost and operating costs.

Plan A.—Lay 8-in. main immediately, 12-in. main in 15 years.	
Present worth of first cost	$13,221
Present worth of operating costs	26,099
Total	$39,320
Plan B.—Lay 15-in. main immediately.	
Present worth of first cost	$10,500
Present worth of operating cost	34,202
Total	$44,702

Thus, from the combined analysis, Plan *A* is seen to be the better. The meaning of these totals is that the particular sum will enable the town to make the initial installation, pay the operating costs, make the additional installation, and pay additional operating costs for the 30 years. Even though this total sum may not be set aside, and it generally is not, it is used as a criterion figure to be compared with a similar figure for some other proposed solution of the problem.

21.8. Income, Return, Yield.—A sewer system is installed in a community or borough by the authority of the regularly constituted officers. It is installed as a service to the community. There is no selling price for the service in the usual meaning of "selling price." Instead, the householders in the community pay for the service in taxes.

It may be said that a certain proportion of the taxes is used to operate the sewer system. The amount allocated to the sewer system will need to be sufficient to cover the operating costs per year and the financial charges for bond issues which are floated to finance the installation.

Since income cannot be figured, return cannot be determined and there will be no yield. Return on the investment could be added to operating cost and a figure thus obtained which, if divided by some unit of service, could be used in the same manner as unit cost of production plus expected return, as discussed in Sec. 10.11. Such a figure would be useful if a comparison were to be made with a privately owned and operated sewer system. Since a sewer system is an enterprise operated for public benefit, such a comparison is not possible.

As was shown previously, a sewer system is operated by community officers and paid for out of taxes. Hence, the amount or proportion of taxes necessary to operate the plant would be a good criterion by which to judge the system. It is impossible to determine the necessary taxes until the method of financing the project has been determined.

21.9. Intangible Analysis.—The principal judgment factor in this problem is the number of times there will be an interruption in the business of the town because of tearing up the street.

These may be listed as follows:

Plan *A:* Two interruptions

Plan *B:* One interruption

Plan *B* is the better from an intangible viewpoint.

21.10. Financial Analysis.—Since a decision may not be made on an economy study alone, or on an economy study plus an intangible study, it is necessary to make a financial study also. The financial requirements may be represented as follows:

For First Cost.

Plan *A*	$ 6,000 immediately 11,250 in 15 years
Plan *B*	$10,500 immediately

Plan *A* requires less immediate cash. The additional cash requirements occur at the end of 15 years. It should be noted that the amount then required is greater than that required now for Plan *B*. However, Plan *A* has one advantage. The immediate outlay is nearly half of that for Plan *B*. Since the new main does not have to be installed for 15 years, there is sufficient time to test the growth of the town. If the growth does not go in the direction of this sewer main, then the installation of the 12-in. main may not be needed.

Plan *B* requires a larger sum immediately than Plan *A*. The major question in this case is whether or not the town council can raise this much money immediately. This will depend upon the legal restrictions on the borrowing capacity of the town. Should the town not grow and the expansion be unnecessary, then too much investment will have been made.

For Operating Cost.—For length of study period 30 years.

Plan *A*	$ 726	yearly for 30 years
	$1,548.75	yearly for last 15 years
Plan *B*	$1,745.50	yearly for 30 years

These operating costs will have to be provided through taxes. Will the tax rate have to be raised to provide for the operating costs?

Thus it is seen that there are three principal viewpoints by which the project can be judged, namely,

1. The effect on bonded debt or first cost provisions
2. The effect on tax rate or operating-cost provisions
3. The intangible items of trouble, confusion, etc.

21.11. A Perpetual Endowment.—The application of the endowment principle to perpetual time will be illustrated by the case of an automobile. The same data will be used as in Sec. 21.2.

The present worth of the annual operating costs of $530 for a perpetual period of time will be equal to the amount of an R sum for an R series of payments for which the yearly payment is $530 and the term is perpetual.

If a table of factors for an R series is consulted, it will be seen that the value of the factor approaches as a limit the amount $33.33 which is equivalent to 1 ÷ 0.03, 0.03 being the interest rate expressed as a decimal.

$$R_p = \$530(1 \div 0.03) = \$17,666$$

In other words, if $17,666 is placed at interest of 3 per cent, it will yield ($17.666)(0.03), or $530 at the end of each year, thus providing for the operating costs and the replacement of the car.

To this amount must be added the first cost.

First cost	$ 800
Present worth of operating cost	17,666
Total	$18,466

This amount will be sufficient to make the original purchase and, as previously explained, to provide funds to operate the car every year and purchase a new one every year perpetually.

21.12. Capitalization.—The method explained in Sec. 21.11 is also called the "method of capitalization." It is so named because it uses the same principle as was used in Chap. 4 for determining the valuation of perpetual earning power.

The method is quite useful when comparing alternative projects. For example, the comparison of the housing and apartment developments used in Chaps. 11 and 12 would be as follows:

Housing Development

First cost: $110,000
Operating cost:
Capitalization of $6,876 for a perpetual period at 3 per cent or $6,876 ÷ 0.03 = $229,200
Total capitalized value:

First cost	$110,000
Operating cost	229,200
Total	$339,200

Apartment Development

First cost: $110,000
Operating cost:
Capitalization of $8,016 for a perpetual period at 3 per cent, or $8,016 ÷ 0.03 = $267,200
Total capitalized value:

First cost	$110,000
Operating cost	267,200
Total	$377,200

On the basis of a comparison of the capitalized costs, the housing development appears to be the better. However, the above analysis shows only part of the picture. If the incomes are capitalized, a different picture will appear.

Housing development:

$$\text{Capitalized income} = \frac{\$15{,}600}{0.03} = \$520{,}000$$

Apartment development:

$$\text{Capitalized income} = \frac{\$16{,}800}{0.03} = \$560{,}000$$

Since the capitalized values of first cost plus operating cost show a preference for the housing development and since the capitalization of income shows a preference for the apartment development, some other comparison must be used, such as the capitalization of return for each project.

Housing development:

$$\text{Capitalized net return} = \frac{\$8{,}724}{0.03} = \$290{,}800$$

Apartment development:

$$\text{Capitalized net return} = \frac{\$8{,}784}{0.03} = \$292{,}800$$

The result of this comparison shows the apartment development to be the better, but by only a very small margin, $2,000, the capitalized value of the difference in return of $60.

Another way of making this comparison with this method is to get the capitalized value of first cost plus operating cost plus expected return.

Housing development:		
First cost	$110,000	
Operating cost	229,220	
Expected return of 10 per cent	366,666	($11,000 ÷ 0.03)
Total capitalized value	$705,866	
Apartment development:		
First cost	$110,000	
Operating cost	267,200	
Expected return of 10 per cent	366,666	
Total capitalized value	$743,866	

The result of this comparison is to show that the easier way to make the expected return of 10 per cent is to choose the housing development. However, a greater income may be possible than the foregoing analysis shows. For this reason no decision should be made until an analysis of probable income has been made.

This method does not give any better results than the method of profit and loss which was used in Chap. 11 and elsewhere. It should be noted, however, that for projects of a public nature in which the earning of profit is not present, this method is excellent. It should be used in all such cases, especially for perpetuities or projects which can be renewed indefinitely and in which the need to earn profit is absent. In cases of a restricted endowment or a perpetuity where it is contemplated actually to set aside a sum of money in a fund, the interest rate should be such as can be readily obtained in financial circles.

21.13. A Typical Case.—Sigvald Johannesson, Engineer of Design, New Jersey State Highway Department, describes[1] the application of the principle of capitalization to the study of a highway development. The problem was the "carrying of the New Jersey State Highway Route 25 from a point near Broadway, Jersey City, to a point west of the Passaic River in the city of Newark."

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[1] Johannesson, S., "Highway Economics," McGraw-Hill Book Company, Inc., New York, 1931.

Three schemes were under consideration. The first proposed a viaduct for the total length with lift bridges across two rivers. The second scheme carried the highway across the rivers by means of tunnels instead of bridges. The third scheme was to be a high level viaduct without recourse to lift bridges or tunnels. Bridges over the rivers were to be high enough to let shipping through.

Table 21.1 is a summarization of the cost data for the three schemes.

Part I is a tabulation of the annual costs that might be charged against the highway. Annual costs such as highway maintenance, highway depreciation, and highway operation were regarded as operating costs to be borne by the state highway department.

Annual costs, such as motor vehicles—distance, motor vehicles—rise and fall, motor vehicles—delays, were operating costs which it was estimated the motorist would have to pay in the operation of his car.

Part II is a tabulation of the construction cost of the highway consisting of right of way, cost of construction of Route 25, and cost of Lincoln Highway Tunnel.

Part III is a tabulation of the capitalized annual cost chargeable to the state, capitalized annual costs chargeable to the motorist, and the construction cost of the highway.

The total of these three items gives a figure which is to be used as a criterion for judging the relative worth-whileness of the three schemes. It should be noted that there is no intention, as with a true perpetual endowment, of setting aside this sum of money for the construction and operation of this highway. This total sum is looked upon as a figure solely useful for gauging the three schemes.

The usefulness of either type of endowment lies in the fact that it is the only way in which first cost and operating cost can be united to get one figure that represents an economic evaluation of the project.

TABLE 21.1.—NEW JERSEY STATE HIGHWAY, ROUTE 25, BROADWAY, JERSEY CITY, TO A POINT WEST OF LINCOLN HIGHWAY, NEWARK*
Comparative Estimate

Item	Scheme 1, lift bridges	Scheme 2, tunnels	Scheme 3, high bridges
Part I			
Physical characteristics:			
Length, in feet	15,050	14,800	14,800
Rise and fall, in feet	58	131	114
Annual costs:			
Highway maintenance	$ 15,330	$ 10,240	$ 52,940
Highway depreciation	264,679	292,180	214,690
Highway operation	136,600	120,900	43,900
Total annual highway costs	$ 416,609	$ 423,320	$ 311,530
Motor vehicles, distance	$ 100,000	0	0
Motor vehicles, rise and fall	532,440	$ 1,202,580	1,046,520
Motor vehicles, delays	798,000	0	0
Total annual motor vehicle costs	$ 1,430,440	$ 1,202,580	$ 1,046,520
Total annual costs	$ 1,847,049	$ 1,625,900	$ 1,358,050
Part II			
Cost of construction and capitalized annual costs:			
Cost of right of way	$ 1,020,000	$ 960,000	$ 710,000
Cost of construction, Route 25	11,389,900	20,454,000	18,915,000
Cost of Lincoln Highway tunnel	6,000,000		
Total right of way and construction	$18,409,900	$21,414,000	$19,625,000
Part III			
Annual highway cost capitalized, 6 per cent	$ 6,943,483	$ 7,055,333	$ 5,192,167
Annual motor vehicle cost capitalized, 6 per cent	23,840,667	20,043,000	17,442,000
Total annual cost, capitalized	$30,784,150	$27,098,333	$22,634,167
Total comparative costs	$49,194,050	$48,512,333	$42,259,167

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* JOHANNESSON, S., "Highway Economics," p. 141, McGraw-Hill Book Company, Inc., New York, 1931.

Problems

A. Data for a commercial hotel were given in Prob. *A* in Chaps. 8 to 11. What is the capitalized cost (perpetual endowment) for the proposition using the expected yield on investment as a percentage?

B. What would be the capitalized cost of the central mix concrete plant, data for which were given in Prob. *B* for Chaps. 8 to 11? Use the expected yield as a percentage.

C. Data for a power plant with uniflow engines were given in Prob. *C* for Chaps. 8 to 11. This power plant is on the property of a college. It is proposed to endow the operation of the power plant for a period of 50 years. What must the total endowment be to provide for the original building of the plant and the endowment of the operating costs if the plant operates at 50 per cent capacity point? The rate of interest to be used is $3\frac{1}{2}$ per cent?

1. What amount must be given to a hospital to enable it to construct a plant having an original cost of $2,000,000, an estimated life of 60 years, and an estimated annual upkeep cost of $140,000, and to have on hand at the end of 60 years $2,000,000 to replace the plant, if money returns $2\frac{1}{2}$ per cent? (From examination for Professional Engineer's license, New York State, Feb. 2, 1938.)

2. A telephone company has four plans set up for cost study, as follows:

Plan 1. Install a 2-duct conduit now, two more ducts in 5 years, two more in 15 years, and two more in 25 years
Plan 2. Install a 4-duct conduit now and four more ducts in 15 years
Plan 3. Install a 6-duct conduit now and two more ducts in 25 years
Plan 4. Install an 8-duct conduit now

Assume installation costs as follows:

2-duct conduit	$2,800 per 1,000 ft.
4-duct conduit	3,200 per 1,000 ft.
6-duct conduit	3,600 per 1,000 ft.
8-duct conduit	4,300 per 1,000 ft.

On the information given, make a comparison of the four plans on a present-worth basis, using an interest rate of 6 per cent, and show which plan is most desirable. (From examination for Professional Engineer's license, New York State, Jan. 27, 1936.)

3. In the preceding problem, the operating costs per year include taxes at 3 per cent of first cost, expected return at 8 per cent, and maintenance at $10 per pair of ducts per 1,000 ft. per year. Depreciation is figured by the straight-line method. It is assumed that, at the end of 60 years, the installation will be no longer needed and that it will be prohibitive to remove it. Make a comparison of the four plans from the operating-cost standpoint. Which is most desirable? Combine operating cost and first cost and determine which is desirable.

4. A wealthy man wishes to set aside a certain sum of money in his will to provide the use of an automobile for his wife for a period of 24 years.

He wishes to know what sum will be required to buy the initial car and then to replace it and succeeding cars every 3 years. The car is to cost $1,200 delivered. The trade-in value at the end of each 3 years is estimated to be $575. Yearly operating costs are estimated to be $325 per year in addition to depreciation. The endowment funds are to be invested at $2\frac{1}{2}$ per cent.

5. A clover-leaf crossing is contemplated at an intersection of two highways, each carrying 1,000,000 vehicles annually. One-half of the traffic approaching the intersection proceeds straight through, one-fourth turns to the right, and the other one-fourth turns to the left. The loss of time due to traffic delays at the crossing is estimated at 4,000,000 vehicle-minutes annually. The estimated cost of right of way and construction of the clover leaf is $200,000.

One-half of the traffic will go straight through, thereby requiring no extra travel distance. One-quarter will turn to the right, thereby, saving 350 ft. of travel. One-quarter will turn to the left, thereby, requiring 1,200 ft. of additional travel. The cost per mile of vehicle travel is 10 cents.

Since additional distance is necessary, additional time will be needed. Assume speed as 20 m.p.h. and cost at 1 cent per vehicle-minute.

Since the overpass is built about 12 ft. above crossing grade and the underpass about 12 ft. below it, there will be additional cost for going grades. One-half will require 12 ft. of rise and fall. One-quarter will require 24 ft. of rise and fall (left turn) and the other quarter (right turn) will require no rise and fall. Value rise and fall at $0.00045 per foot.

Savings in expenses will consist of traffic officer expense which may be valued at four traffic officers per 24-hr. day at $2,500 per year. The motorist will save 4,000,000 vehicle-minutes of time. Value this saving at 1 cent per vehicle-minute.

Make an analysis by capitalizing the costs and compare them with the capitalized savings, using 5 per cent. Is it economically proper to construct the clover leaf?

Assuming all proportions of traffic remain the same, below what traffic density would this proposition be uneconomical?

CHAPTER 22

SOME ASPECTS OF PUBLIC PROJECTS

22.1. Types of Projects.—It was suggested in Chap. 2 that there were two broad divisions or types of engineering projects; namely, those which produced a product and those which rendered a service. They may also be classified with regard to ownership and control, as private, quasi-public, and public.

Private projects are those that are owned and operated by private citizens with a minimum of interference or control by government. Quasi-public and public projects are those which are essential to the public welfare. Such projects include,[1]

- I. Services of transportation (common carriers),
  - *A*. Provision of highways, turnpikes, ferries, and bridges
  - *B*. Vehicular transport over highways by stagecoaches
  - *C*. Provision of artificial waterways, canals, canalized rivers, harbors, and roadsteads
  - *D*. Water transport by towboat, sailing vessel, tramp steamer, and steamship companies having a definite schedule
  - *E*. Railway transport of freight, passengers, mail, and express
  - *F*. Urban and interurban electric railway transit of passengers, freight, mail, and express
  - *G*. Transport by airways of passengers, express, and mail
  - *H*. Pipe lines for transport of oils and natural gas
- II. Services incidental to transportation,
  - *A*. Supply of specialized equipment—sleeping, dining, parlor, refrigerator, or other car service
  - *B*. Stockyards, warehouses, and elevators
  - *C*. Docks and terminals, transfer, lighterage, compress services

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[1] Glaeser, M. G., "Outlines of Public Utility Economics," pp. 8–9' The Macmillan Company, New York, 1927.

III. Services facilitating communication,
    - *A*. Postal service
    - *B*. Telegraph service, wire and wireless, including submarine cable service
    - *C*. Local and long-distance telephone service, wire and wireless
    - *D*. Stock-ticker service
    - *E*. Radio broadcasting and signaling services

IV. Facilities providing power, light, heat, and refrigeration,
    - *A*. Artificial and natural gas works
    - *B*. Electricity supply work
        - 1. Using water power
        - 2. Using steam power

V. Facilities providing water and sanitation in urban communities,
    - *A*. Sewage and garbage disposal plants
    - *B*. Waterworks supplying water for fire protection, domestic, commercial, and industrial purposes

VI. Facilities regulating water supply for agricultural purposes,
    - *A*. Irrigation works
    - *B*. Flood protection works
    - *C*. Drainage works

Services that are essential to the operation of society are generally government owned or regulated. In early civilization, these included highways, postal services, and water supply, but as new developments took place, they have been increased to include sewerage and, in some cases today, electric supply and even transportation. Where these are owned and operated by the government or its agencies, they are called "public" projects. Where they are owned and operated by private means but regulated by government commissions, they are called "quasi-public" projects.

22.2. Economy Factors for Public Projects.—In Chaps., 1, 2, 3, and 7, there were discussed the various factors that enter into the economy analysis of an engineering project. These are development and promotion cost, construction cost, and investment and operating costs.

These factors of cost are applicable to all types of engineering projects whether privately or publicly owned and operated.

There are, however, certain viewpoints held as to the treatment of depreciation and taxes for publicly owned and operated projects which deserve some attention.

22.3. Depreciation for Public Projects.—Depreciation was defined in Chap. 3 as the annual charge that was made as a part of operating costs for the maintenance and recovery of capital. Since a publicly owned and operated utility is owned by the public (a political subdivision, whether Federal, state, county, or municipal), the necessity for maintaining capital is not present. Depreciation was also defined as the annual charge to replace equipment which had worn out through use or become obsolete. Even though the need for maintenance of capital is not present, the need for replacement of plant is present. Hence, the policy set up by the publicly owned utility must be one of maintaining the plant and equipment in good working condition at all times.

There are two methods of taking care of this requirement. One is the method of private industry where a depreciation reserve is set up to maintain the capital and replace or renew pieces of equipment that have lived their useful life. The depreciation charge will be made from the beginning of the enterprise and will be a charge against operating costs.

The other is a replacement policy and method. This policy would charge into annual operating costs sufficient amounts to replace aged equipment and keep the plant operating in a satisfactory manner. During the early years, the charges for replacement will be very small and will increase with age until a time is reached when they will be more or less uniform because of the fact that equipment is reaching the end of its life year by year.

This latter stage is also the one when the depreciation reserve policy and method is used. The chief difference lies in the fact that during the earlier years the charges for replacement are less than the charges for depreciation reserve. In later life, the charges for replacement and depreciation reserve are about equal. Thus operating costs may be lower during the earlier years if the replacement method is used.

22.4. Taxes on Public Projects.—Taxes are charges made by government for the upkeep and maintenance of its procedures. A publicly owned plant may be considered as a true commercial project, entirely similar to a privately owned and operated project. From this viewpoint, taxes would be charged on a publicly owned plant in the same manner as on a private plant. There are some who believe that a publicly owned plant should not charge itself taxes because of the widespread benefits that the community derives from its operation. However, it should be remembered that, when a private plant is taken over by the public and when taxes are not charged against the now publicly owned plant, the taxes formerly paid by the private plant must somehow be made up. The result may be a lowering of the cost of the service to the public but an increase in their taxes, which means practically an unchanged condition as far as the total amount which the public will pay for the service and for taxes.

The belief is growing that publicly owned projects should pay taxes in the same manner as private projects do. The Detroit Street Railways are an example of a municipally owned plant which pays taxes not only to the city, but also to the county and state.[1]

22.5. Return on Investment in Public Projects.—A privately owned project expects to make a return on its investment because the investors forego the use of the invested capital in favor of a return. A publicly owned project need not make a return on its investment. The obvious result of this procedure is to eliminate expected return on investment and thus make the unit minimum selling price less than for a privately owned project.

22.6. Bond Redemption for Public Projects.—It was shown in Sec. 15.10, that the amortization of a bond issue must be considered not as an operating cost but as the buying of equity in the project and hence paid out of the private funds of the investor.

The funds for the amortization of a bond issue for a public project may come from several sources. If the public project is conducted so as to earn a return on its investment, this return may be used to amortize the bond issue. If, in addition, the project maintains a reserve for depreciation, it may use this reserve to amortize the bond issue. This may be possible, since replacements of equipment will be small during the early years of the project's life. Later, a replacement charge must be included in operating costs in order to keep the plant renewed and up to date.

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[1] Buck, A. E., "Municipal Finance," p. 533, The Macmillan Company, New York. 1926.

In any case, amortization of the bond issue for a public project is an operating cost because it is compulsory and must be met out of current income. Because of this compulsion, unit operating costs will be higher during the time that the bond issue is being amortized than afterwards. After the bond issue has been amortized, either the reserve for depreciation or the replacement charge will be sufficient to maintain and renew the plant forever.

22.7. Total and Unit Operating Costs.—The effect of these various viewpoints regarding depreciation, taxes, return on investment, and amortization of the bond issue may be seen by applying the methods to a problem. The data for this problem are as follows:

Power-plant Problem

Total investment		$800,000
Total income per year		174,000
Depreciation charges	$50,000	
Taxes	16,000	
Other operating costs	44,000	
Total operating costs		110,000
Outstanding bond issue, 5 per cent for 30 years		500,000
Yearly interest charge	$25,000	
Sinking fund deposit for amortization at 5 per cent, bonds retired as money becomes available	8,000	
Total annual amortization charges		33,000
Output 11,000,000 kw.-hr. per year		

The various combinations of taxes, depreciation, return, and bond amortization may be summarized as follows:

I. PRIVATE OWNERSHIP

Total income		$174,000
Operating cost:		
Depreciation	$50,000	
Taxes	16,000	
Other operating costs	44,000	110,000
Total net return		$ 64,000
Economic yield on $800,000: 8 per cent		

$$\text{Unit selling price per kilowatt-hour} = \frac{\$174{,}000}{11{,}000{,}000} = \$0.0158, \text{ or } 1.58 \text{ cents per kilowatt-hour}$$

Total net return to company	$ 64,000
Bond interest (financier's share)	25,000
Total net return to stockholders	$ 39,000

$$\text{Yield to stockholders' equity} = \frac{\$39{,}000}{\$800{,}000 - \$500{,}000} \times 100 = 13 \text{ per cent}$$

Total net return to stockholders	$ 39,000
Sinking fund for amortization of principal of bond issue (stockholders buying back equity)	8,000
Return to stockholders after bond issue sinking fund	$ 31,000

Yield to stockholders' equity after taking out sinking fund for bond issue

$$= \frac{\$31{,}000}{\$800{,}000 - \$500{,}000} \times 100 = 10.33 \text{ per cent}$$

II. PUBLIC OWNERSHIP

(Allowed to earn a return commensurate with private ownership)

a. All Operating Costs Same as Private Ownership.—The analysis as made for the private ownership applies throughout.

The balance of net return of $31,000, however, is not paid to any stockholders but is considered as public funds to be used as officials decree for the political divisions (borough, state, nation), to pay operating costs, thereby permitting a reduction in the taxes paid by individual citizens and property owners or to allow a reduction in power rates.

If, as is many times the case, the entire financing is by bond issue, then the charges for amortization of $800,000 will increase to $40,000 for bond interest and $13,333 for sinking fund which reduces the balance of $31,000 to $10,666. It should be noted that the rate per kilowatt-hour remains the same as for private ownership, $0.0158.

b. Taxes Not Charged against Publicly Owned Plant.—If the publicly owned plant is not taxed, the balance of net income will be increased from $31,000 by the addition of the saving in taxes of $16,000 to $47,000. Thus there is no change in the funds which the public officials have at their disposal. If, however, they calculate that their operating costs have been reduced by the amount of the taxes to $110,000 − $16,000, or $94,000, they may also reduce their income from $174,000 to $158,000, thus making it possible to reduce the price per kilowatt-hour to

$0.0143. This, however, is accomplished at a reduction in public revenues equal to the amount of the taxes, or $16,000. These funds must be replaced by new taxation. This is usually such a hard thing to do that this scheme is not feasible.

c. Depreciation Charge Not Made during First 15 *Years but Followed by Replacement Charge at That Time.*

Operating costs:	
Taxes	$ 16,000
Other operating costs	44,000
	$ 60,000
Return on investment	64,000
	$124,000

$$\text{Unit selling price} = \frac{\$124{,}000}{11{,}000{,}000} = \$0.012, \text{ or } 1.12 \text{ cents per kilowatt-hour}$$

With this plan the immediate charge for power may be $0.012 per kilowatt-hour for the first 15 years. At that time a replacement charge will begin which is estimated to equal the depreciation charge of $50,000 at the end of the next 15 years, when the unit selling price would naturally return to $0.0158 per kilowatt-hour. It is harder to increase rates than to lower them; hence, this is a poor program.

d. All Operating Costs Same as Private Ownership. Bonds Paid Off at End of Thirtieth Year.—The situation here is the same as in private ownership when the entire equity rests with the stockholders of the company. When this exists in a publicly owned plant operated as a private plant, the public treasury then has a total of $31,000 as in Case II*a*, increased by the amortization charge of $33,000 to a total of $64,000. This it can use as specified in Case II*a*.

III. PUBLIC OWNERSHIP

(Not allowed to earn a return as in private ownership)

a. All Operating Costs Same as Private Ownership.

Operating costs:	
Depreciation	$ 50,000
Taxes	16,000
Other operating costs	44,000
	$110,000
Bond amortization charges	33,000
	$143,000

$$\text{Unit selling price} = \frac{\$143{,}000}{11{,}000{,}000} = \$0.013, \text{ or 1.3 cents per kilowatt-hour}$$

$$\text{Unit selling price after amortization of bond issue} = \frac{\$110{,}000}{11{,}000{,}000}$$
$$= \$0.01, \text{ or 1.0 cent per kilowatt-hour}$$

In contrast with the situation in Case II*a*, the public is deprived of the use of the saving of either $31,000 or $64,000, whichever the case may be, for public uses but gets its power at a lower rate.

b. Taxes Not Charged against Publicly Owned Plant.—The total operating cost before the bond issue is amortized will be reduced by the amount of the taxes not charged, and will equal $143,000 − $16,000, or $127,000, giving a unit cost per kilowatt-hour of $0.0115.

The total operating cost after the bond issue is amortized will be $110,000 − $16,000, or $94,000, which will give a unit cost per kilowatt-hour of $0.0085.

The disadvantage of this plan is the same as that of Case II*b* in that taxes must be increased by the amount of $16,000.

c. Depreciation Charge Not Made during First 15 *Years, but Followed by Replacement Charge at That Time.*—If the depreciation charge is not made during the first 15 years but a replacement charge is made at that time, the total operating cost before the bond issue is amortized will be equal to $143,000 − $50,000, or $83,000, which will give a unit cost per kilowatt-hour of $0.0075.

At the end of 15 years the replacement charge will begin, estimated to equal the depreciation charge of $50,000 at the end of the next 15 years. At that time the total operating cost will be equal to $143,000 minus the amortization charge of $33,000, or $110,000, and the unit cost per kilowatt-hour will be $0.01.

d. All Operating Costs Same as in Private Ownership. Bonds Paid Off at the End of Thirtieth Year.—Under this situation the total operating cost of $143,000 will be reduced by the sum of $16,000 and $50,000 and will equal $77,000. Unit cost per kilowatt-hour could then be $0.007.

However, as was shown in Case III*c*, a charge for replacement begins at the end of the first 15 years, equaling $50,000 at the end of the next 15 years, or at the end of 30 years of life. Since the bond issue has been amortized at that time, the total operating costs will be \$143,000 − (\$33,000 + \$16,000), or \$84,000, and the unit cost per kilowatt-hour will be \$0.0076.

In view of these varying rates for power and the other factors which have been mentioned, it is difficult to arrive at a conclusion. Perhaps a policy could be summed up as follows:

Case A.—Where there are public and private enterprises of the same nature in competition with each other, it would seem to be good business to let the privately owned and publicly owned utility exist side by side and in competition with each other, charging rates for their services determined on the same basis. The return from the publicly owned utility can then be passed along to its owners, the public, through decreased taxes or through larger services to the community.

Case B.—Where there is no competition between the public enterprise and the private enterprise, it would seem to be best to operate as a private enterprise except for the earning of a return. Charges for services should be set at the highest rate, as shown in Case III*a*, or the rate that is charged while the bond issue is being amortized. Then the savings in bond issue expenses could be used for expansion of the service. Specific examples of this type are sewer systems, roads, and pavements.

22.8. Criteria for Making Decisions.—In Chap. 11 several criteria were explained for judging the worth-whileness of a project. For a privately owned and operated project, the criterion of unit operating cost plus expected return on investment meets the public in a selling price. The financial yield rate on the investor's share in the project meets the investing public as a measure of its interest in taking the risk.

When a project is publicly owned, it falls into either of the two cases discussed in the conclusion of Sec. 22.7, namely, Case *A*, where there is private competition, and Case *B*, where there is no private competition.

For those projects falling into Case *A*, the criteria used by private enterprise may be used only in part. The useful criterion is that of unit operating cost. It must be borne in mind that, as shown in Sec. 22.7, the unit operating cost for such a project includes all the normal operating costs as used by private enterprise plus charges for financing, which in private enterprises are borne by the return to the investor.

One sewage disposal plant for a town is all that is built because it is an essential utility for the public. No competition is allowed. Sewage disposal systems in other communities provide standards for comparison. The operating cost per unit of family or house or individual may be compared with the unit operating cost in other communities similar in size and population make-up. The public can judge then whether or not it is paying too much for its utility.

When several proposals are made to put into effect an extension of the sewer, another method or criterion must be used. The one generally used is the one discussed in Chap. 21; namely, capitalization of a perpetuity. In this method the utility is assumed to have perpetual life; that is, its need is so essential that it will continue to be in demand for an indefinite period. The method is to add to the first cost of the project the capitalized cost of operating costs using a rate of interest that represents the value of money at that time.

When improvements to highways are proposed, such as straightening or removing grades, their justification is made in terms of the savings in cost of car operation that will accrue to those who use the highways. These savings are determined by an analysis of costs of car operation before and after the proposed change. The capitalized value of the savings then represents the allowable investment in first cost for the improvements.

However, if the project was the complete building of a new highway, the method used for the sewer would be used, in which the capitalized first costs or construction costs would be added to the capitalized operating costs and the capitalized savings. The resulting sum would be used as a criterion in deciding which one of the several proposed solutions should be used. In general, the minimum capitalized cost should be selected, although this decision is greatly affected by the results of the intangible and financial analyses.

CHAPTER 23

STUDIES AND REPORTS

23.1. Objectives of Engineering Economic Analysis.—It should be emphasized very strongly that the technique of engineering economic analysis is to uncover and bring to light as many of the relationships as possible between whatever factors may be inherent in the problem or situation.

The choice of any line of action by individuals or groups is dependent upon the sum total of their personalities. By this is meant the sum total of the points of view which they have concerning those factors discussed as belonging to the realm of intangible factors.

It is fallacious, therefore, ever to expect a formula to contain all the intangible factors. Consequently, it is the belief of this writer that each problem should be examined on the basis of the principles of analysis presented in this book. The various approaches are then brought together and evaluated in the light of the experiences and judgments of those whose duty it is to make the decision or choice.

23.2. Kinds of Studies.—The several studies presented in the previous chapters make up the technique of engineering economic analysis. Briefly summarized, they are as follows:

1. First cost and investment. An analysis of the elements that make up the first cost of a project together with the necessary investment in working capital required to operate it.

2. Valuation of first cost. The process by which a money value is placed on those elements of first cost and investment which it is proposed to incorporate in the project.

3. First cost patterns. The relationship between total cost and unit cost as compared with capacity or size of equipment.

4. Estimating first cost. The procedure by which first cost is estimated for those elements which must be assembled or built.

5. Operating cost. An analysis of the elements that make up the cost of operating the project.

6. Estimating operating cost. The procedure by which operating cost is estimated.

7. Operating cost patterns. The relationship between unit operating cost and capacity or rate of production of the project.

8. Income, return, and yield. The relationship existing between first cost and investment, operating cost, income, and financial ownership of the first cost and investment.

9. Economy analysis of a single opportunity. The application of the profit and loss statement procedure to the determination of economic yield.

10. Intangible analysis of a single opportunity. The importance of the consideration of nonmonetary aspects inherent in projects or situations.

11. Financial analysis of a single opportunity. The application of the profit and loss and balance sheet statement to the determination of financial yield for various methods of financing.

12. In-favor-of analysis. The comparison from all angles of alternative opportunities.

13. Problems of capacity and rates of production. The effect of varying rates of production or operation on operating cost and economic and financial yields.

14. Increment cost problems. The application of the "out-of-pocket" viewpoint on first cost and investment, operating cost, and economic financial yields of proposed additions to existing projects.

15. Replacement problems. The adjustment of the economic and financial factors present in the replacement of the equipment.

16. Present vs. future investment. A method of comparing present action with deferred or delayed future action.

The end results of these several studies provide factual data on which choices and decisions are made. These results are as follows:

1. Total first cost and investment
2. Total annual operating cost
3. Unit operating cost at a certain per cent capacity or rate of production
4. Total annual operating cost plus return on first cost and investment
5. Unit operating cost plus return on first cost and investment at a certain per cent capacity or rate of production

6. Total annual income
7. Unit income
8. Total annual return on first cost and investment
9. Per cent economic yield on first cost and investment
10. Total annual financial return on first cost and investment
11. Per cent financial yield on first cost and investment
12. Equal, greater, or lower values for the various factors in an alternative project
13. Values for capacity factor, load factor, diversity factor, demand factor, and power factor
14. Increment values for first cost and investment income, operating costs and return on investment
15. Per cent economic and financial yields on increment investment
16. Required capital funds and their sources
17. Capitalized values for a perpetual project
18. Endowment values for a limited life project

23.3. Choice of Kind of Study.—In the Introduction, the scope of the engineering process was defined as consisting of a possible ten steps:

1. Research on the problem of satisfying the need
2. Invention or adaptation of existing means and methods to perform the task efficiently
3. Preliminary valuation of the project
4. Financial planning of the project
5. Organization for effort and cooperation
6. Promotion of project
7. Design of project
8. Design and construction of plant
9. Supervision of operation and production
10. Sale and distribution of product

Also engineering projects were classified into three types:

1. Equipment required to provide a specified service
2. Method of operation of equipment to provide a specified service
3. Rate of production of equipment

The application of the several kinds of studies and their end results will depend upon the step in the engineering process at which the proposals under consideration occur. It is impossible to specify in advance in any specific case which kind of study is to be used. The selection must depend on the question to be answered. In some cases, the first cost is the only factor wanted; in others, the operating cost may be desired. The answer to the question, "Will it pay?" generally cannot be answered unless the project is examined from all angles by using nearly all the kinds of studies to get enough information on which to base a decision.

In most of the steps of the engineering process, the fourfold analysis by means of

1. The economy analysis
2. The intangible analysis
3. The financial analysis
4. The in-favor-of analysis

is indicated.

23.4. The Analysis.—The method, in general, of studying a particular project involves the following steps:

1. Description of the project, problem, or situation. This step is a statement of the difficulty that demands an answer. It is a statement of such length that the difficulty and general conditions involved are clear.

2. The technical answer to the difficulty. This is a solution prepared by the engineer; it is a technical solution to the difficulty but may not be the economic solution to it. Usually a number of technical solutions are presented from which one may be selected as being the one that best gives an economic solution.

3. Step of the engineering process. An identification of the character of the difficulty.

4. Choice of kind or kinds of study. The selection of the method of analysis and reasons for the selection.

5. The study or studies in detail. This step consists of the actual working out of the several studies in full detail.

6. Recapitulation of results obtained from the studies. This is the assembling of the pertinent end results of the several studies in a form that can be easily studied apart from their derivation.

7. The brief. This is a statement giving in full the argument concerning the relationships of the several factors in the difficulty.

8. The recommendation. This is, in effect, the answer that was sought.

23.5. The Report.—The report is a statement presented to one's superior concerning the problem, project, or difficulty. It is essentially the analysis in reverse. The busy executive wishes to learn the conclusions that have been reached in the quickest possible manner. For this reason, the analysis is presented in reverse, with the recommendation first, then followed and supported by the detailed analysis.

The order of the report should, therefore, be

1. Description of the project, problem, or situation
2. The recommendation
3. The brief, or argument
4. Recapitulation of end results
5. The studies in detail
6. The technical solutions

APPENDICES

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[illegible]

APPENDIX A

INTEREST

A.1. Interest.—Interest may be called the "time value" of money. It is the price that the lender charges the borrower for the use of money.

A.2. Simple Interest.—For some time I have had an account in the savings department of a bank. When I opened the account, I deposited the sum of $1,000. Each year the bank pays me a sum of money for the use of my money. This sum amounts to $30. I do not let this amount stay in the savings account but withdraw it for my own use each year. Thus, I always have $1,000 in the bank and I receive from the bank each year $30. In banking language the bank pays me a simple interest at the rate of 3 per cent.

The elements of simple interest are

1. *Principal* is the sum of money lent or borrowed. P
2. *Interest* is the price or charge for the use of the money. I
3. *Time* is the period of time during which the interest is paid. n
4. *Rate* is the price paid for a unit of money per unit of time. i

NOTE: The symbol for rate is often r.

The equation that connects these elements is

Interest = (principal)(rate)(time)

In terms of symbols the equation is

$$I = (P)(i)(n) \tag{A-1}$$

In terms of the equation, the illustration above would be

$$I = (\$1{,}000)(0.03)(1)$$
$$I = \$30$$

In this case the principal is $1,000, the rate is 3 per cent or 0.03, the time is 1 year, and the interest is $30.

If the \$1,000 had been at interest for ½ year, the equation would have been

$$I = (\$1{,}000)(0.03)(\tfrac{1}{2})$$
$$I = \$15.$$

A.3. Compound Interest.—In the illustration for simple interest, I withdrew my interest each year when the bank paid it. A friend of mine does not withdraw his interest but lets it stay in the bank. Consequently at the beginning of the second year, he has a sum of \$1,030 in the bank. He received for this sum, interest amounting to \$30.90 at the end of the second year. The difference in the interest received, \$30.90 − \$30.00, or \$0.90, represents the price paid for using the interest from the preceding year which was left in the bank. This is really interest on interest. In banking language, this process is called "compound interest."

The elements of compound interest are

1. *Principal*, the original sum of money.
2. *Interest*, the price paid or the charge made for the use of money.
3. *Time*, the period of time during which interest is charged, measured in some specific unit. The unit may be a day, week, month, 3 months, 6 months, or a year.
4. *Rate*, the price paid for a unit of money for a unit of time. It is given as a per cent or so many hundredths of the original amount.
5. *Amount* is the sum of the principal and the interest at the end of any specific number of units of time.

In compound interest, the interest is computed not only on the principal but also on the accumulated interest.

The symbols used in compound interest are the same as those used in simple interest, with the addition of A to represent the amount and n to represent the number of periods of time.

The equation that connects these elements is

$$A = P(1 + i)^n \tag{A-2}$$

The illustration given would be, in terms of the equation,

$$A = (\$1{,}000)(1 + 0.03)^1$$

The amount at the end of the second year would be

$$A = (\$1{,}000)(1 + 0.03)^2$$
$$A = (\$1{,}000)(1.0609) = \$1060.90$$

A.4. Derivation of Compound Interest Equation.—In order to derive the equation for compound interest, use the principles of simple interest as previously developed. For illustration purposes, determine the amount at the end of three periods of time.

At the end of the first period,

$$\text{Interest} = (P)(i)$$
$$\text{Amount} = P + Pi, \text{ or } P(1 + i)$$

At the end of the second period,

$$\text{Interest} = P(1 + i)(i)$$

which is the amount at the end of the first period multiplied by the interest rate i.

Amount is the amount at the end of the first period plus interest for the second period, or

$$\text{Amount} = P(1 + i) + P(1 + i)(i)$$

which equals, when the proper algebraic factoring is done,

$$P(1 + i)(1 + i), \text{ or } P(1 + i)^2$$

At the end of the third period,

$$\text{Interest} = P(1 + i)^2(i)$$

which is the amount at the end of the second period multiplied by the interest rate i. Amount equals the amount at the end of the second period plus the interest for the second period or

$$P(1 + i)^2 + P(1 + i)^2 i$$

which equals, when the proper algebraic factoring is done,

$$P(1 + i)^2(1 + i), \qquad \text{or} \qquad P(1 + i)^3$$

The process can be carried on for any number of periods. An examination of the equations for the amounts at the end of each of the three periods,

First period:

$$A = P(1 + i)^1$$

Second period:

$$A = P(1 + i)^2$$

Third period:

$$A = P(1 + i)^3$$

as shown above, indicates that the problem can be generalized. When this is done, the symbol n replaces the figure representing the number of periods. The equation is then,

$$A = P(1 + i)^n \tag{A-3}$$

It is most important to note the meaning of the symbols i and n. The symbol i represents the interest rate on a unit of money for a period of time n.

If the principal P is placed at interest for 4 years at an interest rate of 6 per cent per year, the interest being paid or compounded each half year, what is the amount A at the end of the fourth year with all interest paid?

The interest rate i in this case is 3 per cent because the interest is paid or computed twice a year.

The number of periods n is 8 because there are 8 half-year periods in the 4 years. The equation is, therefore,

Example:

$$A = P(1 + 0.03)^8$$

When P is \$1,000,

$$A = (\$1{,}000)(1.03)^8$$
$$A = (\$1{,}000)(1.267) = \$1{,}267$$

Period	Amount of beginning of period	Interest during period	Total amount at end of period
1	\$1,000.00	\$30.00	\$1,030.00
2	1,030.00	30.90	1,060.90
3	1,060.90	31.82	1,092.72
4	1,092.72	32.78	1,125.50
5	1,125.50	33.77	1,159.27
6	1,159.27	34.78	1,194.05
7	1,194.05	35.82	1,229.87
8	1,229.87	36.90	1,266.77

A.5. Present Worth.—In Sec. A.4, it was shown how to determine the amount A that a certain principal P will reach when left at compound interest at an interest rate i for n periods of time. Sometimes it is desired to know what certain principal P should be placed at interest rate i for n periods of time in order to equal the amount A. In other words, it is desirable to find the present worth P of the amount A.

The equation for compound interest is

$$A = P(1 + i)^n \tag{A-3}$$

Rearrange the equation so as to solve for the quantity P,

$$P = \frac{A}{(1 + i)^n} \tag{A-4}$$

In Sec. A.3, it was found that, if \$1,000 were placed at 3 per cent interest for two periods, it would amount to \$1,060.90. It is now desired to know what principal P should be placed at 3 per cent interest for two periods so as to equal \$1,060.90. The problem is to find the present worth, two periods back, of \$1,060.90.

Substituting in the equation for the present worth,

$$P = \frac{\$1{,}060.90}{(1 + 0.03)^2}$$

$$P = \frac{\$1{,}060.90}{1.0609} = \$1{,}000$$

A.6. Other Symbols and Formulas.—The terminology of interest varies with the writers. One of the best suggestions is that which substitutes a single symbol for parts of the compound interest and present worth formulas.

The compound interest formula is

$$A = P(1 + i)^n \tag{A-2}$$

If the symbol s is substituted for $1 + i$, the equation becomes

$$A = Ps^n \tag{A-5}$$

The present worth formula is

$$P = \frac{A}{(1 + i)^n} \tag{A-4}$$

If the symbol p is substituted for $\frac{1}{1 + i}$, the equation becomes

$$P = Ap^n \tag{A-6}$$

Note that the symbol p^n is equivalent to $\frac{1}{s^n}$.

A.7. Calculations and Tables.—The calculation of a problem such as $A_{13} = \$1{,}234(1 + 0.04)^{13}$ would be quite laborious if it were necessary to do it in the same manner as the equation for compound interest was derived. Fortunately there are two shorter methods of doing the work.

1. *Logarithms.*—The problem may be restated as

$$A_{13} = (\$1{,}234)(1.04)^{13}$$

The equivalent of $(1.04)^{13}$ is the difficult part of the problem. Using the principles of logarithms it can be done quite easily.

$$\begin{array}{lr} \log 1.04 & = 0.0170 \\ \text{Multiply by 13} & \times\ 13 \\ \hline & 510 \\ & 170 \\ \hline \log\ (1.04)^{13} & = 0.2210 \end{array}$$

Find the antilogarithm of 0.2210.

$$\text{antilog } 0.2210 = 1.663$$

The amount $(A)_{13}$ is, therefore, $(\$1{,}234)(1.663) = \$2{,}052.38$

2. *Tables.*—It is apparent that considerable work is involved when problems must be solved by either arithmetic or logarithms. Tables have been prepared giving the values of s^n and p^n. To solve the previous problems by means of tables, the value of

$(1.04)^{13}$ is found from a table. A three-place table gives 1.665. A four-place table gives 1.6651.

$$A = (\$1{,}234)(1.665)$$
$$A = \$2{,}054.61$$

The difference in the two answers, \$2,052.38 and \$2,054.61, is due to the use of four-place logarithms for the first and factor values from tables which were calculated by using logarithms of many more places than four. Obviously, interest tables with five places are more accurate than those having only three or four.

For most of the purposes of engineering studies, three-place interest tables are satisfactory. In fact, slide-rule calculations are acceptable when performed with care.

A.8. Time Schedule.—This is a graphic device for making clear certain problems in compound amount and present worth, for the solution of problems of equivalence, and for the determination of the basic equations of sinking funds and annuities.

It consists of appropriate scales which indicate time or periods of time during which interest is calculated or paid.

Year	0	1	2	3	4	5	6	7
Interest Periods	n_0	n_1	n_2	n_3	n_4	n_5	n_6	n_7
Dates	1930	1931	1932	1933	1934	1935	1936	1937

A.9. Equivalence.—A man lends \$5,000 in units of \$1,000, which he expects to get back in 2, 4, 6, 8, and 10 years. The amount expected in each case is the compound amount of \$1,000 at 6 per cent for the number of years indicated. Respectively, these amount to \$1,124 for 2 years, \$1,262 for 4 years, \$1,419 for 6 years, \$1,594 for 8 years, and \$1,791 for 10 years. The principle of equivalence states that each of these loans is equivalent to the others because the lender receives the same rate of return or interest rate from each one. The lender can get any one of the above future payments for the initial sum of \$1,000.

The lender may wish after the first year to have his total loan of \$5,000 returned to him at the end of 5 years. He wishes to know what is the single equivalent payment which would equal all of the five separate payments if made at the end of 5 years. It is obvious that the payment equivalent to any of the single payments at the end of 5 years is the compound amount of

$1,000 at that time which is $1,338. Since there were five loans, the equivalent single payment would be 5 × $1,338 or $6,690. Another way to compute this equivalent single payment is to get the compound amounts and present worths of each of the payments to the end of 5 years, as follows:

Year	Original Repayment		Compound Amount	Present Worth
2	$1,124	×	(1.191) = $1,338	
4	1,262	×	(1.060) = 1,338	
6	1,419	×		(0.9434) = $1,338
8	1,594	×		(0.8396) = 1,338
10	1,791	×		(0.7473) = 1,338

The total of the five payments is (5)($1,338) = $6,690

It is sometimes desired to determine the sum of money which will be equivalent to several other sums payable on different dates. Its use consists in the fixing of a common date and the proper relation of the other sums to it, either by use of compound interest or by the use of present worth.

Example: The following sums are payable at the end of the years indicated:

M_1	$ 500	1940
M_2	1,000	1938
M_3	300	1945
M_4	2,000	1948

Three questions may be asked:

1. What is the equivalent sum at the end of the year 1948, interest at 6 per cent per year compounded annually?
2. What is the equivalent sum at the end of the year 1942, interest at 6 per cent per year compounded annually?
3. What is the equivalent sum at the end of the year 1938, interest at 6 per cent per year compounded annually?

Analysis.—In each case the interest rate (i) is 6 per cent or 0.06. In each case (n) represents 1 year. The value of (n) in each case will depend upon the year of reference. In question 1 the year of reference is the end of 1948; in question 2 it is the end of 1942; and in question 3 it is the end of 1938.

Prepare a time schedule as follows:

A. Line 1: Time scale in years.

B. Line 2: Locate sums M_1, M_2, M_3, and M_4 in accordance with the date of their payment.

C. Line 3: Indicate the application of compound amount or present worth by using the appropriate symbols.

D. Line 4: Substitute for symbols, values from tables.

E. Line 5: Complete calculations and summate values.

Years	1938	1939	1940	1941	1942	1943	1944	1945	1946	1947	1948
Sum	M_2		M_1					M_3			M_4

Problem 1

Reference year, 1948

1948 ↓ Point of reference

M_2s^{10} M_1s^8 M_3s^3 M_4s^0

($1,000)(1.791) $1,791

($500)(1.594) 797

($300)(1.191) ... 357

($2,000)(1) 2,000

Summate value, 1948 ..... $4,945

Problem 2

Reference year, 1942

1942 ↓ Point of reference

M_2s^4 M_1s^2 M_3p^3 M_4p^6

($1,000)(1.262) $1,262

($500)(1.124) 562

251 ...... ($300)(0.8396)

1,410 .... ($2,000)(0.705)

$3,485, summate value, 1942

Problem 3

Reference year, 1938

↓

Reference point

M_2s^0 M_1p^2 M_3p^7 M_4p^{10}

$1,000 ...... ($1,000)(1)

445 .... ($500)(0.8900)

199 .... ($300)(0.6651)

1,116 .... ($2,000)(0.5584)

$2,760, summate value, 1938

If, in the study of a situation, it becomes necessary to determine the equivalent sums at the end of each year in the above problem, then it is best to determine the equivalent sum at either end of the time scale and apply either compound amount or present worth calculations to them.

Example: The equivalent amount at the end of 1938 was found to be $2,760. What would be the equivalent sums at the end of each year from 1938 to 1948?

Year	Sum at End of Year				
1938			$2,760		
1939	($2,760)$s^1$	=	($2,760)(1.06)	=	$2,925
1940	($2,760)$s^2$	=	($2,760)(1.124)	=	$3,102
1941	($2,760)$s^3$	=	($2,760)(1.191)	=	$3,287
1942	($2,760)$s^4$	=	($2,760)(1.262)	=	$3,483
1943	($2,760)$s^5$	=	($2,760)(1.338)	=	$3,692
1944	($2,760)$s^6$	=	($2,760)(1.419)	=	$3,916
1945	($2,760)$s^7$	=	($2,760)(1.504)	=	$4,151

Year	Sum at end of Year
1946	$(\$2,760)s^{8} = (\$2,760)(1.594) = \$4,399$
1947	$(\$2,760)s^{9} = (\$2,760)(1.689) = \$4,661$
1948	$(\$2,760)s^{10} = (\$2,760)(1.791) = \$4,943$

The problem may be also solved by determining first the equivalent sum at the end of 1948, in this case \$4,945 (see Prob. 1, page 319).

Year	Sum at End of Year
1948	\$4,945
1938	$(\$4,945)p^{10} = (\$4,945)(0.5584) = \$2,761$
1939	$(\$4,945)p^{9} = (\$4,945)(0.5919) = \$2,926$
1940	$(\$4,945)p^{8} = (\$4,945)(0.6274) = \$3,102$
1941	$(\$4,945)p^{7} = (\$4,945)(0.6651) = \$3,288$
1942	$(\$4,945)p^{6} = (\$4,945)(0.7050) = \$3,485$
1943	$(\$4,945)p^{5} = (\$4,945)(0.7473) = \$3,695$
1944	$(\$4,945)p^{4} = (\$4,945)(0.7921) = \$3,916$
1945	$(\$4,945)p^{3} = (\$4,945)(0.8396) = \$4,151$
1946	$(\$4,945)p^{2} = (\$4,945)(0.8900) = \$4,401$
1947	$(\$4,945)p^{1} = (\$4,945)(0.9434) = \$4,665$

By comparing the amounts arrived at for each year, it will be seen that the amounts by each method for the same year are the same. The slight variations are because of the difference between three-place and four-place factors.

The principle of equivalence may be used to develop the equations which apply to uniform series of payments.

A.10. Uniform Series of Payments.—The following questions may be answered by means of the principles of uniform series of payments:

1. If a certain amount D is paid at the end of each period of time for n periods and each amount allowed to accumulate at compound interest at a certain rate i per period, what will be the total amount S of all the payments and accumulated interest at the end of n periods?

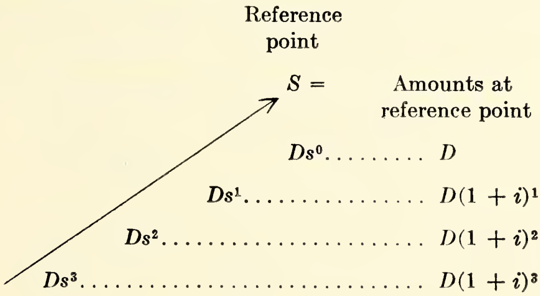
2. If an amount S is to be accumulated at the end of n periods of time by paying a certain amount D at the end of each period of time for n periods and each amount is allowed to accumulate at compound interest at a certain rate i per period, what must be the amount of each periodic payment D?

3. What amount R must be set aside at the beginning of a certain time, drawing compound interest at a rate i so as to be able to withdraw from the amount R and its accumulated interest, an amount D at the end of each interest period for n periods?

4. If an amount R is set aside at the beginning of a certain time, drawing compound interest at a rate i for n periods of time, what amount D is it possible to withdraw from the diminishing fund R at the end of each period for n periods?

A.11. Series Whose End Total Is S.—The basis for the solution of this problem is the principle of equivalence. Draw a time schedule, indicate payments, calculate their value by using as a reference point the end of the last period. The summation of the values of the separate payments will equal the amount looked for, S. Assume four payments.

Periods	0	1	2	3	4
Payments		D	D	D	D


Reference point
$S =$
Amounts at reference point
Ds^0 .... D
Ds^1 .... $D(1 + i)^1$
Ds^2 .... $D(1 + i)^2$
Ds^3 .... $D(1 + i)^3$

In equation form the amount S is equivalent to

$$S = Ds^3 + Ds^2 + Ds^1 + Ds^0$$

Substituting for s the equivalent $(1 + i)$,

$$S = D(1 + i)^3 + D(1 + i)^2 + D(1 + i)^1 + D$$

Since D is common to all terms, we may write

$$S = D[(1 + i)^3 + (1 + i)^2 + (1 + i)^1 + 1]$$

If we substitute for the expression within the brackets the symbol Y, the equation becomes

$$S = DY$$

What is the value of Y?

Since each term except the last one within the brackets is a binomial, it should be possible to expand each one and add.

$$\begin{array}{rl} (1+i)^3 &= 1 + 3i + 3i^2 + i^3 \\ (1+i)^2 &= 1 + 2i + i^2 \\ (1+i)^1 &= 1 + i \\ 1 &= 1 \\ \hline Y &= 4 + 6i + 4i^2 + i^3 \end{array}$$

It is remembered, from the study of the binomial theorem, that this value of Y looks somewhat like the expansion of a binomial. If the expression Y is multiplied by i and 1 added to it, there will result a perfect binomial expansion for $(1 + i)^4$. Multiply by i,

$$iY = 4i + 6i^2 + 4i^3 + i^4$$

Add 1,

$$1 + iY = 1 + 4i + 6i^2 + 4i^3 + i^4$$
$$1 + iY = (1 + i)^4$$

Subtract 1,

$$iY = (1 + i)^4 - 1$$

Divide by i,

$$Y = \frac{(1 + i)^4 - 1}{i}$$

Now substitute the value of Y in the original equation $S = DY$.

$$S = D\left[\frac{(1 + i)^4 - 1}{i}\right]$$

If five payments had been made, the same equation would have been obtained with the exception that the exponent of the expression $(1 + i)$ would have been 5. For six payments, the exponent would have been 6, and so on. The equation can, therefore, be generalized by substituting for the numerical exponent the symbol n which represents the number of periods of time during which interest is paid, at the same time the number of payments.

The generalized equation is

$$S = D\left[\frac{(1 + i)^n - 1}{i}\right] \qquad (A\text{-}7)$$

Example:
Periodic payment: \$100
Interest rate: 5 per cent
Number of periods: 6
Amount wanted: (S)

$$S = \$100\left[\frac{(1 + 0.05)^6 - 1}{0.05}\right]$$

The value of this expression may be found from tables specially prepared for this purpose. Looking in Table B-3 for an interest rate of 5 per cent and six periods, there is found the factor 6.802. Therefore,

$$S = (\$100)(6.802) \text{ or } \$680.20$$

If such a table giving factors of this kind is not available for a series of payments, a table of compound amounts can be used. The value of the factor $(1 + 0.05)^6$ found from Table B-1 is 1.340. Therefore,

$$S = \frac{(\$100)(1.340 - 1)}{0.05} = \frac{(\$100)(0.340)}{0.05} = \$680$$

Period	Amount at beginning of period	Interest during period	Payment at end of period	Total amount at end of period
1	\$000.00	\$000.00	\$100	\$100.00
2	100.00	5.00	100	205.00
3	205.00	10.25	100	315.25
4	315.25	15.76	100	431.01
5	431.01	21.55	100	552.50
6	552.56	27.63	100	680.19

A.12. Amount D for a Series Whose End Total Is S.—The value for the periodic payment may be found from the equation for the S series of uniform payments.

$$S = D\left[\frac{(1 + i)^n - 1}{i}\right] \tag{A-7}$$

Solving this equation for D gives

$$D = S\left[\frac{i}{(1+i)^n - 1}\right] \qquad (A\text{-}8)$$

Example:
Amount of the series S: \$680.20
Interest rate i: 5 per cent
Number of periods n: 6
Payment wanted: (D)

$$D = (\$680.20)\left[\frac{0.05}{(1+0.05)^6 - 1}\right]$$

The value of the expression in brackets may be found from tables specially prepared for this purpose. Looking in Table B-4 for an interest rate of 5 per cent and six periods, the factor is found to be 0.1470. Therefore,

$$D = (\$680.20)(0.1470) = \$99.98$$

If this type of table is not available, the problem may be solved by finding the reciprocal of the factor for the same interest rate and number of periods from Table B-3 for the amount of an S series. It is 6.802. The reciprocal is 1/6.802 or 0.1470.

This is the same value obtained from Table B-4 for values of D in an S series. Still another way to do this problem, if one has only a compound amount table, such as Table B-1, is to find the value of the expression $(1 + 0.05)^6$ and substitute it in the equation. The value is 1.340.

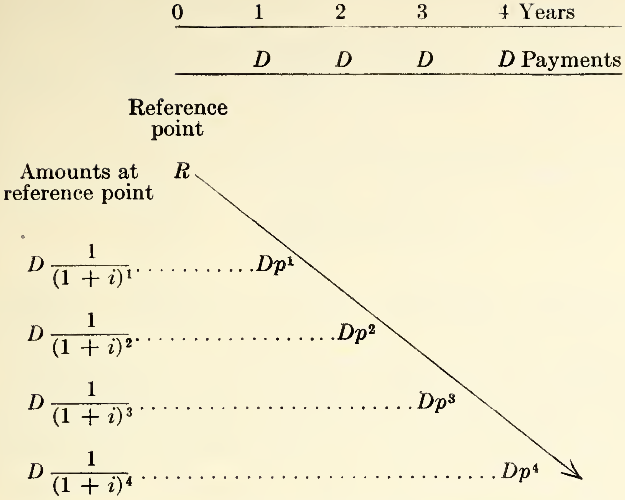
$$\frac{0.05}{1.340 - 1} = 0.1470.$$

Substituting we get

$$D = (\$680.20)\left(\frac{0.05}{1.340 - 1}\right)$$

$$D = (\$680.20)\left(\frac{0.05}{0.340}\right) = \$99.98$$

A.13. Series Whose Initial Total Is R.—The basis for the solution of this problem is also the principle of equivalence. Draw a time schedule, indicate the payments, calculate their value using as a reference point the initial date. The summation of the values of the separate payments will equal the amount looked for, the sum R. Assume four payments.


0 1 2 3 4 Years
D D D D Payments
Reference point
Amounts at reference point
R
$D\frac{1}{(1+i)^1}$ Dp^1
$D\frac{1}{(1+i)^2}$ Dp^2
$D\frac{1}{(1+i)^3}$ Dp^3
$D\frac{1}{(1+i)^4}$ Dp^4

In equation form the amount R is equivalent to

$$R = Dp^1 + Dp^2 + Dp^3 + Dp^4$$

Substituting for p its equivalent, $\frac{1}{1+i}$,

$$R = D\frac{1}{(1+i)^1} + D\frac{1}{(1+i)^2} + D\frac{1}{(1+i)^3} + D\frac{1}{(1+i)^4}$$

Since D is common to all terms, we may write

$$R = D\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \frac{1}{(1+i)^4}\right]$$

If the symbol T is substituted for the expression within the brackets, the equation becomes $R = DT$.

What is the value of T?

By reducing the expression equivalent to T to one having a common denominator,

$$T = \frac{(1+i)^3 + (1+i)^2 + (1+i)^1 + 1}{(1+i)^4}$$

Note that the numerator is the same as the expression Y in Sec. A-11. We can therefore substitute the value of Y for the numerator. Then

$$T = \frac{\dfrac{(1+i)^4 - 1}{i}}{(1+i)^4}$$

Reducing the equation again, we get

$$T = \frac{(1+i)^4 - 1}{i(1+i)^4}$$

If five payments had been considered, the same equation would have been obtained with the exception that the exponent for the expression $(1+i)$ would have been 5; for six payments, the exponent would have been 6, and so on. It is possible to generalize and substitute for the numerical exponent the symbol n which represents the number of periods of time during which interest is paid and at the same time the number of payments. The generalized equation is

$$R = D\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] \qquad (A\text{-}9)$$

Example:

Periodic withdrawal or payment: \$100
Interest rate: 5 per cent
Number of periods: 6
Amount wanted: R

$$R = \$100\left[\frac{(1+0.05)^6 - 1}{0.05(1+0.05)^6}\right]$$

The value of the expression in brackets may be found from tables specially prepared for this purpose. Looking in Table B-5 for an interest rate of 5 per cent for six periods, the factor is 5.076. Therefore,

$$R = (\$100)(5.076), \text{ or } \$507.60$$

If a table giving such factors is not available for this kind of series of payments, a table of compound amounts can be used.

The value of the factor $(1 + 0.05)^6$ found from Table B-1 is 1.340. Then

$$R = (\$100)\left[\frac{1.340 - 1}{0.05(1.340)}\right]$$

$$R = (\$100)\left[\frac{0.340}{0.05(1.340)}\right] = \$507.47$$

Period	Amount at beginning of period	Interest during period	Amount plus interest	Payment or withdrawal during period	Total amount at end of period
1	\$507.60	\$25.38	\$532.98	\$100	\$432.98
2	432.98	21.65	454.63	100	354.63
3	354.63	17.73	372.36	100	272.36
4	272.36	13.63	285.99	100	185.99
5	185.99	9.30	195.29	100	95.29
6	95.29	4.76	100.05	100	0.05

A.14. Amount D for a Series Whose Initial Total Is R.—The value of the periodic withdrawal or payment may be found from the equation for the R series of uniform payments.

$$R = D\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

from which

$$D = R\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \qquad (A\text{-}10)$$

Example:
Amount of series R: \$1,246
Interest rate i: 5 per cent
Number of periods n: 20
Payment wanted: D

$$D = \$1{,}246\left[\frac{0.05(1+0.05)^{20}}{(1+0.05)^{20} - 1}\right]$$

The value of this expression may be found from tables specially prepared for this purpose. Looking in Table B-6 for an interest rate of 5 per cent for twenty (20) periods, we find that the factor is 0.0802. Therefore,

$$D = (\$1{,}246)(0.0802) = \$99.92$$

If this type of table is not available, the problem may be solved by finding the factor for the same interest rate and number of periods from a table for amount R series. Note that the value wanted is the reciprocal of the same expression appearing in the equation for an R series. It is 12.46. The reciprocal is 1/12.46, or 0.0802. This is the same value obtained from the special table for values of D in an R series. Still another way to do this problem, if one has only a table of compound amounts, is to find the value of the expression $(1 + 0.05)^{20}$ and substitute it in the equation. The value is 2.653.

Substituting, we get

$$D = (\$1,246)\left[\frac{0.05(1 + 0.05)^{20}}{(1 + 0.05)^{20} - 1}\right]$$

$$D = (\$1,246)\left[\frac{(0.05)(2.653)}{2.653 - 1}\right] = \$100$$

A.15. Nominal and Effective Interest Rates.—Interest rates, as usually quoted, are called "nominal" interest rates. Interest at 6 per cent means that the rate of interest is on a yearly basis unless some qualifying phrase is attached. Interest at 6 per cent payable semiannually means that interest is paid on the principal sum twice a year. As explained in Sec. A.4, there would be two periods per year and the interest rate per period would be one-half of the rate per year or 3 per cent. This is applicable to any problem in which the principal is compounded in accordance with the formula $A = (1 + i)^n$.

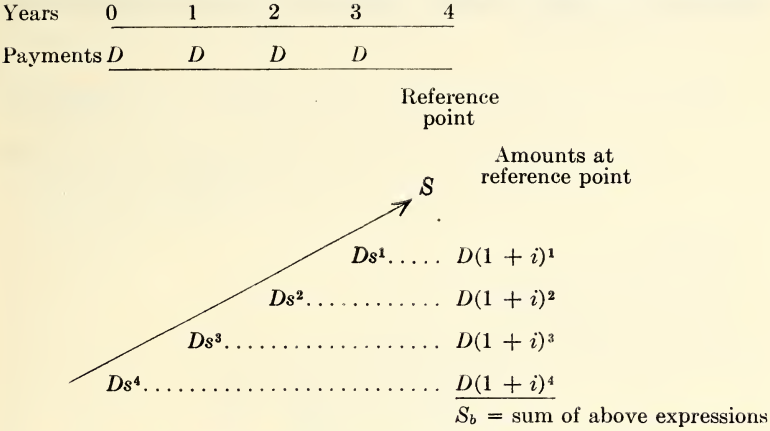
If, however, the problem is one in which interest is being paid on a debt, then there is a difference between the nominal interest rate and what is called the "effective" interest rate.

It is obvious that the amount at the middle of the year is equal to the amount at the beginning of the year multiplied by the compound factor for one period or $(1 + i)^1$. The amount at the end of the year will, therefore, be the amount at the middle of the year multiplied by the compound amount factor for one period or $(1 + i)^1$. Consequently the amount at the end of the year will be equal to the amount at the beginning multiplied by the compound amount factor for two periods, or $(1 + i)^2$. If the interest rate per period were 0.03, the factor for two periods would be $(1.03)^2$ or 1.061. The effective interest rate would therefore be 6.1 per cent per one-year period. Thus the original semiannual rate is converted into an effective annual rate.

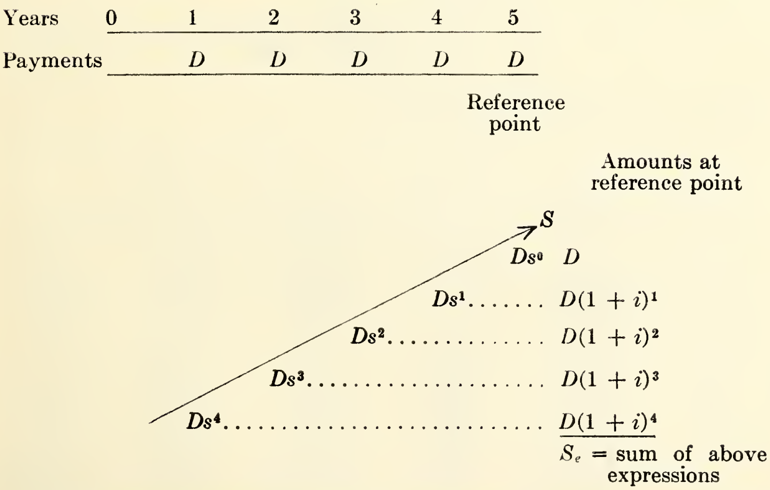
A.16. Uniform Series of Payments—Payments Made at Beginning of Period.—The previous discussion of uniform series has dealt with series in which the payments were made at the end of the interest periods. Sometimes it is necessary to determine the values for a series in which the payments or withdrawals are made at the beginning of the period.

A simple way to extend the principles of uniform series with payments at the end of the periods to one having payments at the beginnings of periods is to compare them on the basis of the equivalence principle.

EQUIVALENCE FOR SERIES OF PAYMENTS MADE AT BEGINNING OF PERIODS


Years 0 1 2 3 4
Payments D D D D
Reference point
Amounts at reference point
S
Ds^1 ..... $D(1 + i)^1$
Ds^2 ..... $D(1 + i)^2$
Ds^3 ..... $D(1 + i)^3$
Ds^4 ..... $D(1 + i)^4$
S_b = sum of above expressions

EQUIVALENCE FOR SERIES OF PAYMENTS MADE AT ENDS OF PERIODS


Years 0 1 2 3 4 5
Payments D D D D D
Reference point
Amounts at reference point
S
Ds^0 D
Ds^1 ..... $D(1 + i)^1$
Ds^2 ..... $D(1 + i)^2$
Ds^3 ..... $D(1 + i)^3$
Ds^4 ..... $D(1 + i)^4$
S_e = sum of above expressions

By comparing the expressions that make up the sums of these series, it is discovered that the only difference between them is that the second one is larger than the first by one payment D. It is also seen that the first series is made up of four payments and the second is made up of five payments. The rule for uniform series of payments, when made at the beginning of the interest periods, may then be stated as follows:

Calculate the amount S for a series consisting of one more payment D than required. From this amount S_e subtract one payment D. The resulting amount is the sum S_b which we wanted to determine.

NOTE: S_e = a series in which the payments are made at the end of each interest period

S_b = a series in which the payments are made at the beginning of each interest period

Example:

Periodic payment: $100

Interest rate: 5 per cent

Number of periods: 6

Calculate the value of S_e for 7 payments.

$$S_e = (\$100)\left[\frac{(1+0.05)^7 - 1}{0.05}\right]$$

$$S_e = (\$100)\left(\frac{1.407 - 1}{0.05}\right)$$

$$S_e = (\$100)\left(\frac{0.407}{0.05}\right) = \$814$$

Subtract the amount of one payment,

$$S_b = \$814 - \$100 = \$714$$

Period	Amount at beginning of period	Payment at beginning of period	Total amount at beginning of period	Interest during period	Total amount at end of period
1	$000.00	$100	$100.00	$ 5.00	$105.00
2	105.00	100	205.00	10.25	215.25
3	215.25	100	316.25	15.76	331.01
4	331.01	100	431.01	21.55	452.56
5	452.56	100	552.56	27.63	580.19
6	580.19	100	680.19	34.00	714.19

The same analysis may be applied to the determination of the amount R that is required by a uniform series when the payments are made at the beginning of the interest periods instead of at the end. The same notation applies to this R as applied to the S series.

The rule is as follows: Calculate the amount R_e for a series consisting of one less payment D than that required. To this amount R_e add one payment D. The resulting amount is the sum R_b which we wished to determine.

Example:
Periodic payment: $100
Interest rate: 5 per cent
Number of periods: 6
Amount wanted: R_b
Calculate the value of R_e for 5 payments.

$$R_e = (\$100)\left[\frac{(1 + 0.05)^5 - 1}{0.05(1 + 0.05)^5}\right]$$
$$R_e = (\$100)\left[\frac{(1.276 - 1)}{0.05(1.276)}\right]$$
$$R_e = (\$100)\left[\frac{0.276}{0.05(1.276)}\right]$$
$$R_e = \$432.60$$

Add the amount of one payment.

$$R_b = \$432.60 + \$100.00 = \$532.60$$

Period	Amount at beginning of period	Payment or withdrawal at beginning of period	Amount left at beginning of period	Interest during period	Total amount at end of period
1	$532.60	$100	$432.60	$21.63	$454.23
2	454.23	100	354.23	17.71	371.94
3	371.94	100	271.94	13.60	285.54
4	285.54	100	185.54	9.28	194.82
5	194.82	100	94.82	4.79	99.61
6	99.61	100	0.00	0.00	0.00

A.17. Solving Interest Problems.—Problems involving compound interest can generally be analyzed into one or several basic concepts or operations such as those listed in the preceding paragraphs of this chapter.

These simple concepts or combinations of them may be shown graphically as an aid to their solution.

I. *Compound Amount.*—What will \$1,000 amount to at compound interest for 6 years at 6 per cent compounded semiannually?

Time Scale	0	1	2	3	4	5	6	Years
	0	2	4	6	8	10	12	Periods

\$1,000 ——————————————→ A

$n_{\text{years}} = 6 \qquad n_{\text{periods}} = 12$

$i_{\text{years}} = 6$ per cent $\qquad i_{\text{periods}} = 3$ per cent

$$A = (\$1{,}000)(1 + 0.03)^{12}$$
$$A = (\$1{,}000)(1.426) = \$1{,}426$$

How many years will it take for \$1,000 to equal \$1,426 if interest is 6 per cent compounded semiannually?

Time Scale	0	? Years
	0	No. periods = 2 × no. years

$P = \$1{,}000$ ——————————→ $A = \$1{,}426$

n years $= x \qquad n$ period $= 2x$

i years $= 6$ per cent $\qquad i$ period $= 3$ per cent

$$A = P(1 + 0.03)^{2x}$$
$$\frac{\$1{,}426}{\$1{,}000} = (1 + 0.03)^{2x}$$
$$1.426 = (1 + 0.03)^{2x}$$

The number 1.426 is the factor used in computing compound amount and represents the amount which \$1 at 3 per cent interest per period equals in $2x$ years. The value for $2x$ is found from Table B-1 for compound amount by finding a factor for two periods in the column for 3 per cent. Opposite the factor will be found 12 in the periods column indicating the number of periods at compound interest. Since $2x$ equals 12 periods, the number of years is 12 ÷ 2 or 6 years.

At what interest rate must \$1,000 be placed at compound interest compounded semiannually so as to equal \$1,426 in 6 years?

Time Scale	0	6 Years
	0	12 Periods

Principal $= \$1{,}000$ ——————————→ $A = \$1{,}426$

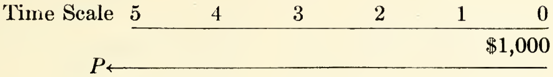
$n_{\text{years}} = 6 \qquad n_{\text{periods}} = 12$

$i_{\text{years}} = y \qquad i_{\text{periods}} = \frac{y}{2}$

$$A = P\left(1 + \frac{y}{2}\right)^{12}$$
$$\frac{\$1{,}426}{\$1{,}000} = \left(1 + \frac{y}{2}\right)^{12}$$
$$1.426 = \left(1 + \frac{y}{2}\right)^{12}$$

The number 1.426 is the factor used in computing compound amount and represents the amount which $1 at $y/2$ rate of interest per period equals in 12 periods. The value $y/2$ is found from an interest table for compound amount by finding in the horizontal line opposite 12 periods the factor 1.426. The column in which the factor is found is the interest rate desired, namely 3 per cent. Since $y/2$ equals 3 per cent, the rate per year will be 3 per cent multiplied by 2 or 6 per cent.

II. *Present Worth.*—What is the present worth of $1,000 for a period of 5 years at 4 per cent interest compounded annually?



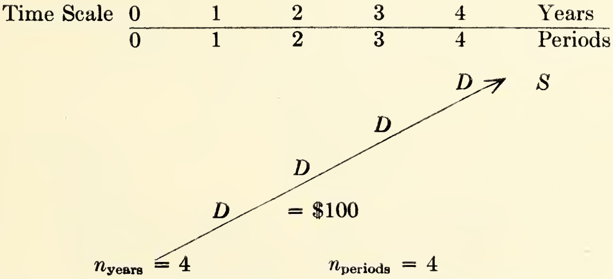
$n_{\text{years}} = 5 \qquad n_{\text{periods}} = 5$

$i_{\text{years}} = 4$ per cent $\qquad i_{\text{periods}} = 4$ per cent

$$P = (\$1{,}000)\left[\frac{1}{(1 + 0.4)^5}\right]$$

$$P = (\$1{,}000)(0.8219) = \$821.90$$

III. *S Series Determination of S.*—The sum of $100 is deposited in a fund at the end of each year for a period of 4 years at 4 per cent compounded annually. What will be the total amount at the end of 4 years?



$n_{\text{years}} = 4 \qquad n_{\text{periods}} = 4$

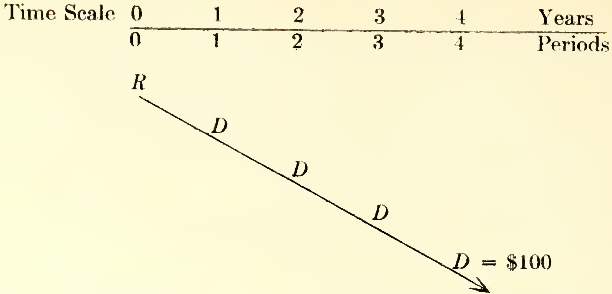
$i_{\text{years}} = 4$ per cent $\qquad i_{\text{periods}} = 4$ per cent

$$S = (\$100)\left[\frac{(1 + 0.04)^4 - 1}{0.04}\right]$$

$$S = (\$100)(4.246) = \$424.60$$

If the amount S is given and D is desired, the diagram is the same but the equation is rearranged. If both S and D are given and either interest rate i or number of periods n is given, the other may be found by using the principles used in solving the compound amount problems.

IV. *R Series. Determination of R.*—The sum of $100 is to be paid out of a fund at the end of each year for a period of 4 years. What amount must be deposited at the beginning of the first year in a fund drawing 4 per cent interest compounded annually?



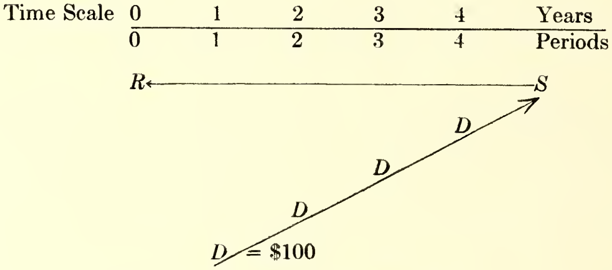
$$n_{\text{years}} = 4 \qquad n_{\text{periods}} = 4$$

$$i_{\text{years}} = 4 \text{ per cent} \qquad i_{\text{periods}} = 4 \text{ per cent}$$

$$R = (\$100)\left[\frac{(1 + 0.04)^4 - 1}{0.04(1 + 0.04)^4}\right]$$

$$R = (\$100)(3.630) = \$363$$

The determination of the value R may be obtained in another manner. Consider the $100 as being paid into a fund. Find the S value and take the present worth of it.



$$n_{\text{years}} = 4 \qquad n_{\text{periods}} = 4$$

$$i_{\text{years}} = 4 \text{ per cent} \qquad i_{\text{periods}} = 4 \text{ per cent}$$

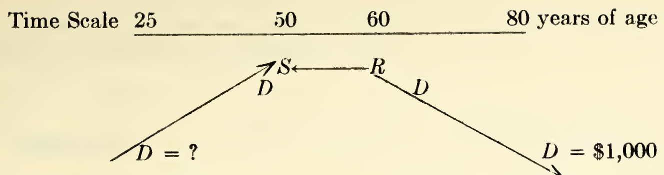
$$S = (\$100)\left[\frac{(1 + 0.04)^4 - 1}{0.04}\right]$$

$$S = (\$100)(4.246) = \$424.60$$

$$R = S\frac{1}{(1 + 0.04)^4}$$

$$R = (\$424.60)(0.8548) = \$362.94$$

V. *Combination Problem.*—I am 25 years old. I wish to have an income at the end of each year from age 60 to age 80. What amount of money must I deposit in a fund from age 25 to age 50 at the end of each year, assuming that the fund draws interest at the rate of 5 per cent per year up to age 60? At that time I transfer the accumulated amount to another fund at interest rate of 2 per cent compounded annually, from which at the end of each year I draw $1,000.



$n_{\text{years}} = n_{\text{periods}} = 25$
$i_{\text{years}} = i_{\text{periods}} = 5$ per cent

$n_{\text{years}} = n_{\text{periods}} = 20$
$i_{\text{years}} = i_{\text{periods}} = 2$ per cent

$n_{\text{years}}\ n_{\text{periods}} = 10$
$i_{\text{years}}\ i_{\text{periods}} = 5$ per cent

$$D = S\left[\frac{i}{(1+i)^n - 1}\right]$$

$$D = S\left[\frac{0.05}{(1+0.05)^{25} - 1}\right]$$

$$D = (\$10{,}037.26)(0.0210) = \$210.78$$

$$R = D\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$R = (\$1{,}000)\left[\frac{(1.02)^{20} - 1}{0.02(1.02)^{20}}\right]$$

$$R = (\$1{,}000)(16.35) = \$16{,}350$$

$$S = R\left[\frac{1}{(1+i)^n}\right]$$

$$S = R\left[\frac{1}{(1.05)^{10}}\right]$$

$$S = (\$16{,}350)(0.6139) = \$10{,}037.26$$

PROBLEMS

SIMPLE INTEREST

1. What is the simple interest for the following conditions?

	Principal	Annual interest rate, per cent	Time, years
a	$1,000	6	1
b	2,500	4	$\frac{1}{2}$
c	3,800	3	$2\frac{1}{2}$
d	550	$2\frac{1}{2}$	$\frac{1}{4}$
e	5,000	$2\frac{3}{4}$	$1\frac{1}{3}$

2. If I expect to get simple interest on a principal, what per cent interest rate per year should I expect in the following situations?

	Principal	Simple interest for period	Time, years
a	$ 2,000	$ 120.00	1
b	2,500	50.00	$\frac{1}{2}$
c	500	37.50	$2\frac{1}{2}$
d	8,400	52.50	$\frac{1}{4}$
e	50,000	1,833.33	$1\frac{1}{3}$

3. If I desire to have simple interest on a principal sum at certain rates of interest, what principal sum must I have under the following conditions?

	Simple interest for period	Interest rate per year, per cent	Time, years
a	$ 240	6	1
b	500	4	½
c	1,200	3	2½
d	100	2½	¼
e	4,400	2¾	1⅓

4. How long must I have the following principal sums at simple interest to receive simple interest amounts under the following conditions?

	Principal	Interest rate per year, per cent	Simple interest amount for the period
a	$10,000	6	$600.00
b	5,000	4	100.00
c	4,500	3	337.50
d	640	2½	4.00
e	3,000	2¾	110.00

COMPOUND INTEREST

5. What is the compound amount for the following conditions?

	Time, years	Principal	Interest rate per year, per cent	Number of interest periods per year
a	1	$3,500	6	1
b	5	1,200	3½	1
c	8½	500	8	2
d	12½	5,600	8	4
e	65	2,300	4½	1
f	10	300	6	3

6. What annual rate of interest is necessary to provide the compound amounts under the following conditions?

	Time, years	Principal	Number of interest periods per year	Compound amounts
a	1	$2,000	1	$2,120.00
b	5	2,500	1	2,897.50
c	8½	5,000	2	9,740.00
d	12½	1,100	4	2,961.20
e	65	150	1	3,576.00
f	10	400	3	724.40

7. How many years will it take to accumulate the compound amounts indicated under the following conditions?

	Principal	Interest rate per year, per cent	Number of interest periods per year	Compound amounts
a	$1,600	6	1	$1,696.00
b	300	4½	1	373.80
c	5,500	4	2	6,704.50
d	750	8	4	1,114.50
e	2,600	6	3	4,180.80
f	250	4½	1	896.00

8. What principal sum is necessary to yield the indicated compound amounts under the following conditions?

	Compound amounts	Interest rate per year, per cent	Number of interest periods per year	Time, years
a	$ 2,100.00	5	1	1
b	1,131.00	2½	1	5
c	826.50	6	2	8½
d	6,730.00	8	4	12½
e	33,685.20	3½	1	65
f	769.67	6	3	10

9. What is the present worth of the amounts under the following conditions?

	Amounts	Interest rate per year, per cent	Time, years	Number of interest periods per year
a	$ 2,400	6	1	1
b	500	4	11	1
c	1,100	5	$9\frac{1}{2}$	2
d	85,600	8	$16\frac{1}{4}$	4
e	65,000	$3\frac{1}{2}$	65	1
f	4,500	6	15	3

EQUIVALENCE

10. What amount on Jan. 1, 1942, is the equivalent of the following amounts using annual interest at 6 per cent compounded semiannually?

$1,000 on Jan. 1, 1940
$1,600 on July 1, 1940
$3,400 on Jan. 1, 1941
$ 500 on July 1, 1941
$2,500 on Jan. 1, 1942

11. What amount on Jan. 1, 1943, is the equivalent of the following amounts using annual interest at 8 per cent compounded semiannually?

$ 4,000 on Jan. 1, 1941
$ 3,600 on July 1, 1942
$ 4,300 on Jan. 1, 1945
$ 2,800 on July 1, 1948
$12,200 on Jan. 1, 1950

12. What amount on Jan. 1, 1940, is the equivalent of the following amounts using annual interest at 8 per cent compounded quarterly?

$3,000 on July 1, 1939
$4,500 on Oct. 1, 1939
$2,200 on Apr. 1, 1941
$1,800 on Jan. 1, 1945
$2,750 on Oct. 1, 1946

SERIES

13. What amounts will the following periodic payments be equal to under the following conditions (payments made at end of interest periods)?

	Periodic payment	Interest rate per year, per cent	Time, years	Number of periodic payments and interest periods per year
a	$ 300	4	10	1
b	250	$2\frac{1}{2}$	25	1
c	100	4	15	2
d	1,200	6	6	3
e	800	8	25	4
f	450	$3\frac{1}{2}$	18	1

14. What periodic payment is necessary in order to accumulate the indicated amounts under the following conditions?

	Interest, per cent	Amount	Time, years	Number of periodic payments and interest periods per year
a	4	$ 30,000	10	1
b	2½	35,000	25	1
c	3	3,000	4	1
d	4	4,000	10	2
e	8	89,000	6	4
f	3½	100,000	25	1

15. How many years will be necessary to accumulate the indicated amounts under the following conditions?

	Accumulated amount	Interest rate per year, per cent	Number of periodic payments and interest periods per year	Periodic payment
a	$ 9,171.20	3	1	$ 800
b	5,487.60	4	1	300
c	24,012.00	4	2	1,200
d	52,701.00	6	3	3,300
e	17,040.00	4½	1	750
f	6,206.40	8	4	160

16. What annual interest rate is required in order to accumulate the indicated amounts under the following conditions?

	Accumulated amount	Number of periodic payments and interest periods per year	Periodic payment	Time, years
a	$ 2,223.00	1	$ 250	8
b	35,815.56	1	860	25
c	36,331.52	2	2,560	6
d	129,402.00	3	1,800	15
e	58,100.00	1	100	75
f	59,780.00	4	350	18¾

17. What periodic withdrawals from a fund at the end of an interest period can be made under the following conditions?

	Fund	Time, years	Interest rate per year, per cent	Number of periodic withdrawals and interest periods per year
a	$100,000	12	4	1
b	65,000	12	3	1
c	12,000	25	4	2
d	4,000	4	4½	1
e	18,000	9½	4	2
f	5,000	15½	5	2

18. What must be the size of a fund so that I can withdraw the indicated periodic amounts from the fund under the following conditions?

	Periodic withdrawal	Time, years	Interest rate per year, per cent	Number of periodic withdrawals and interest periods per year
a	$ 500	13	4	1
b	1,000	20	4	2
c	950	4	3	1
d	2,400	18	2½	1
e	375	35	2	1
f	1,100	15	6	3

19. What annual interest rate must I receive on the indicated funds so that I may withdraw from them the indicated periodic amounts at the end of the interest periods under the following conditions?

	Fund	Time, years	Periodic withdrawal	Number of periodic withdrawals and interest periods per year
a	$ 2,106.00	8	$ 300	1
b	12,678.00	17	600	2
c	12,888.00	15	1,200	1
d	31,060.00	13½	1,500	2
e	26,982.00	6	1,800	3
f	4,537.50	4	1,250	1

20. For how many years may I receive the indicated periodic withdrawals under the following conditions?

	Fund amount	Interest rate per year, per cent	Number of periodic withdrawals and interest periods per year	Periodic withdrawals
a	$15,354.00	3	1	$1,800
b	15,420.00	4	2	1,200
c	62,388.00	2½	1	3,600
d	21,604.80	3½	1	2,400
e	13,680.00	6	3	500
f	15,108.00	4½	1	1,200

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21. What is the compound amount for the conditions indicated?

	Principal	Interest rate per year, per cent	Number of interest periods per year	Time, years
a	$1,600	3.8	1	6
b	5,600	4.75	1	18
c	350	2.5	2	11½
d	2,800	2.75	1	36
e	260	4.8	4	21¼
f	20	6.0	12	12

22. What annual rate of interest is necessary to accumulate the indicated amounts under the following conditions?

	Principal	Number of interest periods per year	Time, years	Accumulated amounts
a	$1,600	1	12	$6,900
b	5,600	1	27	8,000
c	350	2	12½	1,312
d	2,800	1	45	10,000
e	260	4	16¾	338
f	20	12	12	4,000

23. What amounts will the following periodic payments be equal to under the following conditions, payments being made at the end of interest periods?

	Periodic payment	Interest rate per year, per cent	Time, years	Number of periodic payments and interest periods per year
a	$ 300	3.8	6	1
b	150	4.75	18	1
c	800	2.5	11½	2
d	1,100	2.75	36	1
e	250	4.8	21¼	4
f	20	6.0	12	12

24. For how many years may I receive the indicated periodic withdrawal under the following conditions?

	Fund	Interest rate per year, per cent	Number of periodic withdrawals and interest periods	Periodic withdrawals
a	$ 4,095.00	3.8	1	$ 500
b	3,159.00	4.75	1	300
c	6,462.50	2.5	2	250
d	14,124.00	2.75	1	1,100
e	2,775.50	4.8	4	65
f	2,810.00	6.0	12	20

25. I wish to be able to withdraw $1,000 at the end of each interest period for 4 years from a fund which I have been accumulating for 20 years by depositing an amount at the end of each interest period into the fund. If the fund earns interest at the annual rate of 4 per cent compounded annually, what was the periodic deposit during the first 20 years?

26. If, in Prob. 25, the withdrawals and payments are made at the beginning of interest periods, what will the periodic deposit be during the 20 years?

27. If, in Prob. 25, the annual interest during the first 20 years is 3½ per cent and 2 per cent during the 4 years, what will the periodic deposit be during the 20 years?

28. If, in Prob. 25, the annual interest during the first 20 years is 4½ per cent and 2½ per cent during the 4 years, what will the periodic deposit be during the 20 years?

APPENDIX B

INTEREST TABLES

Table B-1.—Values of $(1 + i)^n$ Compound Amount Factor (s^n)
$A = P(1 + i)^n$

n	2%	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	1.020	1.025	1.030	1.035	1.040	1.045	1.050	1.060	1.070	1.080
2	1.040	1.051	1.061	1.071	1.082	1.092	1.103	1.124	1.145	1.166
3	1.061	1.077	1.093	1.109	1.125	1.141	1.158	1.191	1.225	1.260
4	1.082	1.104	1.126	1.148	1.170	1.193	1.216	1.262	1.311	1.360
5	1.104	1.131	1.159	1.188	1.217	1.246	1.276	1.338	1.403	1.469
6	1.126	1.160	1.194	1.229	1.265	1.302	1.340	1.419	1.510	1.587
7	1.149	1.189	1.230	1.272	1.316	1.361	1.407	1.504	1.606	1.714
8	1.172	1.218	1.267	1.317	1.369	1.422	1.477	1.594	1.718	1.851
9	1.195	1.249	1.305	1.363	1.423	1.486	1.551	1.689	1.838	1.999
10	1.219	1.280	1.344	1.411	1.480	1.553	1.629	1.791	1.967	2.159
11	1.243	1.312	1.384	1.460	1.539	1.623	1.710	1.898	2.105	2.332
12	1.268	1.345	1.426	1.511	1.601	1.696	1.796	2.012	2.252	2.518
13	1.294	1.379	1.469	1.564	1.665	1.772	1.886	2.133	2.410	2.720
14	1.319	1.413	1.513	1.619	1.732	1.852	1.980	2.261	2.579	2.937
15	1.346	1.448	1.558	1.675	1.801	1.935	2.079	2.397	2.759	3.172
16	1.373	1.485	1.605	1.734	1.873	2.022	2.183	2.540	2.952	3.426
17	1.400	1.522	1.653	1.795	1.948	2.113	2.292	2.693	3.159	3.700
18	1.428	1.560	1.702	1.857	2.026	2.208	2.407	2.854	3.380	3.996
19	1.457	1.599	1.754	1.923	2.107	2.308	2.527	3.026	3.617	4.316
20	1.486	1.639	1.806	1.990	2.191	2.412	2.653	3.207	3.870	4.661
21	1.516	1.680	1.860	2.059	2.279	2.520	2.786	3.400	4.141	5.034
22	1.546	1.722	1.916	2.132	2.370	2.634	2.925	3.604	4.430	5.437
23	1.577	1.765	1.974	2.206	2.465	2.752	3.072	3.820	4.741	5.871
24	1.608	1.809	2.033	2.283	2.563	2.876	3.225	4.049	5.072	6.341
25	1.641	1.854	2.094	2.363	2.666	3.005	3.386	4.292	5.427	6.848
26	1.673	1.900	2.157	2.446	2.772	3.141	3.556	4.549	5.807	7.396
27	1.707	1.948	2.221	2.532	2.883	3.282	3.733	4.822	6.214	7.988
28	1.741	1.996	2.288	2.620	2.999	3.430	3.920	5.112	6.649	8.627
29	1.776	2.046	2.357	2.712	3.119	3.584	4.116	5.418	7.114	9.317
30	1.811	2.098	2.427	2.807	3.243	3.745	4.322	5.743	7.612	10.06
31	1.848	2.150	2.500	2.905	3.373	3.914	4.538	6.088	8.145	10.87
32	1.885	2.204	2.575	3.007	3.508	4.090	4.765	6.453	8.715	11.74
33	1.922	2.259	2.652	3.112	3.648	4.274	5.003	6.841	9.325	12.68
34	1.961	2.315	2.732	3.221	3.794	4.466	5.253	7.251	9.978	13.69
35	2.000	2.373	2.814	3.334	3.946	4.667	5.516	7.686	10.68	14.79
40	2.208	2.685	3.262	3.959	4.801	5.816	7.040	10.29	14.97	21.72
45	2.438	3.038	3.782	4.702	5.841	7.248	8.985	13.76	21.00	31.92
50	2.692	3.437	4.384	5.585	7.107	9.033	11.47	18.42	29.46	46.90
55	2.972	3.889	5.082	6.633	8.646	11.26	14.64	24.65	41.32	68.91
60	3.281	4.400	5.892	7.878	10.52	14.03	18.68	32.99	57.95	101.3
65	3.623	4.978	6.830	9.357	12.80	17.48	23.84	44.14	81.27	148.8
70	4.000	5.632	7.918	11.11	15.57	21.78	30.43	59.08	114.0	218.6
75	4.416	6.372	9.179	13.20	18.95	27.15	38.83	79.06	159.9	321.2
80	4.875	7.210	10.64	15.68	23.05	33.83	49.56	105.8	224.2	472.0
85	5.383	8.157	12.34	18.62	28.04	42.16	63.25	141.6	314.5	693.5
90	5.943	9.229	14.30	22.11	34.12	52.54	80.73	189.5	441.1	1,019.0
95	6.562	10.44	16.58	26.26	41.51	65.47	103.0	253.5	618.7	1,497.0
100	7.245	11.81	19.22	31.19	50.50	81.59	131.5	339.3	867.7	2,200.0

TABLE B-2.—VALUES OF $\frac{1}{(1+i)^n}$—PRESENT WORTH FACTOR (p^n)

$$P = \frac{A}{(1+i)^n}$$

n	2%	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	0.9804	0.9756	0.9709	0.9662	0.9615	0.9569	0.9524	0.9434	0.9346	0.9259
2	0.9612	0.9518	0.9426	0.9335	0.9246	0.9157	0.9070	0.8900	0.8734	0.8573
3	0.9423	0.9286	0.9151	0.9019	0.8890	0.8763	0.8638	0.8396	0.8163	0.7938
4	0.9238	0.9060	0.8884	0.8714	0.8548	0.8386	0.8227	0.7921	0.7629	0.7350
5	0.9057	0.8839	0.8626	0.8420	0.8219	0.8025	0.7835	0.7473	0.7130	0.6806
6	0.8880	0.8623	0.8375	0.8135	0.7903	0.7679	0.7462	0.7050	0.6663	0.6302
7	0.8706	0.8413	0.8131	0.7860	0.7599	0.7348	0.7107	0.6651	0.6227	0.5835
8	0.8535	0.8207	0.7894	0.7594	0.7307	0.7032	0.6768	0.6274	0.5820	0.5403
9	0.8638	0.8007	0.7664	0.7337	0.7026	0.6729	0.6446	0.5919	0.5439	0.5002
10	0.8203	0.7812	0.7441	0.7089	0.6756	0.6439	0.6139	0.5584	0.5083	0.4632
11	0.8043	0.7621	0.7224	0.6849	0.6496	0.6162	0.5847	0.5268	0.4751	0.4289
12	0.7885	0.7436	0.7014	0.6618	0.6246	0.5897	0.5568	0.4970	0.4440	0.3971
13	0.7730	0.7254	0.6810	0.6394	0.6006	0.6543	0.5303	0.4688	0.4150	0.3677
14	0.7579	0.7077	0.6611	0.6178	0.5775	0.5400	0.5051	0.4423	0.3878	0.3405
15	0.7430	0.6905	0.6419	0.5969	0.5553	0.5167	0.4810	0.4173	0.3624	0.3152
16	0.7284	0.6736	0.6232	0.5767	0.5339	0.4945	0.4581	0.3936	0.3387	0.2919
17	0.7142	0.6572	0.6050	0.5572	0.5134	0.4732	0.4363	0.3714	0.3166	0.2703
18	0.7002	0.6412	0.5874	0.5384	0.4936	0.4528	0.4155	0.3503	0.2959	0.2502
19	0.6864	0.6255	0.5703	0.5202	0.4746	0.4333	0.3957	0.3305	0.2765	0.2317
20	0.6730	0.6103	0.5537	0.5026	0.4564	0.4146	0.3769	0.3118	0.2584	0.2145
21	0.6598	0.5954	0.5375	0.4856	0.4388	0.3968	0.3589	0.2942	0.2415	0.1987
22	0.6468	0.5809	0.5219	0.4692	0.4220	0.3797	0.3418	0.2775	0.2257	0.1839
23	0.6342	0.5667	0.5067	0.4533	0.4057	0.3634	0.3256	0.2618	0.2109	0.1703
24	0.6217	0.5529	0.4919	0.4380	0.3901	0.3477	0.3101	0.2470	0.1971	0.1577
25	0.6095	0.5394	0.4776	0.4231	0.3751	0.3327	0.2953	0.2330	0.1842	0.1460
26	0.5976	0.5262	0.4637	0.4088	0.3607	0.3184	0.2812	0.2198	0.1722	0.1352
27	0.5859	0.5134	0.4502	0.3950	0.3468	0.3047	0.2678	0.2074	0.1609	0.1252
28	0.5744	0.5009	0.4371	0.3817	0.3335	0.2916	0.2551	0.1956	0.1504	0.1159
29	0.5631	0.4887	0.4243	0.3687	0.3207	0.2790	0.2429	0.1846	0.1406	0.1073
30	0.5521	0.4767	0.4120	0.3563	0.3083	0.2670	0.2314	0.1741	0.1314	0.0994
31	0.5412	0.4615	0.4000	0.3442	0.2965	0.2555	0.2204	0.1643	0.1228	0.0920
32	0.5306	0.4538	0.3883	0.3326	0.2851	0.2445	0.2099	0.1550	0.1147	0.0852
33	0.5202	0.4427	0.3770	0.3213	0.2741	0.2340	0.1999	0.1462	0.1072	0.0789
34	0.5100	0.4319	0.3660	0.3105	0.2636	0.2239	0.1904	0.1379	0.1002	0.0730
35	0.5000	0.4214	0.3554	0.3000	0.2534	0.2143	0.1813	0.1301	0.0937	0.0676
40	0.4529	0.3724	0.3066	0.2526	0.2083	0.1719	0.1420	0.0972	0.0668	0.0460
45	0.4102	0.3292	0.2644	0.2127	0.1712	0.1380	0.1113	0.0727	0.0476	0.0313
50	0.3715	0.2909	0.2281	0.1791	0.1407	0.1107	0.0872	0.0543	0.0339	0.0213
55	0.3365	0.2572	0.1968	0.1508	0.1157	0.0888	0.0683	0.0406	0.0242	0.0145
60	0.3048	0.2273	0.1697	0.1269	0.0951	0.0713	0.0535	0.0303	0.0173	0.0099
65	0.2761	0.2009	0.1464	0.1069	0.0781	0.0572	0.0419	0.0227	0.0123	0.0067
70	0.2500	0.1776	0.1263	0.0900	0.0642	0.0459	0.0329	0.0169	0.0088	0.0046
75	0.2265	0.1569	0.1089	0.0758	0.0528	0.0368	0.0258	0.0127	0.0063	0.0031
80	0.2051	0.1387	0.0940	0.0638	0.0434	0.0296	0.0202	0.0095	0.0045	0.0021
85	0.1858	0.1226	0.0811	0.0537	0.0357	0.0237	0.0158	0.0071	0.0032	0.0014
90	0.1683	0.1084	0.0699	0.0452	0.0293	0.0190	0.0124	0.0053	0.0023	0.0010
95	0.1524	0.0958	0.0603	0.0381	0.0241	0.0153	0.0097	0.0039	0.0016	0.0007
100	0.1380	0.0846	0.0520	0.0321	0.0198	0.0123	0.0076	0.0029	0.0012	0.0005

TABLE B-3.—VALUE OF $\frac{(1+i)^n - 1}{i}$

$$S = D\left[\frac{(1+i)^n - 1}{i}\right]$$

n	2%	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	2.020	2.025	2.030	2.035	2.040	2.045	2.050	2.060	2.070	2.080
3	3.060	3.076	3.091	3.106	3.122	3.137	3.153	3.184	3.215	3.246
4	4.122	4.153	4.184	4.215	4.246	4.278	4.310	4.375	4.440	4.506
5	5.204	5.256	5.309	5.362	5.416	5.471	5.526	5.637	5.751	5.867
6	6.308	6.388	6.468	6.550	6.633	6.717	6.802	6.975	7.153	7.336
7	7.434	7.547	7.662	7.779	7.898	8.019	8.142	8.394	8.654	8.923
8	8.583	8.736	8.892	9.051	9.214	9.380	9.549	9.897	10.26	10.64
9	9.755	9.955	10.16	10.37	10.58	10.80	11.03	11.49	11.98	12.49
10	10.95	11.20	11.46	11.73	12.01	12.29	12.58	13.18	13.82	14.49
11	12.17	12.48	12.81	13.14	13.49	13.84	14.21	14.97	15.78	16.65
12	13.41	13.80	14.19	14.60	15.03	15.46	15.92	16.87	17.89	18.98
13	14.68	15.14	15.62	16.11	16.63	17.16	17.71	18.88	20.14	21.50
14	15.97	16.52	17.09	17.68	18.29	18.93	19.60	21.02	22.55	24.21
15	17.29	17.93	18.60	19.30	20.02	20.78	21.58	23.28	25.13	27.15
16	18.64	19.38	20.16	20.97	21.82	22.72	23.66	25.67	27.89	30.32
17	20.01	20.86	21.76	22.71	23.70	24.74	25.84	28.21	30.84	33.75
18	21.41	22.39	23.41	24.50	25.65	26.86	28.13	30.91	34.00	37.45
19	22.84	23.95	25.12	26.36	27.67	29.06	30.54	33.76	37.38	41.45
20	24.30	25.54	26.87	28.28	29.78	31.37	33.07	36.79	41.00	45.76
21	25.78	27.18	28.68	30.27	31.97	33.78	35.72	39.99	44.87	50.42
22	27.30	28.86	30.54	32.33	34.25	36.30	38.51	43.39	49.01	55.46
23	28.84	30.58	32.45	34.46	36.62	38.94	41.43	47.00	53.44	60.89
24	30.42	32.35	34.43	36.67	39.08	41.69	44.50	50.82	58.18	66.76
25	32.03	34.16	36.46	38.95	41.65	44.57	47.73	54.86	63.25	73.11
26	33.67	36.01	38.55	41.31	44.31	47.57	51.11	59.16	68.68	79.95
27	35.34	37.91	40.71	43.76	47.08	50.71	54.67	63.71	74.48	87.35
28	37.05	39.86	42.93	46.29	49.97	53.99	58.40	68.53	80.70	95.34
29	38.79	41.86	45.22	48.91	52.97	57.42	62.32	73.64	87.35	104.0
30	40.57	43.90	47.58	51.62	56.08	61.01	66.44	79.06	94.46	113.3
31	42.38	46.00	50.00	54.43	59.33	64.75	70.76	84.80	102.1	123.3
32	44.23	48.15	52.50	57.33	62.70	68.67	75.30	90.89	110.2	134.2
33	46.11	50.35	55.08	60.34	66.21	72.76	80.06	97.34	118.9	146.0
34	48.03	52.61	57.73	63.45	69.86	77.03	85.07	104.2	128.3	158.6
35	49.99	54.93	60.46	66.67	73.65	81.50	90.32	111.4	138.2	172.3
40	60.40	67.40	75.40	84.55	95.03	107.0	120.8	154.8	199.6	259.1
45	71.89	81.52	92.72	105.8	121.0	138.8	159.7	212.7	285.7	386.5
50	84.58	97.48	112.8	131.0	152.7	178.5	209.3	290.3	406.5	573.8
55	98.59	115.6	136.1	160.9	191.2	227.9	272.7	394.2	575.9	848.9
60	114.1	136.0	163.1	196.5	238.0	289.5	353.6	533.1	813.5	1,253.
65	131.1	159.1	194.3	238.8	295.0	366.2	456.8	719.1	1,147.	1,847.
70	150.0	185.3	230.6	288.9	364.3	461.9	588.5	967.9	1,614.	2,720.
75	170.8	214.9	272.6	348.5	448.6	581.0	756.7	1,301.	2,270.	4,003.
80	193.8	248.4	321.4	419.3	551.2	729.6	971.2	1,747.	3,189.	5,887.
85	219.1	286.3	377.9	503.4	676.1	914.6	1,245.	2,343.	4,479.	8,656.
90	247.2	329.2	443.3	603.2	828.0	1,145.	1,595.	3,141.	6,287.	12,724.
95	278.1	377.7	519.3	721.8	1,013.	1,433.	2,041.	4,209.	8,824.	18,702.
100	312.2	432.5	607.3	862.6	1,238.	1,791.	2,610.	5,638.	12,382.	27,485.

TABLE B-4.—VALUES OF $\dfrac{i}{(1+i)^n - 1}$

$$D = S\left[\frac{i}{(1+i)^n - 1}\right]$$

n	2%	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.4951	0.4938	0.4926	0.4914	0.4902	0.4890	0.4878	0.4854	0.4831	0.4808
3	0.3268	0.3251	0.3235	0.3219	0.3204	0.3188	0.3172	0.3141	0.3111	0.3080
4	0.2426	0.2408	0.2390	0.2373	0.2355	0.2337	0.2320	0.2286	0.2252	0.2219
5	0.1922	0.1903	0.1884	0.1865	0.1846	0.1828	0.1810	0.1774	0.1739	0.1705
6	0.1585	0.1566	0.1546	0.1527	0.1508	0.1489	0.1470	0.1434	0.1398	0.1363
7	0.1345	0.1325	0.1305	0.1285	0.1266	0.1247	0.1228	0.1191	0.1156	0.1121
8	0.1165	0.1145	0.1125	0.1105	0.1085	0.1066	0.1047	0.1010	0.0975	0.0940
9	0.1025	0.1005	0.0984	0.0965	0.0945	0.0926	0.0907	0.0870	0.0835	0.0801
10	0.0913	0.0893	0.0872	0.0852	0.0833	0.0814	0.0795	0.0759	0.0724	0.0690
11	0.0822	0.0801	0.0781	0.0761	0.0742	0.0723	0.0704	0.0668	0.0634	0.0601
12	0.0746	0.0725	0.0705	0.0685	0.0666	0.0647	0.0628	0.0593	0.0559	0.0527
13	0.0681	0.0660	0.0640	0.0621	0.0601	0.0583	0.0565	0.0530	0.0497	0.0465
14	0.0626	0.0605	0.0585	0.0566	0.0547	0.0528	0.0510	0.0476	0.0443	0.0413
15	0.0578	0.0558	0.0538	0.0518	0.0499	0.0481	0.0463	0.0430	0.0398	0.0368
16	0.0537	0.0516	0.0496	0.0477	0.0458	0.0440	0.0423	0.0390	0.0359	0.0330
17	0.0500	0.0479	0.0460	0.0440	0.0422	0.0404	0.0387	0.0354	0.0324	0.0296
18	0.0467	0.0447	0.0427	0.0408	0.0390	0.0372	0.0356	0.0324	0.0294	0.0267
19	0.0438	0.0418	0.0398	0.0379	0.0361	0.0344	0.0328	0.0296	0.0268	0.0241
20	0.0412	0.0391	0.0372	0.0354	0.0336	0.0319	0.0302	0.0272	0.0244	0.0219
21	0.0388	0.0368	0.0349	0.0330	0.0313	0.0296	0.0280	0.0250	0.0223	0.0198
22	0.0366	0.0347	0.0328	0.0309	0.0292	0.0276	0.0260	0.0231	0.0204	0.0180
23	0.0347	0.0327	0.0308	0.0290	0.0273	0.0257	0.0241	0.0213	0.0187	0.0164
24	0.0329	0.0309	0.0291	0.0273	0.0256	0.0240	0.0225	0.0197	0.0172	0.0150
25	0.0312	0.0293	0.0274	0.0257	0.0240	0.0224	0.0210	0.0182	0.0158	0.0137
26	0.0297	0.0278	0.0259	0.0242	0.0226	0.0210	0.0196	0.0169	0.0146	0.0125
27	0.0283	0.0264	0.0246	0.0229	0.0212	0.0197	0.0183	0.0157	0.0134	0.0115
28	0.0270	0.0251	0.0233	0.0216	0.0200	0.0185	0.0171	0.0146	0.0124	0.0105
29	0.0258	0.0239	0.0221	0.0205	0.0189	0.0174	0.0160	0.0136	0.0115	0.0096
30	0.0247	0.0228	0.0210	0.0194	0.0178	0.0164	0.0151	0.0127	0.0106	0.0088
31	0.0236	0.0217	0.0200	0.0184	0.0169	0.0154	0.0141	0.0118	0.0098	0.0081
32	0.0226	0.0208	0.0190	0.0174	0.0160	0.0146	0.0133	0.0110	0.0091	0.0075
33	0.0217	0.0199	0.0182	0.0166	0.0151	0.0137	0.0125	0.0103	0.0084	0.0069
34	0.0208	0.0190	0.0173	0.0158	0.0143	0.0130	0.0118	0.0096	0.0078	0.0063
35	0.0200	0.0182	0.0165	0.0150	0.0136	0.0123	0.0111	0.0090	0.0072	0.0058
40	0.0166	0.0148	0.0133	0.0118	0.0105	0.0093	0.0083	0.0065	0.0050	0.0039
45	0.0139	0.0123	0.0108	0.0095	0.0083	0.0072	0.0063	0.0047	0.0035	0.0026
50	0.0118	0.0103	0.0089	0.0076	0.0066	0.0056	0.0048	0.0034	0.0025	0.0017
55	0.0101	0.0087	0.0073	0.0062	0.0052	0.0044	0.0037	0.0025	0.0017	0.0012
60	0.0088	0.0074	0.0061	0.0051	0.0042	0.0035	0.0028	0.0019	0.0012	0.0008
65	0.0076	0.0063	0.0051	0.0042	0.0034	0.0027	0.0022	0.0014	0.0009	0.0005
70	0.0067	0.0054	0.0043	0.0035	0.0028	0.0022	0.0017	0.0010	0.0006	0.0004
75	0.0059	0.0047	0.0037	0.0029	0.0022	0.0017	0.0013	0.0008	0.0004	0.0003
80	0.0052	0.0040	0.0031	0.0024	0.0018	0.0014	0.0010	0.0006	0.0003	0.0002
85	0.0046	0.0035	0.0026	0.0020	0.0015	0.0011	0.0008	0.0004	0.0002	0.0001
90	0.0041	0.0030	0.0023	0.0017	0.0012	0.0009	0.0006	0.0003	0.0002	0.0001
95	0.0036	0.0027	0.0019	0.0014	0.0010	0.0007	0.0005	0.0002	0.0001	0.0000
100	0.0032	0.0023	0.0017	0.0012	0.0008	0.0006	0.0004	0.0002	0.0001	0.0000

TABLE B-5.—VALUES OF $\frac{(1+i)^n-1}{i(1+i)^n}$

$$R = D\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$$

n	2%	$2\frac{1}{2}$%	3%	$3\frac{1}{2}$%	4%	$4\frac{1}{2}$%	5%	6%	7%	8%
1	0.980	0.975	0.970	0.966	0.961	0.957	0.952	0.943	0.935	0.926
2	1.942	1.927	1.913	1.900	1.886	1.873	1.859	1.833	1.808	1.783
3	2.884	2.856	2.829	2.802	2.775	2.749	2.723	2.673	2.624	2.577
4	3.808	3.762	3.717	3.673	3.630	3.588	3.546	3.465	3.387	3.312
5	4.713	4.646	4.580	4.515	4.452	4.390	4.329	4.212	4.100	3.993
6	5.601	5.508	5.417	5.329	5.242	5.158	5.076	4.917	4.767	4.623
7	6.472	6.349	6.230	6.115	6.002	5.893	5.786	5.582	5.389	5.206
8	7.325	7.170	7.020	6.874	6.733	6.596	6.463	6.210	5.971	5.747
9	8.162	7.971	7.786	7.608	7.435	7.269	7.108	6.802	6.515	6.247
10	8.983	8.752	8.530	8.317	8.111	7.913	7.722	7.360	7.024	6.710
11	9.787	9.514	9.253	9.002	8.760	8.529	8.306	7.887	7.499	7.139
12	10.58	10.26	9.954	9.663	9.385	9.119	8.863	8.384	7.943	7.536
13	11.35	10.98	10.63	10.30	9.986	9.683	9.394	8.853	8.358	7.904
14	12.11	11.69	11.30	10.92	10.56	10.22	9.899	9.295	8.745	8.244
15	12.85	12.38	11.94	11.52	11.12	10.74	10.38	9.712	9.108	8.559
16	13.58	13.06	12.56	12.09	11.65	11.23	10.84	10.11	9.447	8.851
17	14.29	13.71	13.17	12.65	12.17	11.71	11.27	10.48	9.763	9.122
18	14.99	14.35	13.75	13.19	12.66	12.16	11.69	10.83	10.06	9.372
19	15.68	14.98	14.32	13.71	13.13	12.59	12.09	11.16	10.34	9.604
20	16.35	15.59	14.88	14.21	13.59	13.01	12.46	11.47	10.59	9.818
21	17.01	16.18	15.42	14.70	14.03	13.40	12.82	11.76	10.84	10.02
22	17.66	16.77	15.94	15.17	14.45	13.78	13.16	12.04	11.06	10.20
23	18.29	17.33	16.44	15.62	14.86	14.15	13.49	12.30	11.27	10.37
24	18.91	17.88	16.94	16.06	15.25	14.50	13.80	12.55	11.47	10.53
25	19.52	18.42	17.41	16.48	15.62	14.83	14.09	12.78	11.65	10.67
26	20.12	18.95	17.88	16.89	15.98	15.15	14.38	13.00	11.83	10.81
27	20.71	19.46	18.33	17.29	16.33	15.45	14.64	13.21	11.99	10.94
28	21.28	19.96	18.76	17.67	16.66	15.74	14.90	13.41	12.14	11.05
29	21.84	20.45	19.19	18.04	16.98	16.02	15.14	13.59	12.28	11.16
30	22.40	20.93	19.60	18.39	17.29	16.29	15.37	13.76	12.41	11.26
31	22.94	21.40	20.00	18.74	17.59	16.54	15.59	13.93	12.53	11.35
32	23.47	21.85	20.39	19.07	17.87	16.79	15.80	14.08	12.65	11.43
33	23.99	22.29	20.77	19.39	18.15	17.02	16.00	14.23	12.75	11.51
34	24.50	22.72	21.13	19.70	18.41	17.25	16.19	14.37	12.85	11.59
35	25.00	23.15	21.49	20.00	18.66	17.46	16.37	14.50	12.95	11.65
40	27.36	25.10	23.11	21.36	19.79	18.40	17.16	15.05	13.33	11.92
45	29.49	26.83	24.52	22.50	20.72	19.16	17.77	15.46	13.61	12.11
50	31.42	28.36	25.73	23.46	21.48	19.76	18.26	15.76	13.80	12.23
55	33.17	29.71	26.77	24.36	22.11	20.25	18.63	15.99	13.94	12.32
60	34.76	30.91	27.68	24.94	22.62	20.64	18.93	16.16	14.04	12.38
65	36.20	31.96	28.45	25.52	23.05	20.95	19.16	16.29	14.11	12.42
70	37.50	32.90	29.12	26.00	22.39	21.20	19.34	16.38	14.16	12.44
75	38.68	33.72	29.70	26.41	23.68	21.40	19.48	16.46	14.20	12.46
80	39.74	34.45	30.20	26.75	23.92	21.57	19.60	16.51	14.22	12.47
85	40.71	35.10	30.63	27.04	24.11	21.70	19.68	16.55	14.24	12.48
90	41.59	35.67	31.00	27.28	24.27	21.80	19.75	16.58	14.25	12.49
95	42.38	36.17	31.32	27.48	24.40	21.88	19.81	16.60	14.26	12.49
100	43.10	36.61	31.60	27.66	24.50	21.95	19.85	16.62	14.27	12.49

TABLE B-6.—VALUES OF $\frac{i(1+i)^n}{(1+i)^n-1}$

$$D = R\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

n	2%	2½%	3%	3½%	4%	4½%	5%	6%	7%	8%
1	1.0200	1.0250	1.0300	1.0350	1.0400	1.0450	1.0500	1.0600	1.0700	1.0800
2	0.5150	0.5188	0.5226	0.5264	0.5302	0.5340	0.5378	0.5454	0.5531	0.5608
3	0.3468	0.3501	0.3535	0.3569	0.3603	0.3638	0.3672	0.3741	0.3811	0.3880
4	0.2626	0.2658	0.2690	0.2723	0.2755	0.2787	0.2820	0.2886	0.2952	0.3019
5	0.2122	0.2152	0.2184	0.2215	0.2246	0.2278	0.2310	0.2374	0.2439	0.2505
6	0.1785	0.1815	0.1846	0.1877	0.1908	0.1939	0.1970	0.2034	0.2098	0.2163
7	0.1545	0.1575	0.1605	0.1635	0.1666	0.1697	0.1728	0.1791	0.1856	0.1921
8	0.1365	0.1395	0.1425	0.1455	0.1485	0.1516	0.1547	0.1610	0.1675	0.1740
9	0.1225	0.1255	0.1284	0.1314	0.1345	0.1376	0.1407	0.1470	0.1535	0.1601
10	0.1113	0.1143	0.1172	0.1202	0.1233	0.1264	0.1295	0.1359	0.1424	0.1490
11	0.1022	0.1051	0.1081	0.1111	0.1141	0.1172	0.1204	0.1268	0.1334	0.1401
12	0.0946	0.0975	0.1005	0.1035	0.1066	0.1097	0.1128	0.1193	0.1259	0.1327
13	0.0881	0.0910	0.0940	0.0971	0.1001	0.1033	0.1065	0.1130	0.1197	0.1265
14	0.0826	0.0855	0.0885	0.0916	0.0947	0.0978	0.1010	0.1076	0.1143	0.1213
15	0.0778	0.0808	0.0838	0.0868	0.0899	0.0931	0.0963	0.1030	0.1098	0.1168
16	0.0737	0.0766	0.0796	0.0827	0.0858	0.0890	0.0923	0.0990	0.1059	0.1130
17	0.0700	0.0729	0.0760	0.0790	0.0822	0.0854	0.0887	0.0954	0.1024	0.1096
18	0.0667	0.0697	0.0727	0.0758	0.0790	0.0822	0.0855	0.0924	0.0994	0.1067
19	0.0638	0.0668	0.0698	0.0729	0.0761	0.0794	0.0827	0.0896	0.0968	0.1041
20	0.0612	0.0641	0.0672	0.0704	0.0736	0.0769	0.0802	0.0872	0.0944	0.1019
21	0.0588	0.0618	0.0649	0.0680	0.0713	0.0746	0.0780	0.0850	0.0923	0.0998
22	0.0566	0.0596	0.0627	0.0659	0.0692	0.0725	0.0760	0.0830	0.0904	0.0980
23	0.0547	0.0577	0.0608	0.0640	0.0673	0.0707	0.0741	0.0813	0.0887	0.0964
24	0.0529	0.0559	0.0590	0.0623	0.0656	0.0690	0.0725	0.0797	0.0872	0.0950
25	0.0512	0.0543	0.0574	0.0607	0.0640	0.0674	0.0710	0.0782	0.0858	0.0937
26	0.0497	0.0528	0.0559	0.0592	0.0626	0.0660	0.0696	0.0769	0.0846	0.0925
27	0.0483	0.0514	0.0546	0.0579	0.0612	0.0647	0.0683	0.0757	0.0834	0.0914
28	0.0470	0.0501	0.0533	0.0566	0.0600	0.0635	0.0671	0.0746	0.0824	0.0905
29	0.0458	0.0489	0.0521	0.0554	0.0589	0.0624	0.0660	0.0736	0.0814	0.0896
30	0.0446	0.0478	0.0510	0.0544	0.0578	0.0614	0.0651	0.0726	0.0806	0.0888
31	0.0436	0.0467	0.0500	0.0534	0.0569	0.0604	0.0641	0.0718	0.0798	0.0881
32	0.0426	0.0458	0.0490	0.0524	0.0559	0.0596	0.0633	0.0710	0.0791	0.0875
33	0.0417	0.0449	0.0482	0.0516	0.0551	0.0587	0.0625	0.0703	0.0784	0.0869
34	0.0408	0.0440	0.0473	0.0508	0.0543	0.0580	0.0618	0.0696	0.0777	0.0863
35	0.0400	0.0432	0.0465	0.0500	0.0536	0.0573	0.0611	0.0690	0.0772	0.0858
40	0.0366	0.0398	0.0433	0.0468	0.0505	0.0543	0.0583	0.0665	0.0750	0.0839
45	0.0339	0.0373	0.0408	0.0445	0.0483	0.0522	0.0563	0.0647	0.0735	0.0826
50	0.0318	0.0353	0.0389	0.0426	0.0466	0.0506	0.0548	0.0634	0.0725	0.0817
55	0.0301	0.0337	0.0373	0.0412	0.0452	0.0494	0.0537	0.0625	0.0717	0.0812
60	0.0288	0.0324	0.0361	0.0401	0.0442	0.0485	0.0528	0.0619	0.0712	0.0808
65	0.0276	0.0313	0.0351	0.0392	0.0434	0.0477	0.0522	0.0614	0.0709	0.0805
70	0.0276	0.0304	0.0343	0.0385	0.0427	0.0472	0.0517	0.0610	0.0706	0.0804
75	0.0259	0.0297	0.0337	0.0379	0.0422	0.0467	0.0513	0.0608	0.0704	0.0802
80	0.0252	0.0290	0.0331	0.0374	0.0418	0.0464	0.0510	0.0606	0.0703	0.0802
85	0.0246	0.0285	0.0326	0.0370	0.0415	0.0461	0.0508	0.0604	0.0702	0.0801
90	0.0240	0.0280	0.0323	0.0367	0.0412	0.0459	0.0506	0.0603	0.0702	0.0801
95	0.0236	0.0276	0.0319	0.0364	0.0410	0.0457	0.0505	0.0602	0.0701	0.0800
100	0.0232	0.0273	0.0316	0.0362	0.0408	0.0456	0.0504	0.0602	0.0701	0.0800

APPENDIX C

SELECTED BIBLIOGRAPHY

AGG, T. R.: "Construction of Roads and Pavements," McGraw-Hill Book Company, Inc., New York, 1940.

ALFORD, L. P.: "Cost and Production Handbook," The Ronald Press Company, New York, 1934.

ANDERSON, J. W.: "Diesel Engines," McGraw-Hill Book Company, Inc., New York, 1935.

BABBITT, H. E.: "Sewerage and Sewage Treatment," John Wiley & Sons, Inc., New York, 1922.

BABBITT, H. E., and J. J. DOLAND: "Water Supply Engineering," McGraw-Hill Book Company, Inc., New York, 1939.

BALDERSTON, C. C., V. S. KARABASZ, and R. P. BRECHT: "Management of an Enterprise," Prentice-Hall, Inc., New York, 1935.

BARROWS, H. K.: "Water Power Engineering," McGraw-Hill Book Company, Inc., 1927.

BATEMAN, J. H.: "Highway Engineering," John Wiley & Sons, Inc., New York, 1928.

BLANCHARD, A. H., and R. L. MORRISON: "Elements of Highway Engineering," John Wiley & Sons, Inc., New York, 1928.

BONBRIGHT, J. C.: "Valuation of Property," McGraw-Hill Book Company, Inc., New York, 1937.

BROWN, H. W.: "Electrical Equipment," McGraw-Hill Book Company, Inc., New York, 1931.

CLARK, J. M.: "Studies in the Economics of Overhead Costs," University of Chicago Press, Chicago, 1923.

COOMBS, R. D.: "Pole and Tower Lines," McGraw-Hill Book Company, Inc., New York, 1916.

DAUGHERTY, R. L.: "Centrifugal Pumps," McGraw-Hill Book Company, Inc., New York, 1915.

DAVIDSON, J. B.: "Agricultural Machinery," John Wiley & Sons, Inc., New York, 1913.

DEGLER, H. E.: "Internal Combustion Engines," John Wiley & Sons, Inc., New York, 1939.

EIDMANN, F. L.: "Economic Control of Engineering and Manufacturing," McGraw-Hill Book Company, Inc., New York, 1931.

EIGELBERNER, J.: "The Investigation of Business Problems," McGraw-Hill Book Company, Inc., New York, 1926.

FERNALD, R. H., and G. A. ORROK: "Engineering of Power Plants," McGraw-Hill Book Company, Inc., New York, 1937.

Fish, J. C. L.: "Engineering Economics," McGraw-Hill Book Company, Inc., New York, 1923.

Fox, G.: "Electric Drive Practice," McGraw-Hill Book Company, Inc., New York, 1928.

Gaffert, G. A.: "Steam Power Stations," McGraw-Hill Book Company, Inc., New York, 1937.

Gebhardt, G. E.: "Steam Power Plant Engineering," John Wiley & Sons, Inc., New York, 1928.

Goldman, O. B.: "Financial Engineering," John Wiley & Sons, Inc., New York, 1923.

Grant, E. L.: "Principles of Engineering Economy," The Ronald Press Company, New York, 1938.

Grant, H. D.: "Simplified Mathematics for Accountants and Executives," McGraw-Hill Book Company, Inc., New York, 1934.

Greene, A. M.: "Heating, Ventilating and Air Conditioning," John Wiley & Sons, Inc., New York, 1915.

Harding, C. F., and D. T. Canfield: "Business Administration for Engineers," McGraw-Hill Book Company, Inc., New York, 1937.

Harrison, J. L.: "Economics of Construction Management," Gillette Publishing Company, Chicago, 1932.

Higbie, H. H.: "Lighting Calculations," John Wiley & Sons, Inc., New York, 1934.

Hool, G. A.: "Reinforced Concrete Construction," McGraw-Hill Book Company, Inc., New York, 1927.

Israelson, O. W.: "Irrigation Principles and Practice," John Wiley & Sons, Inc., New York, 1932.

Johannesson, S.: "Highway Economics," McGraw-Hill Book Company, Inc., New York, 1931.

Justine, J. D. W., and W. G. Mervine: "Power Supply Economics," John Wiley & Sons, Inc., New York, 1934.

Knoppel, C. E.: "Managing for Profit," McGraw-Hill Book Company, Inc., New York, 1937.

Koshkin, S. J.: "Modern Materials Handling," John Wiley & Sons, Inc., New York, 1932.

Lester, B.: "Marketing Industrial Equipment," McGraw-Hill Book Company, Inc., New York, 1935.

Lloyd, T. C.: "Electrical Equipment," John Wiley & Sons, Inc., New York, 1930.

Loew, E. A.: "Electrical Power Transmission," McGraw-Hill Book Company, Inc., New York 1928.

Lovell, A. H.: "Generating Stations," McGraw-Hill Book Company, Inc., New York, 1935.

MacIntire, H. J.: "Refrigeration Engineering," John Wiley & Sons, Inc., New York, 1940.

Mackenzie, D. H.: "Mathematics of Finance," McGraw-Hill Book Company, Inc., New York, 1938.

Marston, A., and T. R. Agg: "Engineering Valuation," McGraw-Hill Book Company, Inc., New York, 1936.

McCullough, C. B.: "Economics of Highway Bridge Types," Gillette Publishing Company, Chicago, 1929.

Morrow, L. W. W.: "Electric Power Stations," McGraw-Hill Book Company, Inc., New York, 1927.

Moyer, J. A.: "Oil Fuels and Burners," McGraw-Hill Book Company, Inc., New York, 1937.

Moyer, J. A., and R. U. Fittz: "Air Conditioning," McGraw-Hill Book Company, Inc., New York, 1933.

Moyer, J. A., and J. F. Wostrel: "Industrial Electricity and Wiring," McGraw-Hill Book Company, Inc., New York, 1930.

Nash, L. R.: "Economics of Public Utilities," McGraw-Hill Book Company, Inc., New York, 1925.

Peele, R.: "Compressed Air Plant," John Wiley & Sons, Inc., New York, 1930.

Pickels, G. W.: "Drainage and Flood Control Engineering," McGraw-Hill Book Company, Inc., New York, 1941.

Powers, W. L., and T. A. Teeters: "Land Drainage," John Wiley & Sons, Inc., New York, 1932.

Rautenstrauch, W.: "The Design of Manufacturing Enterprise," Pitman Publishing Corporation, New York, 1941.

———: "The Successful Control of Profits," B. C. Forbes Publishing Company, New York, 1930.

Raymond, F. E.: "Quantity and Economy in Manufacture," McGraw-Hill Book Company, Inc., New York, 1931.

Reyneau, P. O., and H. P. Seelye: "Economics of Electrical Distribution," McGraw-Hill Book Company, Inc., New York, 1922.

Rogers, P. L.: "Power Factor Economics," John Wiley & Sons, Inc., New York, 1939.

Seelye, H. P.: "Electrical Distribution Engineering," McGraw-Hill Book Company, Inc., New York, 1930.

Staley, W. W.: "Mine Plant Design," McGraw-Hill Book Company, Inc., New York, 1936.

Stansel, N. R.: "Industrial Electric Heating," John Wiley & Sons, Inc., New York, 1933.

Steel, E. W.: "Water Supply and Sewerage," McGraw-Hill Book Company, Inc., New York, 1938.

Still, A.: "Elements of Electrical Design," McGraw-Hill Book Company, Inc., New York, 1924.

Stubbs, F. W.: "Estimates of Costs of Construction," John Wiley & Sons Inc., New York, 1938.

Tarboux, J. G.: "Electric Power Equipment," McGraw-Hill Book Company, Inc., New York, 1932.

Tournier, E. J.: "Materials Handling Equipment," McGraw-Hill Book Company, Inc., New York, 1929.

Tyler, C.: "Chemical Engineering Economics," McGraw-Hill Book Company, Inc., New York, 1926.

Underwood, G.: "Estimating Construction Costs," McGraw-Hill Book Company, Inc., New York, 1930.

VILBRANDT, F. C.: "Chemical Engineering Plant Design," McGraw-Hill Book Company, Inc., New York, 1942.

VOSKUIL, W. H.: "Economics of Water Power Development," McGraw-Hill Book Company, Inc., New York, 1928.

WADDELL, J. A. L.: "Economics of Bridge Works," John Wiley & Sons, Inc., New York, 1921.

WEBB, W. L.: "Economics of Railroad Construction," John Wiley & Sons, Inc., New York, 1915.

WELLINGTON, A. M.: "The Economic Theory of Railway Location," John Wiley & Sons, Inc., New York, 1887.

WILCOX, E. A.: "Electric Heating," McGraw-Hill Book Company, Inc., New York, 1928.

WILLIAMS, C. C.: "Design of Railway Location," John Wiley & Sons, Inc., New York, 1917.

WOODRUFF, E. B., and H. B. LAMMERS: "Steam Plant Operation," McGraw-Hill Book Company, Inc., New York, 1935.

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